ELECTRICAL MEASURING INSTRUMENTS

PART ONE

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Electrical Measuring Instruments

PART ONE

General Principles and Electrical Indicating Instruments

SECOND EDITION

revised by

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PREFACE TO THE SECOND EDITION

.E first edition of this book was published in 1924 and has since
in considerable demand by all concerned in the design, manu-
ure and use of electrical instruments. In the years which have
ɔs ;d considerable advances have been made, mainly in the
duction of new materials, which have involved changes in the
ns, but not necessarily the principles of operation of the
ıments in common use. The time has then arrived when a
;d edition is desirable, in order that the original information
d be combined with the latest methods of construction.
ortunately the original authors were unable to undertake this
sk and so it has been undertaken by the revising author by
ient with Dr. Drysdale. Consequently full responsibility
.new edition must be accepted by the revising author.

endeavour has been made to retain as much of the original
rmation as possible and to include, as far as space will allow,
s of the latest forms of instruments, together with any advances
in the theory of design and performance. The revising author
reatly indebted to his various friends in the instrument industry
.he help which they have given, and also to various firms in the
rument industry, who have provided information and samples
their instruments. Among these firms are: Elliott Bros. Ltd.,
ett, Edgecumbe & Co. Ltd., Evershed & Vignoles Ltd., Ernest
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.td., Ferranti Ltd., English Electric Co. Ltd., Salford Electrical
uments Ltd., Sangamo-Weston Ltd., Westinghouse Electric
oration, Nalder Bros & Thompson Ltd., Record Electric Co.
, Crompton & Co. Ltd., M.I. (Pullen) Ltd., Cambridge Instru-
t Co. Ltd., Automatic Coil Winder Ltd., Westinghouse Brake
Taylor Electrical Instrument Co. In particular thanks are

due to the officials of the Science Museum for kindly providing photographs of instruments in the collection at the Museum and for permission to reproduce them.

One branch of measurement which has grown extensively is that involving the use of electronic techniques. Since, however, this book deals only with fundamental instruments, it is felt that inclusion of these would be out of place and consequently only slight reference is made to them.

Since this text was revised there has been a change from the International Units to the so-called Absolute Units. A note on this is given in an appendix.

10th January, 1952. G. F. T.

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CHAPTER 1

GENERAL ELECTRICAL PRINCIPLES

In measuring the magnitude of any phenomenon we make use of the effects produced by it. For the measurement of a mass, for example, we avail ourselves of the attraction of gravity for it—that is, we determine its weight ; or we might utilize the force required to give it a certain acceleration. Again, for the measurement of temperature we make use of the expansion of solids, liquids, or gases, the change in electrical resistance of a wire, or the thermo-electromotive force produced at the junction of two conductors, etc. Indicating instruments in which the amount of the quantity to be measured is directly shown by the position of a pointer on a graduated scale are most convenient for the majority of everyday measurements. Considerable use is also made of instruments which trace a continuous record of the quantity to be measured on a continuously moving chart.

For the production of such instruments some effect is employed which enables the phenomenon to be measured to produce a mechanical force or torque tending to move the index or pointer along its scale ; this is resisted by a controlling force or torque which tends to move the index in the opposite direction, towards some zero position. The actual displacement of the index, or deflection, is the resultant of these two forces, and is greater the magnitude of the deflecting force or of the phenomenon to be measured.

Effects of the Electric Current

When an electric current traverses a circuit it produces certain effects, any of which may be made use of in connection with its measurement. Taking these in convenient sequence, rather than in the chronological order of their discovery, they may be enumerated under three headings :—

(*a*) Magnetic.
(*b*) Thermal.
(*c*) Chemical.

(a) *Electro-magnetic Effects and Instruments.*—The magnetic effects of the electric current are the most important, and give rise to the

great majority of electrical instruments. Firstly, we have the discovery of Oersted in 1819, that when a current is passed along a wire held parallel to a magnetic needle, the needle turns aside, or tends to set itself at right-angles to the wire, due to the formation of a circular magnetic field round the conductor. The controlling force in this case is provided by the earth's magnetic field (Fig. 1.1).

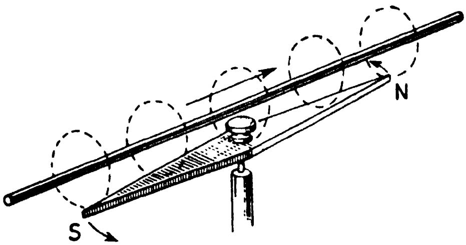


FIG. 1.1.—Deflection of a magnetic needle by a current-carrying conductor.

This phenomenon was utilized by Schweigger to produce his "multiplier," or galvanoscope, in which a magnetic needle was delicately pivoted within a rectangular coil. In 1825 Nobili greatly increased the sensitiveness of such instruments by employing an astatic system of two magnetic needles rigidly fixed one above the other with opposite poles pointing in the same direction. In Fig. 1.2 a pair of coils is arranged to deflect the magnetic system, so placed that the lower magnet swings within them while the upper magnet is just above their upper horizontal sides: the movement is indicated by a long, light pointer attached to the needles and moving over a horizontal circular scale. In this way the three chief means for increasing the sensitiveness of deflecting instruments were employed: (1) increase in the deflecting force by using two magnetic needles; (2) elimination of the controlling force (since the two needles assist each other as regards the magnetic field produced by the current, and oppose one another as regards the controlling force of the earth's field); and (3) increase in pointer length.

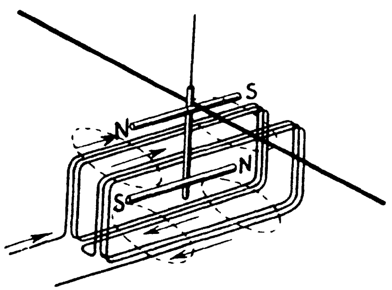


FIG. 1.2.—Nobili's astatic galvanometer.

Sir Wm. Thomson (afterwards Lord Kelvin) put practically the final touches to the sensitive moving-needle galvanometer in 1858 by the production of this four-coil astatic mirror instrument, which is shown diagrammatically in Fig. 1.3. By employment of the reflecting mirror a weightless pointer many feet in length can be obtained. This, and the instruments described above, were intended rather for the detection of very small currents than for the accurate measurement of large ones, but in 1837 Pouillet had devised the sine and tangent forms of galvanometer, afterwards developed by von Helmholtz into accurate measuring instruments. In 1879 Ayrton and Perry applied this principle in producing the first portable direct-reading ammeter (Fig. 1.4), the controlling field being increased by the addition of a large permanent magnet, so as to make the instrument nearly independent of external magnetic fields. About the same time Deprez also produced a galvanometer constructed on very similar principles.

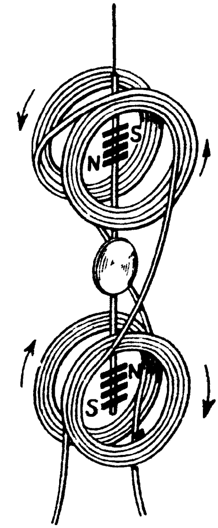


FIG 1 3—Principle of Kelvin's astatic galvanometer.

Since action and reaction are equal and opposite, it follows that the torque exerted by a current-carrying conductor on a magnetic needle must be accompanied by an equal and

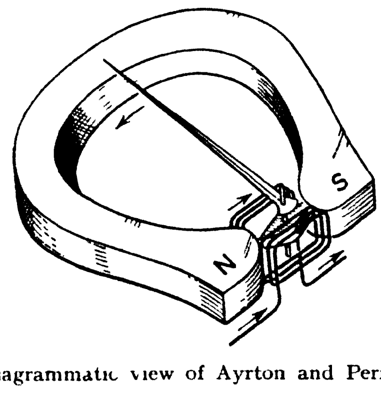


FIG 1 4—Diagrammatic view of Ayrton and Perry's ammeter.

opposite torque produced by the needle on the conductor, and this effect was independently discovered by Faraday in 1821, when he

found that a conductor carrying a current would continuously rotate around a magnet pole if free to do so (Fig. 1.5). This introduces us to the important class of moving coil permanent magnet instruments. In 1836 Sturgeon employed a coil suspended in the field of a permanent magnet as a galvanometer, and twenty years later Highton adopted it as a telegraphic receiving instrument. Sir Wm. Thomson, however, greatly improved this form of instrument in his Syphon Recorder in 1867, notably by introducing a cylindrical iron

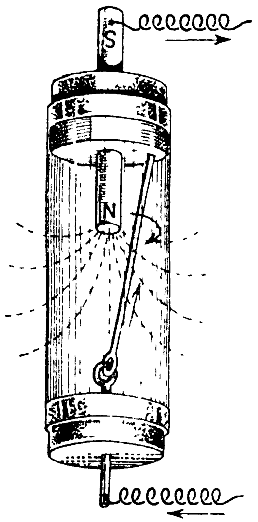


FIG. 1.5.—Experiment to show the rotation of a current-carrying conductor in a magnetic field.

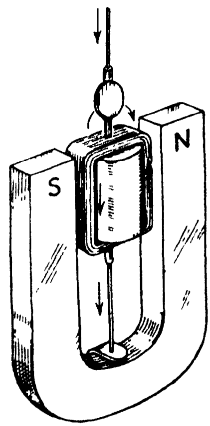


FIG. 1.6.—Diagram to illustrate the principle of the d'Arsonval galvanometer.

core inside, but clear of the coil, so as to increase the strength of the magnetic field in which the vertical sides moved.

In 1882 d'Arsonval revived the moving coil as a mirror galvanometer in a convenient and sensitive form (Fig. 1.6), and Weston, in 1888, produced the first direct-reading portable commercial instrument of this type (Fig. 1.7), the prototype of the most accurate ammeters and voltmeters for direct current measurement now extant.

By reducing the moving coil to a pair of tightly stretched strips moving in an intense magnetic field, Blondel, in 1891, produced an oscillograph for delineating the wave shape of an alternating

current, and Duddell about the same time constructed a similar instrument, while Einthoven in 1901 further simplified the principle by employing a single, fine stretched conductor in a powerful field as a sensitive galvanometer of very short period.

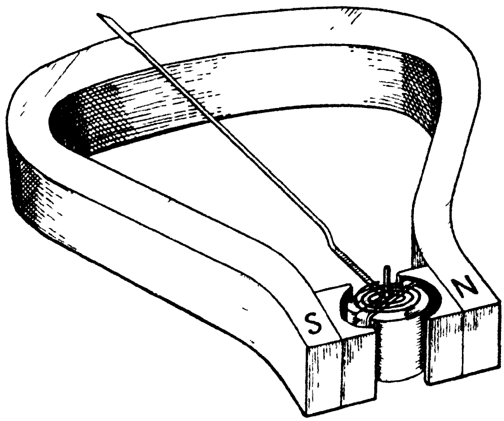


FIG. 1.7.—The Weston moving coil instrument.

The fact, discovered by Arago and Davy in 1820, that a current circulating in a coil of wire magnetizes iron rods placed near the coil, and tends to draw them into it, gives rise to a very simple class of commercial indicating instruments variously known as "electro-magnetic," "soft-iron," or "moving-iron" instruments.

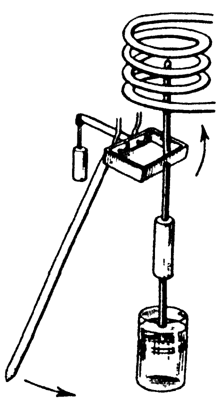


FIG. 1.8.—Principle of Kelvin's moving iron ammeter.

The simplest of these is the direct-pull type, of which Lord Kelvin's Ampere Gauge, 1885 (Fig. 1.8), is a good example. A small piece of soft iron suspended from a pivoted arm is sucked into a coil, and the tilting of the arm is indicated by a pointer moving over a scale. Variations of this were the Schuckert instruments (Fig. 1.9), in which the iron moved from the centre towards the inner edge of the coil, where the field is stronger, and the Nalder (Fig. 1.10) and other types of "repulsion" instruments, in which two pieces of iron lying side by side in the same coil are magnetized in the same manner and repel one another.

In 1820, immediately after the announcement of Oersted's discovery of the effect of a current upon a magnetic needle, Ampère described his brilliant researches upon the action of current-carrying conductors upon one another, the simplest of which was, that two currents flowing side by side attracted each other if in the same direction, and repelled each other if in opposite directions. This is the foundation of the important class of "dynamometer" instruments originated by Weber in 1843, and afterwards developed by

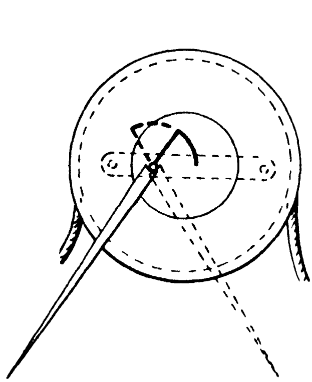


FIG. 1.9.—Principle of the Schuckert moving iron ammeter.

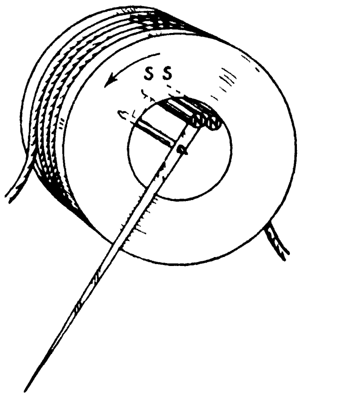


FIG. 1.10.—Repulsion type moving-iron ammeter

Siemens (Fig. 1.11) and by Joule and Lord Kelvin (1883) as standard current balances (Fig. 1.12). In 1881 Professors Ayrton and Perry constructed a dynamometer instrument with current and pressure coils as a wattmeter for measuring electrical power, and this was developed as a commercial instrument by Siemens in 1884.

As a matter of fact, one single principle underlies all electromagnetic instruments, i.e. that the current-carrying circuit tends to enclose as large a magnetic field as possible. In the moving-needle galvanometers the magnetic needle turns so that more of its lines of force pass through the coil, while in the moving-coil instrument the coil sets itself so as to enclose as much of the field of the magnet as possible. In the soft-iron instruments the iron moves so as to increase the magnetic flux produced by the coil, and in the dynamometer the moving coil turns so that its magnetic effect increases that of the fixed coil. If the conducting circuit is made entirely of flexible material in a uniform field it will become circular in order to enclose the maximum possible area, while, if the current flows

in a liquid conductor, such, for instance, as mercury contained in a long trough, it will actually try to reduce its section so as to shorten the path of the lines of force round it. To such an extent

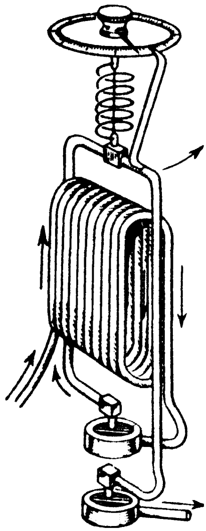


FIG. 1.11.—Diagram of Siemens' electro-dynamometer movement.

is this the case that it is difficult to pass a large current along such a conductor owing to the tendency of the mercury to contract and break the circuit (Fig. 1.13). This phenomenon, apparently first observed by P. Bary in 1900, and termed by Carl Hering the " pinch

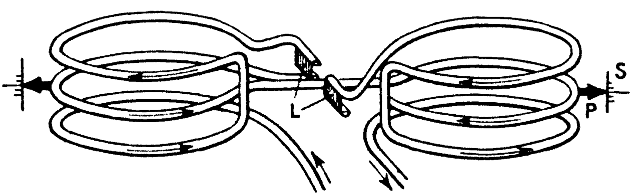


FIG. 1.12.—Principle of operation of the Kelvin current balance.

phenomenon," was utilized in 1907 by Dr. Northrup in his ingenious standard instrument for heavy currents.

(b) *Thermal Effects and Instruments.*—The thermal or " hot-wire " instruments depend upon the simple principle that the current heats the circuit through which it passes, the amount of heat developed being proportional to the square of the current strength and to the

resistance of the circuit, as found by Joule in 1843. In the great majority of hot-wire instruments the heating of the circuit is indicated by its expansion, a magnifying device being employed to obtain a large deflection of the pointer with the small expansion produced in the wire through which the current passes. The first practical instrument of this type was devised by Major Cardew in

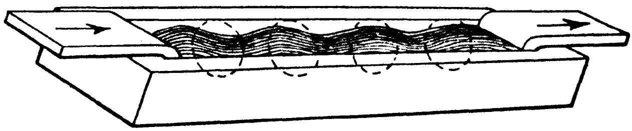


FIG. 1.13.—Sketch showing the " pinch " phenomenon in a trough of mercury.

1883 (Fig. 1.14), in which the current passed through a long, fine platinum-silver wire, whose expansion was communicated to the pointer through a pulley and spur gearing. An improvement in this device was employed by Hartmann and Braun and others, who utilized the change in the sag of a comparatively short wire, and employed the sag of a second system to magnify the movement, as shown in Fig. 1.15.

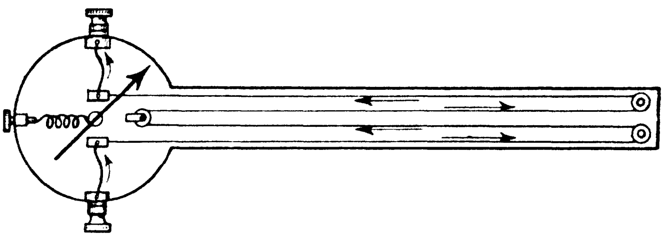


FIG. 1.14.—Diagram of Cardew's hot-wire voltmeter.

It is, however, interesting to note that Professors Ayrton and Perry described a sagging hot-wire instrument in 1892, and magnified the movement with one of their twisted strip springs. Instead of employing the expansion of the conductor, some inventors have employed other devices to indicate its temperature. The simplest of these is the " crossed thermo-junction " (Fig. 1.16), in which an alternating current is made to heat the junction of two wires (such as copper and constantan) and thus produce a thermo E.M.F., which is indicated on an ordinary galvanometer. Another very sensitive device is the thermo-galvanometer of W. Duddell (Fig. 1.17),

where the current to be measured passes through a short conductor of high resistance which, becoming heated, radiates to a thermo-couple suspended above it. This thermo-couple is closed on a loop

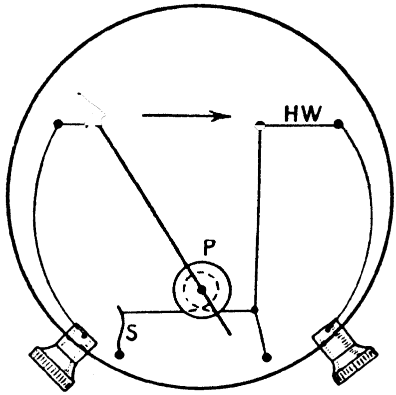


FIG. 1.15.—The Hartmann and Braun hot-wire meter.

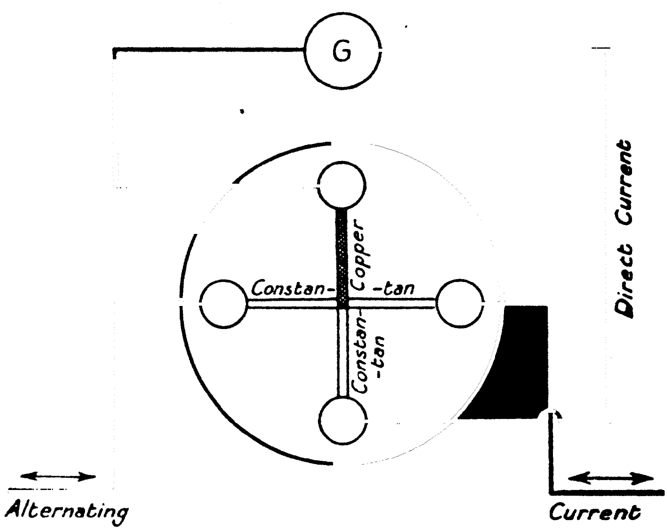


FIG. 1.16.—Principle of measurement of A.C. by the cross thermo-junction.

or coil, and is free to turn in the field of a powerful permanent magnet (it is, in fact, a Boys' radio-micrometer), and the resulting deflections are interpreted in terms of the current through the heater.

(c) *Chemical Effects and Instruments.*—Of electro-chemical measuring instruments there are comparatively few, in spite of the fact that the first electrical measurements were made by their aid after 1833, when Faraday enunciated the laws of electrolysis. When an electric

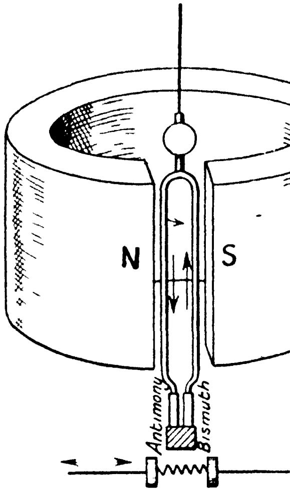


FIG. 1.17—Duddell's Thermo-galvanometer

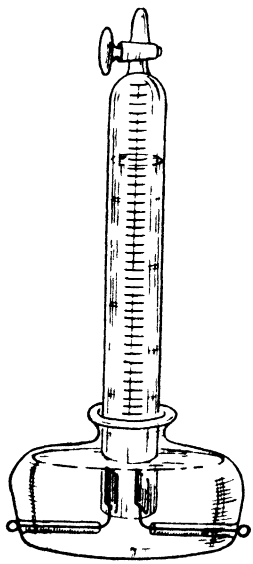


FIG. 1.18—The water voltameter.

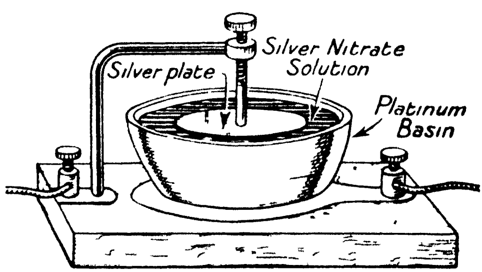


FIG. 1.19.—Diagram of silver voltameter.

current passes through an electrolyte or conducting compound liquid it produces chemical decomposition, which is shown either by the liberation of gas (Fig. 1.18), as observed by Carlisle and Nicholson in 1800, or by the deposition of metal at the point where the current leaves the liquid, noticed by Cruikshank in the same year.

As the amount of gas liberated or the metal deposited in a given time is very closely proportional to the current over wide limits, this device, or voltameter, as it is called, has been adopted to give the legal standard of current, the International Ampere being defined

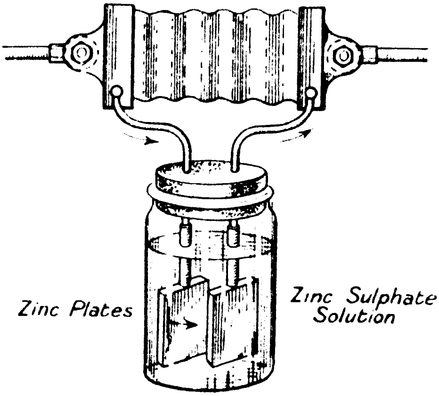


FIG. 1.20.—Principle of Edison's electrolytic supply meter.

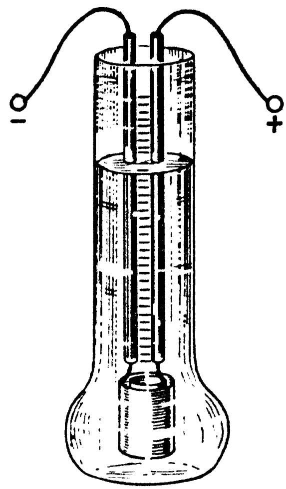


FIG. 1.21.—Bastian supply meter.

as that steady current which will deposit 1·11800 milligrammes of silver per second from a solution of silver nitrate in water (Fig. 1.19). As the total amount of chemical action in a given time is proportional to the product of the current strength and the time, or to the total quantity of electricity which has passed, the voltameter is really a quantity meter, and was developed as a supply meter by Edison in 1879 (Fig. 1.20) using the deposition of zinc, while Bastian in 1898 employed the liberation of gas for the same purpose (Fig. 1.21), and Wright perfected the mercury quantity meter. Since chemical action gives rise conversely to an electromotive force, we have the various forms of primary and secondary cells which afford us the most convenient practical standards of E.M.F.

In 1872 Latimer Clark produced his standard cell having electrodes of mercury and zinc in contact with saturated zinc sulphate solution, the depolarizer being mercurous sulphate paste. The E.M.F. of this cell has been given as 1·4328 International Volts at 15° C., and it enables very accurate measurements of electrical P.D. to be made by the aid of the potentiometer, the principle of which was due to Poggendorf in 1841, and was put into practical form by Fleming and Crompton in 1893. A considerable advance on the Clark Cell was made by Dr. Weston in 1893 by substituting cadmium and cadmium sulphate for zinc and zinc sulphate. The E.M.F. of the cadmium cell is 1·0184 International Volts at 15° C., and its temperature variation is very much less than that of the Clark Cell, being only 0·004% per 1° C.

Electrostatic Effects and Instruments

The remaining class of electrostatic instruments do not depend for their operation upon any property of the electric current, but on the mechanical attraction which exists between any two bodies between which there is a difference of potential. This principle has been made use of by Beccana, Volta, Binnet and others to produce

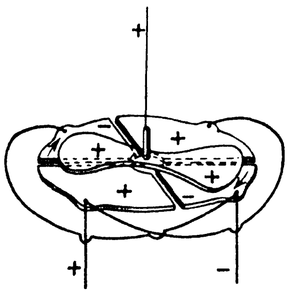


FIG. 1.22.—Principle of the Kelvin quadrant electrometer.

electroscopes, and crude forms of electrometers for measuring pressure and quantity were devised by Bohnenberger, Henley and others. But it is almost entirely to Lord Kelvin that we owe the modern electrometers and electrostatic voltmeters, which he divided into two classes : the symmetrical and attracted-disc forms. In 1856 he devised the divided-ring electrometer, but soon afterwards improved it to form his well-known quadrant instrument (Fig. 1.22), in which the flat aluminium needle turns from one pair of quadrants

to the other. This was afterwards put into commercial form in 1887 as an indicating electrostatic voltmeter (Fig. 1.23), for use on pressures between 400 and 10,000 volts. In 1890 a further increase of sensitiveness was obtained by making it multicellular and mounting a number of needles on the same axis (Fig. 1.24), so that it served to indicate potential differences of from 60 to 150 volts.

The attracted-disc electrometer was first devised by Sir Wm. Snow Harris in 1834, who simply weighed the attraction between two discs, but Lord Kelvin worked out the theory of the instrument,

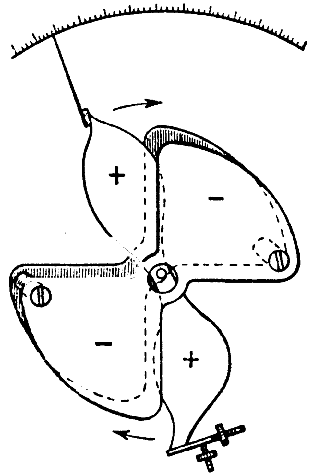


FIG. 1.23.—Principle of Kelvin's electrostatic voltmeter.

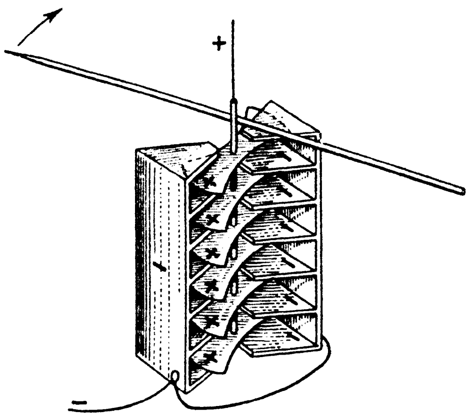


FIG. 1.24.—Kelvin's multicellular voltmeter.

introduced the important improvement of the "guard ring" and put it into practical shape as an "absolute electrometer" (Fig. 1.25) in 1853. This instrument enabled the voltage to be deduced from the dimensions of its parts and the force of attraction without reference to any other standard.

Just as the fact that a current-carrying circuit endeavours to increase the magnetic flux linked with it is the fundamental principle underlying all electro-magnetic instruments, so the fundamental principle actuating all electrostatic instruments is that a charged body tends to move so as to increase the electrostatic field associated with it. Where no independent electrostatic field exists the system will tend to move so as to increase its capacity, and advantage has

been taken of this in the ingenious forms of moving dielectric voltmeters of Arno and Burrows, in which the moving element is a cylinder of insulating material of high inductive capacity which tends to turn between fixed metal plates so as to bridge as nearly as possible the space between them.

A further effect of high potential difference is that of spark or disruptive discharge, the length of the spark depending upon the P.D., and upon the form of the surfaces between which it passes. This is sometimes employed as a means of estimating high voltages.

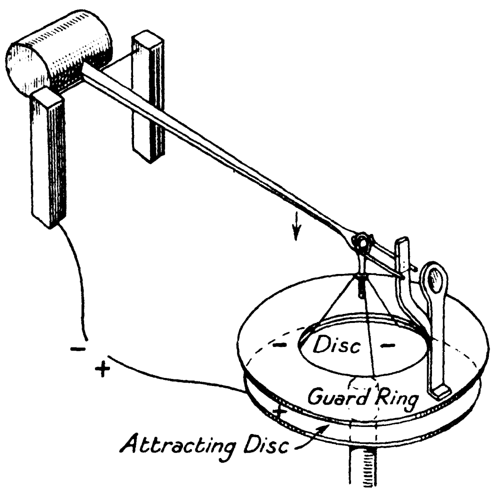


FIG. 1.25.—Kelvin's attracted-disc electrometer.

The foregoing description, though necessarily incomplete, comprises the essential types of indicating instruments and is summarized in Table I. Besides these instruments, however, there are a large number of comparison instruments, which enable two quantities of the same or related kinds to be compared. Such, for example, are the Wheatstone and Kelvin bridges for comparing the values of two resistances, the potentiometer for comparing E.M.F.'s or P.D.'s with the E.M.F. of a standard cell, and the various devices for comparing inductances and capacities, etc.

Direct Current and Alternating Current Instruments

A special feature of electrical supply which differentiates it from practically every other power supply, is the employment of alter-

nating currents, in which the P.D. and current continuously and rapidly reverse their direction. The ordinary commercial frequencies range from 25 to 133 cycles per second, with a few odd cases of an even lower frequency, while the frequencies used in radio-communication run up to millions of cycles per second.

It is obvious that, in order to obtain a steady reading with such a current or P.D. the instrument must be non-polarized, i.e. the deflection must be in the same direction with the current flowing in either direction through it. This excludes all permanent magnet instruments, whether of the moving-needle or moving-coil type, as these reverse their deflection with reversal of current. It also excludes all chemical devices except the mixed gas voltmeter.

All other types of instrument, i.e. the soft-iron ammeter and voltmeter, the dynamometer, the thermal and electrostatic instruments, serve equally well for direct or alternating current measurement when properly designed. There is also a special class of " induction " instruments which will only operate with alternating currents. In these the moving element has currents circulating in it which are produced by the E.M.F.'s induced by the alternating magnetic field, which is in turn produced by the current being measured, and the reaction between this field and the induced currents provides the necessary deflecting torque.

Effective Value of an Alternating Current or P.D.

For most purposes it is necessary to know the average power supplied in the case of an alternating current. If a current passes through a resistance, such as the filament of an incandescent lamp, the power communicated to the lamp is I^2R watts. If the current, instead of being steady, is varying in any way, the average power supplied will be the average of I^2R, and, if the resistance is constant in value, this means that the average value of the power supplied is R multiplied by the average value of I^2. But if we had a steady current I_1 passing through the same resistance and giving the same power, this will be $I_1{}^2R$, which must be equal to R multiplied by the mean or average square of the alternating current. Consequently $I_1{}^2$ is equal to the mean I^2, from which we have $I_1 = \sqrt{(\text{mean } I^2)}$. The effective value of an alternating current is therefore the " root-mean square " (R.M.S.) value of the current. As the power applied to the resistance is also given by V^2/R, or is proportional to V^2, it

can be proved, in the same way, that the effective value of an alternating P.D. is the R.M.S. value of the voltage.

If we consider any of the principles laid down in the classification other than the permanent magnet and chemical ones, we see that they naturally lend themselves to indicating the effective values of the current of P.D. The attraction of a solenoid for a piece of soft-iron is approximately proportional to the square of the current in the coil. The attraction of two wires connected in series but placed parallel to one another, through which a current is flowing, is proportional to the square of the current; the heat developed in a hot-wire instrument follows the same law; while the electrostatic attraction between two bodies is proportional to the square of the P.D. between them. Hence, if the periodic time of the moving portion of any of these instruments is large in comparison with that of the alternating supply (so that the pointer is affected by the average force instead of following its variation), the deflection depends on the average square of the current or P.D. Consequently, if such an instrument is calibrated with a direct current, it indicates the effective or R.M.S. value of the alternating current.

In dealing with the various instruments in detail it will be convenient to divide them into two classes: first, the indicating instruments used in commercial engineering work, and secondly, the sensitive and standard instruments used in precision testing.

In the first section may be included ammeters, voltmeters, wattmeters, quantity and energy supply meters, phase and frequency meters, and their accessories. The second comprises delicate galvanometers, current standards, standard resistances and cells, Wheatstone and standardizing bridges and capacities and their measuring devices. These will be dealt with in the second volume. In Table II are given the principal electrical quantities with their practical and scientific units, and the instruments employed for the measurement of each.

TABLE I.—*Classification of Electrical Phenomena and Instruments.*

Phenomenon.	Manifestation.	Discoverer.	Instruments. Laboratory.	Instruments. Indicating.
		I.—*Electric Current.*		
(A) **Magnetic**	(*a*) Deflects permanent magnet	Oersted, 1819	Moving needle galvanometers. Schweigger's multiplier, 1820. Nobili's astatic, 1825. Pouillet sine and tangent, 1837. Kelvin mirror, 1858. Deprez magnetic control, 1879	Ayrton and Perry ammeter, 1879.
	(*b*) Tends to move conductor in magnetic field	Faraday, 1921	Moving coil galvanometer. Sturgeon, 1836. Kelvin siphon recorder, 1867. d'Arsonval galvanometer, 1882. Blondel oscillograph, 1891. Duddell oscillograph, 1897. Einthoven string galvanometer, 1901	Weston voltmeter, 1888.
	(*c*) Magnetizes and attracts iron core	Arago and Davy, 1820	Ayrton and Perry ohmmeter, 1882. Blondel oscillograph, 1893	Ayrton and Perry magnifying spring, 1884. Kelvin ampere gauge, 1885. Schuckert attraction ammeter. Nalder repulsion ammeter, 1885.
	(*d*) Attracts or repels second current	Ampère, 1820	Weber dynamometer, 1843. Joule's current weigher	Ayrton and Perry wattmeter, 1881. Kelvin balance, 1883. Siemen's dynamometer, 1883.
	(*e*) Tends to reduce section of conductor	Bary, 1901	Northrup absolute current meter, 1907	..
	(*f*) Induces currents in neighbouring conduc-	Faraday, 1830	Fleming's cymometer. Heterodyne wavemeter	Ferraris induction instruments.

2

TABLE I—*continued*.

Phenomenon.	Manifestation.	Discoverer.	Instruments. Laboratory.	Instruments. Indicating.
(B) Thermal	(*g*) Heats circuit	..	Duddell thermogalvanometer, 1904. Irwin oscillograph, 1907	Cardew voltmeter, 1883. Hartmann and Braun hot-wire instruments.
	(*h*) Decomposes water	Carlisle and Nicholson, 1800.	Gas voltmeter	Bastian supply meter.
(C) Chemical	(*i*) Deposits metals	Cruikshank, 1800	Copper and silver voltameters	Edison supply meter, 1879.
	(*j*) Produces standard polarisation E.M.F.	Volta, 1800	Latimer Clark cell, 1872. Weston cadmium cell, 1893	..
II.—*Potential Difference.*				
(D) Kinetic	(*k*) Produces current proportional to P.D. in fixed resistance	Cavendish, 1781. Ohm, 1827	Any galvanometer or ammeter of suitable constant resistance	Voltmeters.
(E) Electrostatic force	(*l*) Attracts or repels other charged bodies	Gilbert	Electrometers, Henley, 1772. Kelvin quadrant, 1851. Volta attracted disc	Electrostatic voltmeters, Kelvin, 1867. Ayrton and Mather. Jona.
	(*m*) Tends to increase capacity of system	..	..	..
(F) Discharge	(*n*) Disruptive discharge	Hawksbee, 1707	Spark gauges	..
	(*c*) In vacuum	Edison, Fleming, Lee de Forest, etc.	Valve amplifiers	Vacuum tube voltmeters.

TABLE II.—*Electrical Quantities, Units and Instruments.*

Quantity Measured.	Symbol and defining relation.	Practical unit.	Practical definition.	Ratio to C.G.S. unit.	Definition of C.G.S. unit, electro-magnetic.	Instrument.
Current	I	Ampere	International ampere deposits 1·11800 mgms. silver per sec.	10^{-1}	Produces magnetic force of 2π gilberts at centre of coil 1 cm. radius	Ammeter.
P.D.	V	Volt	1 — Clark cell = 1·4328 volts at 15° C.	10^{8}	Alteration of flux at rate of 1 linkage per second	Voltmeter. Potentiometer.
Resistance	$R = \frac{V}{I}$	Ohm	International ohm, 106·3 cm. of mercury weighing 1·44521 grammes at 0° C.	10^{9}	..	Ohmmeter. Wheatstone Bridge.
Power	$W = VI$	Watt	1 ampere at 1 volt	10^{7}	1 erg per second	Wattmeter.
Quantity	$Q = \int I dt.$	Coulomb	1 ampere-second	10^{-1}	1 C.G.S. unit of current per 1 second	Quantity meter.
Energy	$E = \int W dt.$	Jouie	1 watt-second	10^{7}	1 erg = 1 dyne-cm.	Energy meter.
Inductance	L	Henry	10^{8} linkages per ampere	10^{9}	1 linkage per C.G.S. unit of current	Inductance bridge.
Capacity	$C = \frac{Q}{V}$	Farad	1 volt charges with 1 coulomb	10^{-9}	1 C.G.S. unit of P.D. charges with C.G.S. unit of quantity	Capacity bridge.
Magnetizing force	H	Gilbert	10/2 magnetic force at centre of coil 1 cm. radius	1	Force of 1 dyne on unit magnetic pole	Magnetometer.
Induction	B	Gauss	..	1	No. of C.G.S. lines of force per sq. cm.	Permeammeter.
Permeability	μ	..	..	1	..	..
Flux	$\Phi = AB$	Maxwell	..	1	Total number of C.G.S. lines of force passing through circuit	Flux meter (with search coil). Bismuth spiral.
Phase	φ	Degree or radian.	..	..	..	Phase meter. Power factor indicator.
Waveform	..	Sine curve. (stand. form)	..	..	..	Oscillograph, etc.
Frequency	f	Cycles per second	..	..	..	Frequency meter. Wavemeter.

CHAPTER 2

MECHANICAL DESIGN AND CONSTRUCTION

IN every indicating instrument there are essentially two systems, one of which is fixed and the other capable of relative movement. It is desirable that the movement of this latter system should bear some simple relation to the quantity measured, that its movement should follow the changes in the quantity in such a way that the readings can be obtained in a minimum of time, and that the expenditure of energy within the instrument itself shall be as small as possible.

To fulfil these conditions it is therefore necessary that the action of the movement should depend upon a simple law—that there should be slightly less than critical damping to the motion, and that such quantities as mechanical friction, heating and magnetic hysteresis should be reduced to the lowest possible value.

The reduction of mechanical friction is entirely a matter of correct design, and is dependent on the method of supporting the movement. In general three methods are employed in commercial instruments, viz.: (*a*) Pivoting, (*b*) knife-edge suspension, (*c*) filar or thread suspension.

PIVOTING

The most generally adopted method is the first, in which the movement is provided with one or more finely-polished pivots in the axis of rotation which engage in hollow-ground jewels or other suitable bearings supported from some fixed part of the instrument.

Such a scheme of support is both robust and tolerably independent of levelling, and when carefully constructed and adjusted should show very little frictional resistance to motion.

The form of pivot adopted varies very considerably, and much depends upon the mass of the movement to be supported, for, in order that the friction shall be small, the area of contact between the pivot and jewel must be small; on the other hand, the life of the bearing is in a great measure dependent upon the pressure per unit area of the contact surface, and with heavy movements this pressure may reach very high values.

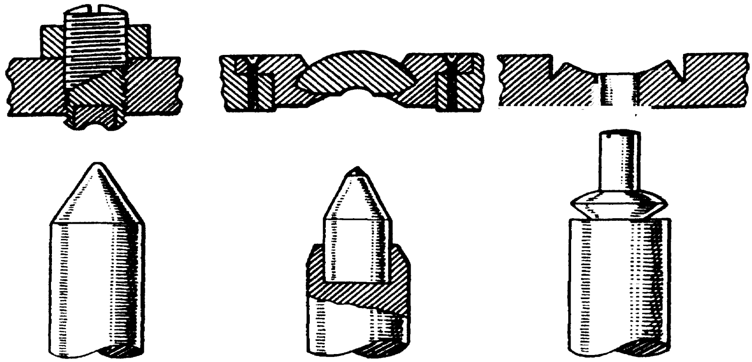
Consider, for instance, the case of a vertical scale instrument whose movement weighs two grammes, and let the area of contact of each of the two pivots be 1/1,000th of a square millimetre : the pressure will then be approximately 100 kg. per square centimetre, or a little more than half a ton per square inch. In actual instruments the contact pressures may be considerably higher than this, as will be shown later.

Cylindrical or Parallel Pivots.—In the case of cylindrical pivots, if the force due to the weight of the moving system is F, then the force of friction is $\mu_k F$ where μ_k is the friction coefficient and the torque is $\mu_k Fa$ where a is the radius of the pivot. It is evident, therefore, that the pivot should be of the smallest possible diameter consistent with good mechanical construction and the bending or shearing strength of the pivot itself.

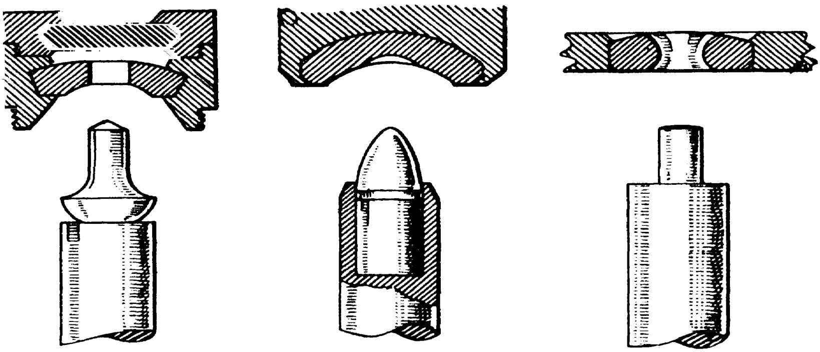
This type of bearing is employed in most cases where the spindle receives a positive drive, as, for instance, in the wheel trains of recorder clocks and meter-counter trains. In most cases watchmakers' practice is followed, the pivot being turned on the end of the steel spindle ; the reduction in diameter being such that the finished diameter is equal to the bearing length, while the total length of the pivot is about 1·7 times the diameter (Fig. 2.1).

Many good workmen are proud of producing a perfectly flat shoulder, but it is doubtful if this is any real advantage, as the variable friction is likely to be more troublesome in such bearings (see counter-train friction in Part *II*), and it is probably better slightly to round off the shoulder, particularly where the pivot works in " through " holes in brass plates (Fig. 2.1*c*). It is usual to groove the spindle just behind the shoulder to prevent the lubricant being drawn away by surface tension from the bearing.

When the endstone jewel is employed (Fig. 2.1*d*) the shoulder is turned conical, or perhaps more curved than a true cone, and is then chamfered back or grooved where it joins the spindle to obviate the ill-effects of surface tension. The parallel portion or point of the pivot passes through the pierced jewel or ringstone, and allows the blunt end to bear against the endstone. This end face of the pivot is usually slightly rounded, but some makers prefer the end quite flat so that the moment of friction shall be more nearly the same in the vertical and horizontal positions of the spindle, and a further modification is to employ a hollow end, but this may tend to serious cutting of the stone.

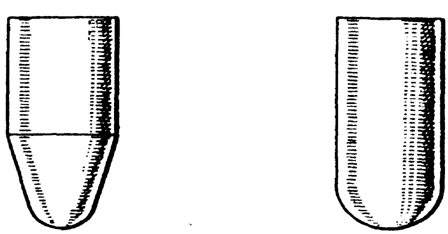


(a) (b) (c)



(d) (e) (f)

FIG. 2.1.—Types of pivot and jewel bearings.



(a) (b)

FIG. 2.2.—Conical and hemispherical pivots.

Conical Pivots.—The most usual form of pivot used in electrical measuring instruments is a cone having a radius at the tip (Fig. 2.2*a*), or a cylinder with a hemispherical end (Fig. 2.2*b*). When this pivot is in contact with a jewel and carries a load which is the whole or part of the weight of the moving system, then theoretically both jewel and pivot are deformed under elastic compression. If the load acts along the axis of the pivot, then the maximum compression will occur on the axis at the point of contact of jewel and pivot, and the maximum pressure will also occur at this point. Hertz has investigated the conditions relating to the elastic contact of two spheres, and his results are applicable to a pivot with a spherical end in contact with a spherical cup jewel.

Let r_1 = radius of pivot in centimetres.
r_2 = radius of jewel in centimetres.
a = radius of the area of contact in centimetres.
P = total load on pivot in dynes.
ψ_1 = constant depending on the elastic constants of the pivot.
ψ_2 = constant depending on the elastic constants of the jewel.
p = pressure at any point on the surface of contact distant x cm., from centre, in dynes per square centimetre.
p_m = maximum pressure on surface of contact in dynes per square centimetre.
p_a = average pressure over surface of contact in dynes per square centimetre.
C_f = torque in dyne-centimetres required to overcome friction between pivot and jewel.
μ = coefficient of friction between pivot and jewel.

The formulae of Hertz can be written for this case as follows :—

$$a = \sqrt[3]{\frac{3P(\psi_1 + \psi_2)}{16\left(\frac{1}{r_1} - \frac{1}{r_2}\right)}} \quad . \quad . \quad . \quad (1)$$

$$p = \frac{3P}{2\pi} \frac{\sqrt{a^2 - x^2}}{a^3} \quad . \quad . \quad . \quad . \quad (2)$$

$$p_m = \frac{3P}{2\pi a^2} \quad . \quad . \quad . \quad . \quad . \quad (3)$$

The quantity ψ_1 is defined as

$$\psi_1 = \frac{4n_1 + 3k_1}{n_1 (n_1 + 3k_1)}$$

where n_1 = modulus of rigidity of the material of the pivot in dynes per square centimetre.

k_1 = bulk modulus of rigidity in dynes per square centimetre.

The quantity ψ_2 is similarly defined for the jewel, but since in a sapphire/steel combination, which is the usual one in modern instruments, it can safely be assumed that all deformation occurs on the pivot, ψ_2 is taken as zero. Stott states that for Stubbs' steel $n_1 = 8{\cdot}90 \times 10^{11}$ dynes per square centimetre and $k_1 = 18 \times 10^{11}$ dynes/cm.^{2}, so that $\psi_1 = 0{\cdot}1601 \times 10^{-11}$ cm.^{2}/dyne. Since ψ_2 is zero the value of $(\psi_1 + \psi_2)$ can be taken as $0{\cdot}1601 \times 10^{-11}$ cm.^{2}/dyne or $1{\cdot}632 \times 10^{-9}$ cm.^{2}/gramme. This figure may vary with the material of the pivot, and Hertz states it always lies between 32/9 and 4 times the reciprocal of Young's modulus for the material. The figure given above is used in the following calculations, and since ψ always appears in the formulae as the one-third or two-thirds power, variations in ψ do not produce proportionate changes in the results.

If the jewel is made of some material other than sapphire and it is no longer justifiable to assume no deformation of the jewel material, then the value of ψ must be adjusted accordingly.

Another function which occurs in the above formulae is $1/(1/r_1 - 1/r_2)$. This can be converted to

$$\frac{1}{1/r_1 - 1/r_2} = \frac{r_1 r_2}{r_2 - r_1} = 1 \left(\frac{r_2/r_1}{r_2/r_1 - 1} \right) = r_1 \gamma$$

where $\gamma = \dfrac{r_2/r_1}{r_2/r_1 - 1}$. So equation (1) can be rewritten

$$a = \left[\frac{3p\,(\psi_1 + \psi_2)\,\gamma\, r_1}{16} \right]^{\frac{1}{3}} \quad . \qquad (4)$$

The above equations are all in absolute units, and are not in the most convenient form for practical calculation. The load on the pivot is usually given in grammes, and the radius of the pivot is given in thousandths of an inch. Putting in the value $0{\cdot}1601 \times 10^{-11}$ sq. cm./dyne for $(\psi_1 + \psi_2)$ and giving the radius of the

circle of contact in thousandths of an inch, then equation (4) can be rewritten as

$$27{\cdot}63\, a_1 = (W r_1\, \gamma)^{\frac{1}{3}} \quad . \quad . \quad . \quad (5)$$

where a_1 = radius of circle of contact in thousandths of an inch.
r_1 = radius of pivot in thousandths of an inch.
W = load on pivot in grammes.

Again, the maximum pressure is required in tons per square inch and if the value for a is substituted in equation (3), and the various quantities converted into practical units, then the pressure in tons per sq. in. is given by

$$P_{max} = 358\, W^{\frac{1}{3}}\, (\gamma\, r_1)^{\frac{2}{3}} \quad . \quad . \quad . \quad (6)$$

Now designers are also interested in the torque required to overcome the friction between the pivot and jewel, and Stott has shown that the friction torque in dyne-cm. is given by

$$C_f = 3\pi\, \mu P a/16 \quad . \quad . \quad . \quad . \quad (7)$$

where all quantities are again in absolute units and μ is the co-efficient of friction between the pivot and jewel. When the value for a has been substituted and the various quantities expressed in practical units, this becomes

$$188{\cdot}3\, C_f = W^{\frac{4}{3}}\, (\gamma\, r_1)^{\frac{1}{3}} \quad . \quad . \quad . \quad . \quad (8)$$

C_f still being in dyne-cm. For convenience μ has been taken as 0·1. This will be discussed later.

The equations (5), (6) and (8) are in a practical form and lend themselves very readily to representation by a nomogram.*

The equations are very similar in form and if written logarithmically become

$$\log r_1 + \log \gamma + \log W - 3 \log (27{\cdot}63a) = 0 \quad . \quad . \quad (9)$$
$$2 \log r_1 + 2 \log \gamma - \log W + 3 \log (P_{max}/358) = 0 \quad . \quad . \quad (10)$$
$$\log r_1 + \log \gamma + 4 \log W - 3 \log (188{\cdot}3\, C_f) = 0 \quad . \quad . \quad (11)$$

It is necessary to consider the range of values of r_1, γ and W which it is desired to cover in the proposed nomogram. The radius

* *Journal of Scientific Instruments*, Vol. 24, No. 9, September, 1947.

of the pivot r_1 may vary from one to forty thousandths of an inch, and the load W from 0·1 to 100 grammes. The values of γ are determined by the ratio r_2/r_1. One limit is represented by a flat jewel which gives an infinite value to the ratio r_2/r_1 and so makes $\gamma = 1$. There is actually a limit to the ratio r_2/r_1 for which the equations hold. This will be discussed later, but for the moment an upper value for $\gamma = 11$, corresponding to $r_2/r_1 = 1.1$, will suffice.

Consider now the pressure equation (10) as typical of all three. There are four variables and the form of the equation is such that a set-square index nomogram can be drawn to represent it. Since

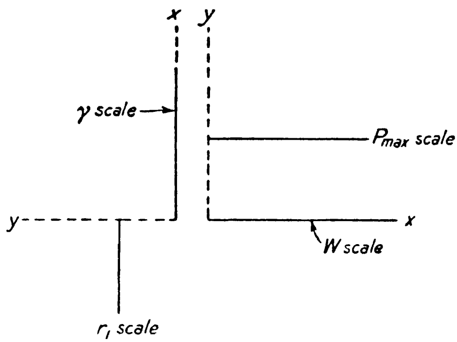


FIG. 2.3.—Rough sketch of nomogram for 4 variables.

the three equations (9)–(11) are so similar it is further possible to draw one combined nomogram having a number of common scales which will give the solution to all three equations, with one setting of the index. The full theory of the set square index nomogram is given in books on the subject of nomography, but reference to Fig. 2.3 will show the general type. The four scales are plotted in two pairs, each to its own set of rectangular co-ordinates, the two sets of co-ordinates being at right angles as shown. Fig. 2.3 actually shows the arrangement of the scales for equation (10). The actual positional arrangement of the two sets of axes is unimportant and can be arranged in any convenient manner, provided they are maintained at right angles.

Fig. 2.4 gives the completed nomogram for equations (9)–(11), and the original has been arranged so that the effective scales are

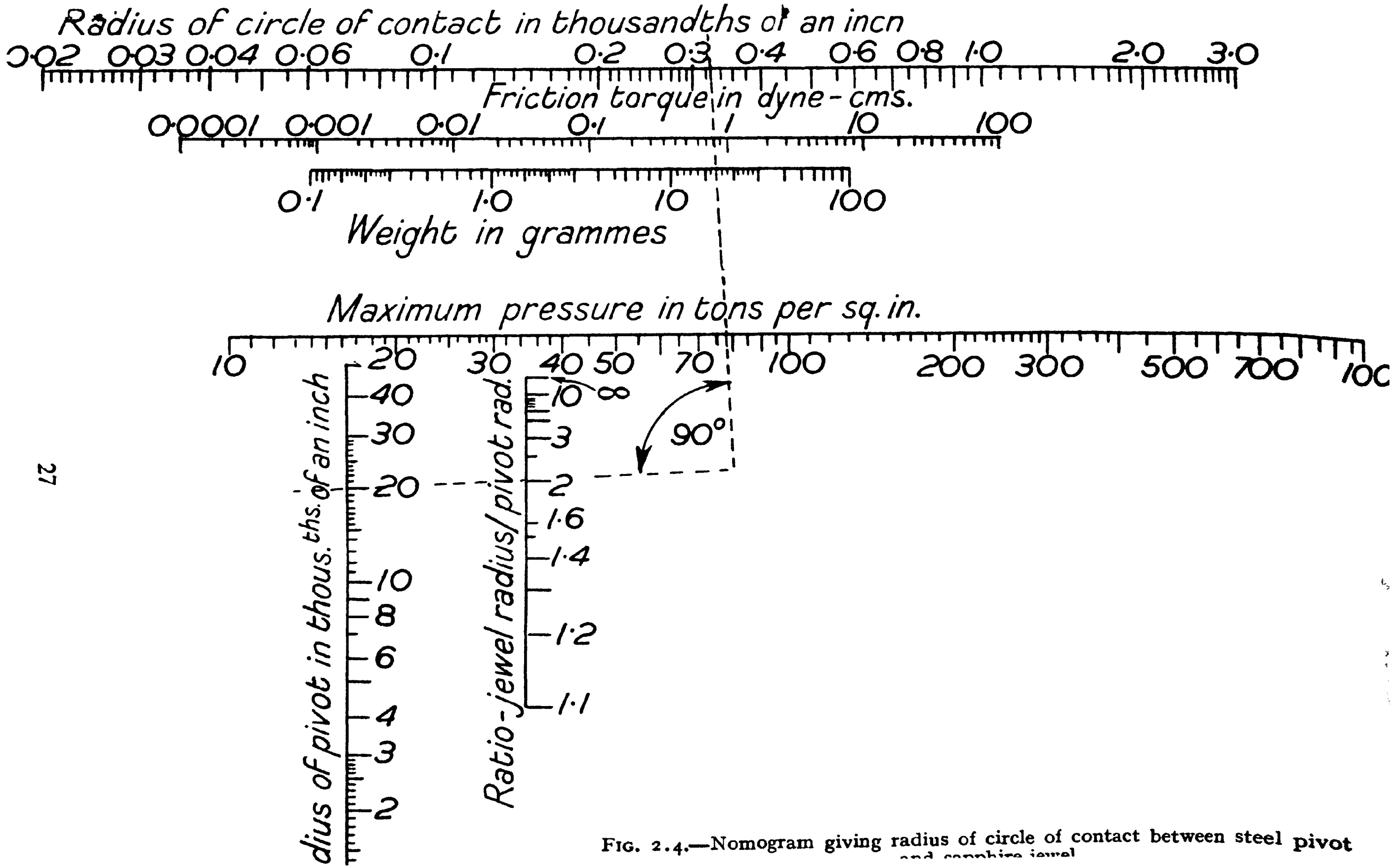


FIG. 2.4.—Nomogram giving radius of circle of contact between steel pivot and sapphire jewel

at least 6 in. in length. To assist in the construction of this nomogram, the following are the co-ordinates of the various scales :

Scale.	*x ordinate.*	*y ordinate.*
r_1 (radius of pivot)	$3{\cdot}636 \log N_1$	4
γ	$6 - 3{\cdot}636 \log \gamma$	2
W (load on pivot)	$2 (1 + \log W)$	4
P_{max}	$6 \log P_{max} - 6{\cdot}7233$	1·8
a_1 (radius of contact)	$6 \log a_1 + 7{\cdot}34808$	5·1
C_f	$1{\cdot}5 \log C_f + 4{\cdot}5873$	4·275

Effect of Impact.

The discussion, up to the present point, has been concerned with static loading on the pivot. This is of great importance in considering the actual performance of an instrument, but as regards possible damage to the jewel and pivot there is no doubt but that the impact conditions are of even greater importance. All instruments are subjected to vibration or shock at some time or other, whether in transport, normal handling or actual operation, and it becomes necessary therefore to investigate the forces developed under these conditions. Here again the equations of Hertz provide a convenient starting point.

Hertz gives the following equations for the equivalent load on impact:

$$P_m = k_2 (\beta_m)^{\frac{3}{2}} \quad . \quad . \quad . \quad . \quad (12)$$

where

$$\beta_m = \left[\begin{matrix} 5v^2 \\ 4k_1 k_2 \end{matrix} \right] \quad . \quad . \quad . \quad (13)$$

β is a measure of the change in distance between two fixed points, one in the pivot and the other in the jewel, clear of the regions of elastic deformation and β_m is the maximum value of β. Under static conditions β is given by

$$\beta = 3P (\psi_1 + \psi_2) \quad . \quad . \quad . \quad (14)$$

In equations (12) and (13) :

v = relative velocity of pivot and jewel at moment of impact.
k_1 = 1/(mass of moving body).
k_2 = constant derived from equations (1) and (14).

To determine the value of k_2 it is necessary to substitute the value of a from equation (1) in equation (14). On doing this and rearranging

$$P = \frac{16\ \beta^{\frac{3}{2}}}{3\ (\psi_1 + \psi_2)\ (1/r_1 - 1/r_2)^{\frac{1}{2}}}$$

hence

$$k_2 = \frac{16}{3\ (\psi_1 + \psi_2)\ (1/r_1 - 1/r_2)^{\frac{1}{2}}} \quad . \quad . \quad (15)$$

The equivalent load on the pivot is therefore from equation (12) making all the necessary substitutions

$$P_m = \left[\frac{16}{3\ (\psi_1 + \psi_2)\ (1/r_1 - 1/r_2)^{\frac{1}{2}}}\right]^{\frac{2}{5}} \left[\frac{5^2 v\ M}{64}\right]^{\frac{3}{5}} \text{dynes} \ . \quad (16)$$

The maximum pressure is given by equation (3) and substituting in this the equivalent load from (16) and the value of a from (1) leads to the equation

$$p_m = \frac{3^{\frac{1}{5}} \ . \ 16^{\frac{4}{5}} \ . \ 5^{\frac{1}{5}} \ . \ v^{\frac{2}{5}} \ . \ M^{\frac{1}{5}} \ . \ (1/r_1 - 1/r_2)^{\frac{3}{5}} \text{ dynes per sq. cm.}}{2\pi \ . \ 64^{\frac{1}{5}} \ . \ (\psi_1 + \psi_2)^{\frac{4}{5}}} \quad (17)$$

In equations (16) and (17), M is the mass of the moving body in grammes. Now if d cm. is the distance, the moving system drops, then $v^2 = 2gd$. If this is substituted in equation (17), d is now expressed in inches, r_1 in thousandths of an inch and p_m in tons per sq. in., then equation (17) becomes

$$P_{mpx} = \frac{3830{\cdot}5\ (Md)^{\frac{1}{5}}}{(\gamma\, r_1)^{\frac{3}{5}}} \text{ tons per sq. in.} \quad . \quad . \quad . \quad . \quad (18)$$

where γ has the same meaning as before and the value $0{\cdot}1601 \times 10^{-11}$ has been substituted for $(\psi_1 + \psi_2)$.

The values of maximum pressure developed on impact are considerably greater than those due to static pressure, and it is interesting to determine the equivalent static load. W_e which will give the same maximum pressure as equation (18). To do this equations (6) and (18) must be equated with W_e in place of W in equation (6).

This leads to

$$W_e = 1225\,(Md)^{\frac{3}{5}}\,(\gamma\, r_1)^{\frac{1}{5}} \quad . \quad . \quad . \quad (19)$$

The two equations (18) and (19) are similar in form and if expressed in logarithms are :

$$3 \log \gamma + 3 \log r_1 - \log Md + 5 \log (P_{max}/3830{\cdot}5) = 0 \quad . \quad (20)$$

$$\log \gamma + \log r_1 + 3 \log Md - 5 \log (W_e/1225) = 0 \quad . \quad (21)$$

By a similar process to that described for the static conditions it is possible to construct a common set square index nomogram, from which, given the values of r_1, the ratio r_2/r_1, and the product Md, it is possible to read off the values of the maximum pressure in tons per sq. in. and the equivalent static load in grammes.

The complete nomogram is given in Fig. 2.5, and the co-ordinates of the various scales, again arranged to be at least 6 inches long in the original, are given in the following table :

Scale.	*x ordinate.*	*y ordinate.*
r_1	$3{\cdot}636 \log r_1$	4
γ	$6 - 3{\cdot}636 \log \gamma$	2
Md	$1{\cdot}5 \log Md + 4{\cdot}5$	4
P_{max}	$7{\cdot}5 \log P_{max} - 14{\cdot}9447$	1·525
W_e	$10 \log W_e - 16{\cdot}1813$	4·275

It is generally considered that for the type of steel used for pivots, the crushing strength is about 500 tons per sq. in., so that if a factor of safety of 2 is employed, the maximum pressure should not exceed 250 tons per sq. in. If this value is inserted in equation (18) for P_{max}, the following equation results, giving the relationship between r_1, γ, M and d.

$$r_1^{\,3}\,\gamma^3 = 844{,}490\, Md \quad . \quad . \quad . \quad (22)$$

or $3 \log r_1 + 3 \log \gamma - \log M - \log (844{,}490\, d) = 0$.

This can also be put in the form of a nomogram, similar to Figs. 2.4 and 2.5 and is given in Fig. 2.6. The co-ordinates of the various scales are given in the following table :

Scale.	*x ordinate.*	*y ordinate.*
r_1	$3{\cdot}636 \log r_1$	4
γ	$6 - 3{\cdot}636 \log \gamma$	2
M	$2\,(1 + \log M)$	4
d	$-\,0{\cdot}1468 - 2 \log d$	0·767.

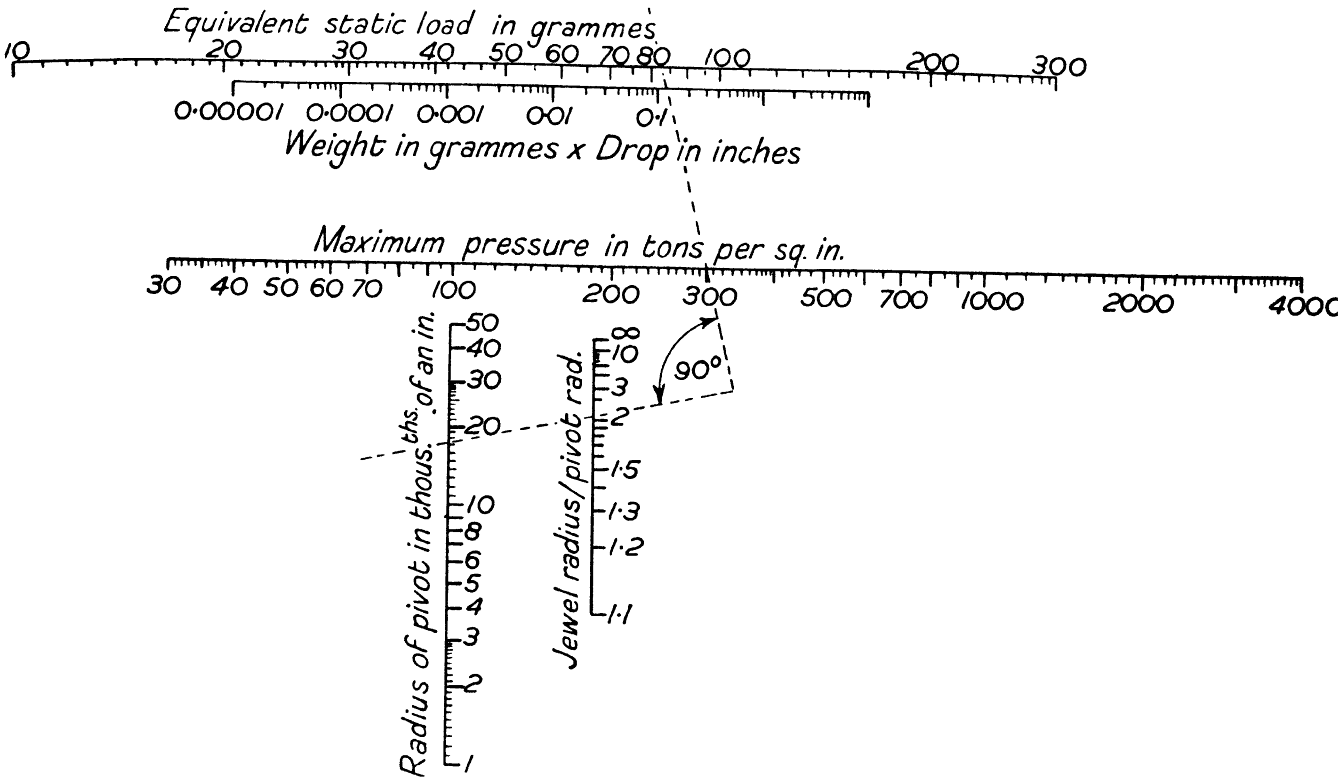


FIG. 2.5.—Nomogram giving maximum pressure on impact between steel pivot and flat sapphire jewel

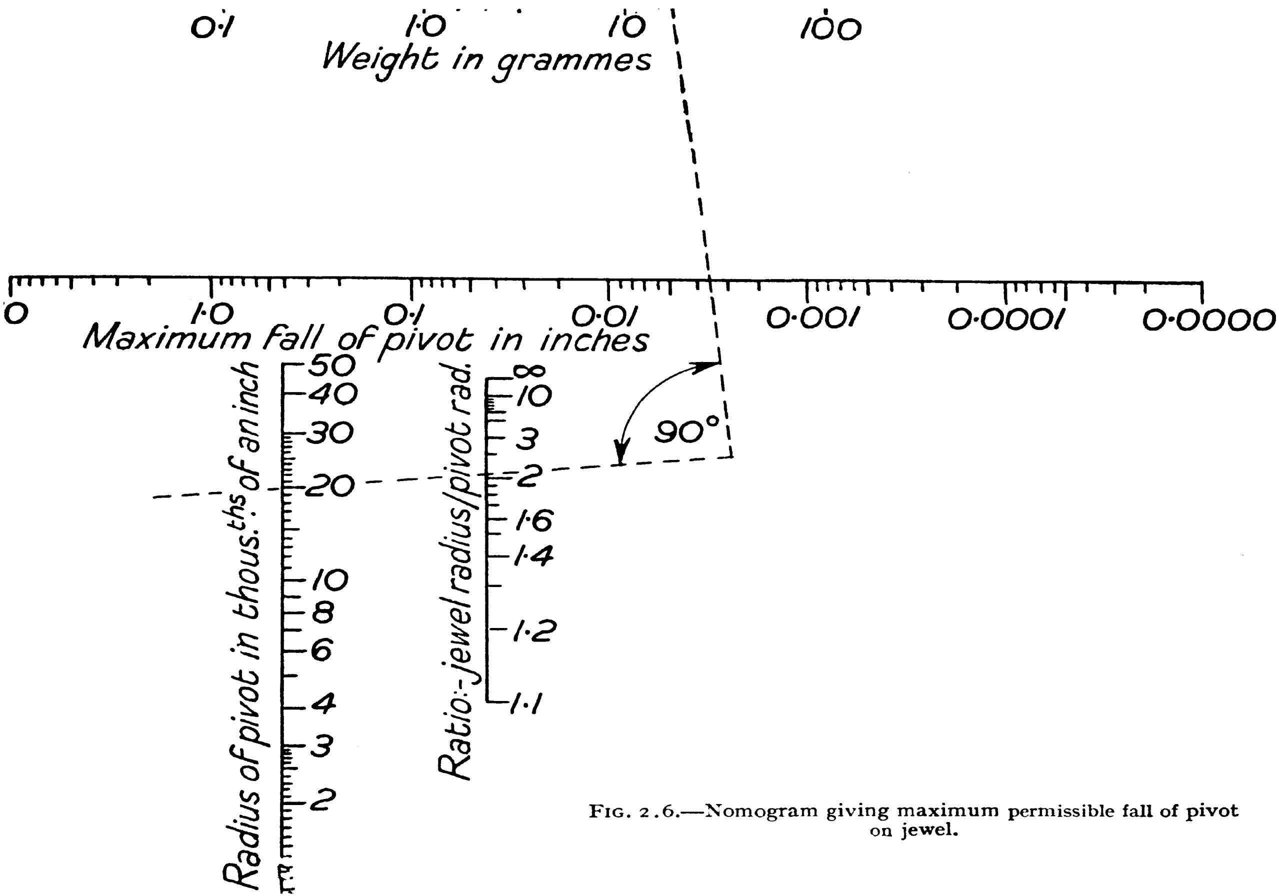


FIG. 2.6.—Nomogram giving maximum permissible fall of pivot on jewel.

With this nomogram, knowing the values of the pivot radius, the jewel radius and the mass of the movement it is possible to read off the maximum distance the moving system may fall on to the jewel, if the resulting pressure is not to exceed 250 tons/sq. in.

It will be noted that care has been taken to distinguish between the load on the pivot in the static case, and the mass of the movement in the impact case, different symbols being employed. This has been done because the load on the pivot is not always that due to the mass of the moving system. In the static case, the operating conditions, the movement may be subjected to forces which materially increase the load on the pivot above that due to the mass of the moving element alone. In dealing with the static case, the total load on the pivot must always be considered.

Coefficient of Friction.

In calculating the friction torque scale for Fig. 2.4, it has been assumed that the coefficient of friction is 0·1. If the coefficient has some other value then the value of friction torque must be adjusted in proportion. Shotter has given some experimental values obtained as a result of a large number of experiments and these are as follows :

For stationary movement :

Pivot and jewel dry	0·191
Pivot and jewel lubricated with meter oil No. 2 .	0·163

For rotating movement :

Pivot and jewel dry	0·163
Pivot and jewel lubricated with meter oil No. 2 .	0·136

Thus, for indicating instruments with dry jewels and pivots, the friction torque values given by Fig. 2.5 must be multiplied by 1.93, while if the jewel is lubricated with meter oil No. 2, the multiplying factor is 1·63. For a rotating instrument, such as an integrating watt hour meter, the multiplying factors will be 1·63 dry and 1·36 lubricated.

Use of nomograms

As an example of the use of the nomograms, consider first the case of an integrating watt-hour meter, the movement weighing

17 grammes, with a pivot radius of 0·020 in., and a jewel radius of 0·048 in. The ratio jewel radius/pivot radius is 2·4, so referring to Fig. 2.5, the first line of the index is made to pass through the value 20 on the pivot radius scale, and the value 2.4 on the scale jewel radius/pivot radius. The second line, at right angles to the first, is then made to pass through the value 17 on the weight scale and then passes through the corresponding values on the other three scales. These are :

Maximum pressure . .	86 tons per square inch.
Radius of circle of contact .	0·305 thousandths of an inch.
Friction torque . .	0·86 dyne-cm.

Since this is a rotating movement and the jewel is probably lubricated, the theoretical value of the friction torque is $1{\cdot}36 \times 0{\cdot}86 = 1{\cdot}17$ dyne-cm.

Again referring to Fig. 2.6, and using the same values of pivot radius, ratio jewel radius/pivot radius and weight of movement, the maximum safe value for the fall of the movement is given as 0·0023 in.—a very small value.

With such a meter a fall of 0·012 in. is possible, and an application of the figures to Fig. 2·5, with weight $\times$ distance $= 17 \times 0{\cdot}012 = 0{\cdot}204$, gives a maximum pressure of 335 tons per sq. in., and an equivalent static load of 92 grammes.

For a second example, an indicating instrument may have a movement weighing 0·2 grammes, a pivot radius of 0·0015 in., and a jewel radius of 0·0047 in.

The ratio jewel radius/pivot radius is thus $0{\cdot}0047/0{\cdot}0015 = 3{\cdot}15$. An application of these figures to the nomograms of Figs. 2.4 and 2.6 gives the following results :

Maximum pressure . .	123 tons per square inch.
Radius of circle of contact .	0·027 thousandths of an inch.
Friction torque . .	0·00073 dyne-cm.
Maximum safe drop . .	0·000048 in.

In this case the movement is stationary and the jewel will probably be dry, so that the theoretical friction torque will be $1{\cdot}91 \times 0{\cdot}00073 = 0\ 00139$ dyne-cm. Such a movement may be capable of dropping 0·010 in., in which case weight $\times$ distance $= 0{\cdot}2 \times$

0·010 = 0·002. On applying the figures to the nomogram of Fig. 2.5 the following results are obtained:

Maximum pressure . .	700 tons per square inch.
Equivalent static load .	40 grammes.

It is quite evident that dangerous pressures may easily be developed on impact, and it is not therefore surprising that steps are often taken to minimise the effects of shock in transport, such as the use of spring jewels, movement locking devices and the like. Experience shows that only a slight shock is sufficient to deform the pivot of an indicating instrument, and it is clear from the above that very high pressures are very easily produced by a comparatively slight shock.

Limitations of the Formulae and Nomograms.

There is a definite limitation to the applicability of these formulae and nomograms, although, in general, this can be ignored in practical cases. In developing his formulae, Hertz made the assumption that the surface of contact does not differ materially from a plane, and states in his paper that for this to be true the radius of the surface of contact a must not be greater than $r_1/10$. Applying this limitation means that

$$\frac{r_1}{r_2} < 1 - \frac{W}{21{\cdot}1\ r_1^2}$$

where W is in grammes and r_1 and r_2 are in thousandths of an inch.

This condition is complied with in most practical cases. Taking the case of the watt-hour meter mentioned in the previous section the condition is

$$\frac{r_1}{r_2} < 1 - \frac{17}{21{\cdot}1 \times 20^2} < 0{\cdot}998$$

and the ratio is actually 1/2·4 or 0·416. So the condition is easily met. Again for the indicating instrument, the condition becomes

$$\frac{r_1}{r_2} < 1 - \frac{0{\cdot}2}{21{\cdot}2 \times 1{\cdot}5^2} < 0{\cdot}996$$

and the ratio is actually 1/3·15 or 0·318, so that the condition is

again easily met. It may safely be stated that the conditions would be very unusual for this condition not to be complied with.

It is quite clear that great care is necessary in the design and manufacture of pivots if they are to withstand the great stresses which may be produced. With the heavier types of movement the cone pivot is turned approximately to an angle of 60°, but some manufacturers prefer a cone of smaller angle. An ellipsoidal form of pivot (Fig. 2.1*c*) is also sometimes employed, this being the

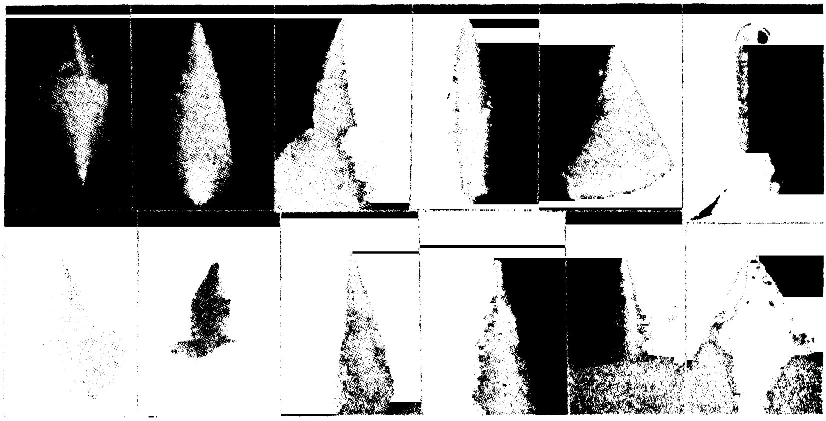


FIG. 2.7.—Photo-micrographs of pivots taken from various commercial indicating instruments.

usual form of agate pivots. Fig. 2.7 shows a set of photo-micrographs of pivots just as they were taken from commercial indicating instruments. They are all photographed to the same magnification, and no attempt was made to clean them before exposure.

Manufacture of Pivots

When a steel spindle is employed the pivots are turned directly on its ends, hardened, and ground true by means of a former wheel of carborundum or fine emery. A rough finish is then given with an arkansas slip and oil, followed by a steel burnisher, or by means of a bell-metal tool charged with tripoli or rouge, followed by a tool of tin with diamantine powder. In all cases, however, the pivot must be brought to its correct form before the final polishing operation, which should not be expected to remove any metal.

The final polishing is effected with putty powder, vienna lime, ground magnesium oxide, or some similar polishing medium served to a hardwood tool.

The inserted pivot is now more commonly used. In this case the pivot is formed out of a short length of the material in the manner described above, or by the process known as rumbling. In this the pivot is ground to shape but left with a fine point, and is then placed in a rotating barrel or rumbler, with a mixture of water and a polishing medium, such as rouge. The rubbing action of

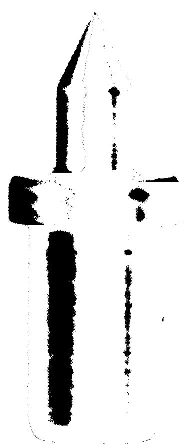


FIG. 2.8.—Completed pivot for insertion in shaft.

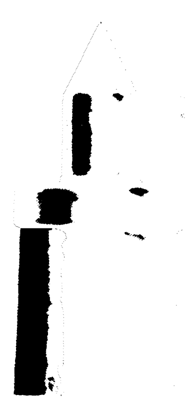


FIG. 2.9.—Pivot inserted in shaft.

the pivots on one another results in a highly polished finish, and at the same time the pointed end is rubbed down to the required radius. The time of the rumbling operation has to be carefully controlled to obtain the right degree of polish and the correct radius on the end. The pivot is inserted in the ends of the shaft, or in a suitable mount, which is drilled for its reception. Fig. 2.8 is a photo-micrograph of a completed pivot of this kind, and Fig. 2.9 the pivot inserted in its shaft. The length of the finished pivot is usually some 2½ times its diameter, the part inserted in the shaft being about 1½ diameters, but in some cases, particularly where the pivot is to be inserted into aluminium, a much longer shank

is allowed. In order to assist in retaining the pivot securely a little groove is turned in the shank, just below the level of the sleeve in which it is inserted, and when the shaft is finished the metal is forced into this groove by means of a hollow punch with rounded edges which rest on the sloping shoulders of the staff, and is given a series of light blows with a hammer.

The difference in radii between the end of the pivot and the jewel cup varies a great deal ; with fairly light movements the radius of the jewel cup is often about four times the radius of the pivot, but with very heavy movements a much smaller difference may be employed.

The material of the pivot should be carefully chosen, for if too hard it may damage the jewel surface and produce pitting and undue wear. If too soft it may deform and produce either an expanded or mushroom tip or become irregular in shape. The material must also be perfectly homogeneous, as otherwise it will chip, or when polished show minute cracks and cavities, which will shorten the life of the bearing. It would seem from the observations of Lieut. Bray that it is essential to make the jewel and pivot of two dissimilar materials which differ in hardness by about two points on Moh's scale of hardness, the softer material not falling below 6 in this scale.

Usually the pivot is of the softer material, and should be tough as well as homogeneous ; it should also be capable of taking a very high polish. Hard drawn compressed carbon steel, chrome steel, tungsten steel and tungsten itself have all been employed, but the alloy steels and tungsten polish indifferently. This may be, however, because the correct polishing medium has yet to be found. The natural alloys, like iridasmine (osmi-iridium), have also been used, but are found to vary very considerably in hardness. Stellite, a cobalt chromium alloy with tungsten and molybdenum, has also given satisfactory results in cases where a steel pivot would not be permissible because of rusting. Stainless steel has also been employed in such cases.

When the movement is heavy and the forces comparatively small, or there is excessive vibration, probably the best construction is to make the bearing entirely of jewels ; but the jewel pivot is difficult and expensive to prepare, and unless very carefully examined during manufacture is liable to defects which result in eventual fracture. Lieut. Bray suggests that the best material for such pivots is cloudy

chalcedony, or the translucent parts of agate selected from regions in the materials which are not strongly banded.

The following is a brief outline, based on Lieut. Bray's work, of the method of manufacture of such pivots. Rectangular blocks of the material are first cut with a lapidary's wheel, and these are individually cemented to a flat chuck or gripped in a special form of chuck with wooden jaws, and are then turned into rough cylinders by means of a grinding wheel run on a special form of slide-rest, which permits of a certain amount of flexibility and personal control.

These roughly-ground cylinders are then placed between two horizontal copper discs rotating in opposite directions at different speeds, between which is a fixed brass plate containing concentric rings of radial rectangular apertures, and each slot accommodates a single jewel cylinder, which is thus constrained to turn about its own axis : the copper discs are charged with moist diamond dust, and kept an accurate distance apart by a micrometer adjustment. The little cylinders are thus ground truly to diameter ; they are then sorted by means of gauge rolls, and each is then rechecked and the pivot formed on its end. The rough forming is done by a profiled carborundum wheel, and corrected by a grooved copper wheel charged with diamond dust, the final shaping being effected with a hard wood profile wheel served with tripoli, great care being taken that during these final stages the tools do not alter with wear.

Polishing may be done by hand, employing a grooved stick of hard wood and putty powder, or better, by means of a multiple spindle polishing machine in which the jewels are each mounted on a vertical spindle rotated at very high speed, whilst the polishing tool is mounted over it, maintained in contact by spring pressure, and is then given a reciprocating motion by means of an eccentric and connecting rod. Throughout the polishing operations, in all cases, scrupulous cleanliness is absolutely essential to success, and the greatest care should be taken to see that the finished pivot presents a perfectly continuous and symmetrical curvature from base to summit, with a surface in a uniform state of high polish and free from pips, cavities, flaws or cracks.

A very similar process to that just described is used in the manufacture of jewel cups described in a later section.

For most commercial instruments at the present time the pivots are made of steel such as Stubbs' steel or silver steel.

JEWELS

The jewel stones are usually some variety of corundum, agate, or diamond. Corundum chemically consists of alumina ($Al_2 O_3$), but in the natural minerals there are usually silica, ferric oxide and water. The mineral occurs in a great variety of colours, and is very largely used for jewellery. The Oriental ruby and the sapphire, particularly the pale blue shades, are the best stones for jewel cups. The sapphire, with the single exception of diamond, is the hardest mineral known, being 9 on Mohs' scale of hardness, and it has a specific gravity of about 4, and is next to diamond in transparency to X-rays.

The red or ruby stones are somewhat softer than the blue sapphire, but are quite suitable stones to employ with steel pivots. Agate is one of the many varieties of quartz, and is made up of a series of minute layers. Almost any of the varieties of chalcedony which are homogeneous and not strongly banded are suitable, but cloudy chalcedony and translucent unbanded agate are best. Flint, jasper, hornstone and chrysoprase are the most troublesome on account of their brittleness, and will crack and splinter very readily during working.

Great advances have been made in recent years in the production of artificial or synthetic sapphires, so much so that all modern instruments are fitted with synthetic sapphire jewels. For miniature instruments, where the moving system is very light, considerable use has been made of toughened glass with some success.

Diamond is the hardest substance known: and is employed in instruments with very heavy movements such as some forms of polyphase watt-hour meters.

Forms of Jewel Cup

In general three forms of jewel are employed in instrument construction, viz. the jewel cup, the ring and endstone, and the pierced jewel, ring stone or jewel hole. In the first class several forms are adopted, as shown in Fig. 2.10. The plain spherical cup (*a*) is undoubtedly the best to manufacture and can be finished very perfectly, but such jewels demand great accuracy in the construction of the rest of the movement, particularly in respect to balance, as there is a considerable tendency for the pivot to "ride up." Better centralization is obtained by using a secondary cup

jewel (*b*), but the life of the pivot is liable to be much shorter in this case, owing to breakdown of the edge between the two cups produced by the pivot riding up, and the collection of dust so produced in the secondary cup and the riding of the pivot over the edge invariably leads to its deformation or fracture. The conical hole jewel (*c*) is an attempt to meet this difficulty. Here the sides of the cup are a true cone, and the bottom is finished to a spherical curvature. Such jewels are very common in indicating instruments, and the dimensions are now standardized in B.S. 904—1940.

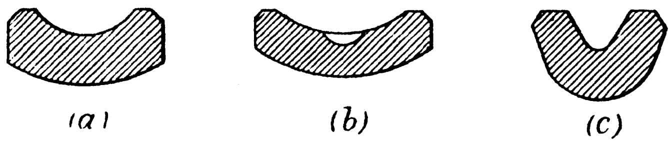


FIG. 2.10.—Forms of jewel cup.

The outer surface of all types of jewel is invariably flat and should be polished, particular attention having been paid to the edge where it joins the surface of the bowl; this edge should be slightly rounded and well polished, so as to minimize the risk of its breaking down and producing a cutting sediment in the cup. The outer edge should be bevelled off so that the rim of the setting may be planished over and hold the stone firmly. The back face of the jewel is sometimes curved, but most often is left flat: in any case the setting should be so machined that the jewel beds properly.

Manufacture of Jewels. Natural Stones

The crystals are examined under a low-power microscope, and only those not showing inclusions, striations and other imperfections should be selected for cutting into plates of suitable thickness by means of a lapidary's wheel charged with diamond dust. In the case of sapphire the plates should then be cleaned and placed in a shallow glass-bottom tank, covered with a solution of methylene iodide having a refractive index of 1·74 (approximately that of the crystal), and each should then be examined under the microscope between crossed Nicols; any imperfections will then show up as dark markings, while rotation between the Nicols will show internal strain. The most uniform stones only should be selected. In the case of agates the immersion liquid should be oil of cloves of refrac-

tive index 1·533. Polishing is done throughout with diamond dust of progressive fineness obtained by the process of elutriation. The early processes are similar to those described for pivots, and the rough discs, after sorting according to size, are checked, and the back face turned with a diamond tool; they are then reversed and the cup rough-ground by means of a hemispherical copper tool charged with diamond dust. This tool is rotated at very high speed from a separate countershaft, and three grindings with successively finer powder are usual. The front surface is then polished with a little rotating copper tool and moist diamond dust. Polishing is best effected by the multiple spindle machine similar to that described for polishing agate pivots, the polisher in this case being hemispherical and made of copper, bell metal, or tin.

The final stages of the polishing should be carefully worked to prevent the formation of ring-like ridges or an undulating surface. The first is produced by a want of variety in the motion of the polisher, and the latter by vibration in, or communicated to, the machine. Care should also be taken that the polisher runs true, as otherwise a pip or bump will be left at the bottom of the cup.

The finished stones should be thoroughly cleansed with alcohol and then examined under a microscope between crossed Nicols; the surface should then be seen in a uniform state of polish and be free from cavities, striations, and the other imperfections mentioned above. Suspected jewels will often reveal their imperfections if the interiors are explored with a finely polished needle used by an operator with a delicate sense of touch.

Ring stones or jewel holes are made in a very similar fashion, the little plates of jewel being chucked after being ground flat, and the hole drilled half-way through from each face, the polishing being effected by a pointed copper tool. Fig. 2.11 shows a photo-micrograph of both surfaces of such a jewel, and it will be seen that on one side the stone has a parting plane, which just escapes making the jewel useless. The endstone jewel shown in Fig. 2.1*d* is used for fine work employing parallel pivots; the two components are set independently and the settings assembled as shown, the two stones being slightly separated to assist in the retention of the lubricant when this is used. Some makers finish the inner face of the endstone to a slight curvature, but on the whole a perfectly flat surface gives the best results. This type of jewel is usually cupped on the side towards the shoulder of the pivot. On no account

should jewels be stored in a loose condition, as they are very liable to abrade one another, and a really successful jewel should have a

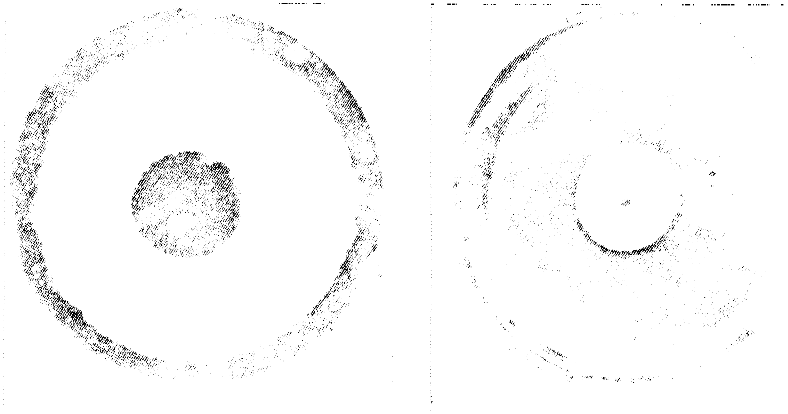


FIG. 2.11.—Photo-micrograph of ring jewel or jewel hole.

continuously polished surface. Shallow trays with a separate recess for each jewel is the best method of storing, or failing this, each should be separately enclosed in soft paper.

Synthetic Sapphire

The earliest attempts to make synthetic sapphire appear to have been made by Gaudin in 1837, but his experiments were not very satisfactory, and the modern method is due to Verneuil, who published a description in 1904. His process, modified only in detail, is in use to-day. The sapphire is made from aluminium oxide, which is obtained from the ignition of ammonia alum of a suitable degree of purity. The process used for the formation of the sapphire "boule" consists essentially in passing this powdered aluminium oxide through an oxy-hydrogen flame, fusing it, and then allowing to solidify and build up like a stalagmite on a refractory support.

A diagram of the apparatus is given in Fig. 2.12. Oxygen is fed into a hopper *A* which extends downwards by a vertical tube *B*, ending in a jet. A larger tube *C* surround this tube *B*, and hydrogen is fed into *C* so that the combination of *B* and *C* forms an oxy-hydrogen blow-pipe. The flame is directed downwards into the cylindrical central hole through the refractory brick *D*, which is split into two halves vertically, for convenience in handling. A refractory member *E* projects up into the furnace so formed, and is supported

on a table F; this can be raised or lowered by turning a handwheel. Fine alumina powder is placed in the suspended canister G, closed at its lower end by a wire gauge cap. The burner is lighted and the small hammer H started so that it is lifted and allowed to stroke the peg beneath at 2- or 3-second intervals. The canister G is suspended from a flexible diaphragm, and at each blow of the hammer a small amount of powder falls through the gauge, and is carried down in the oxygen stream. The alumina is melted in passing through the flame and is caught on the top of the member E, which is in a cooler part of the furnace. Here a cone of sintered materials builds up, and as the temperature rises its apex just melts and forms a tiny spherical globule which grows upwards as more powder falls. As the globule grows in height the lower part is screened from the flame and solidified; the sphere becomes a stem whose tip may be made to spread by supplying more heat until it resembles a mushroom.

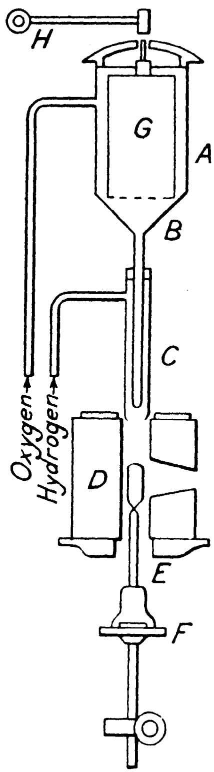


FIG. 2.12.—Verneuil synthetic sapphire furnace.

After a time the increasing heat supply is checked and the boule grows upwards without further increase in diameter. Finally the powder feed is stopped, the gas supply is cut off, and the boule is allowed to cool in the furnace. A convenient size of boule is about 50–60 mm. in length, 20–25 mm. in diameter, and weighs about 300 carats; this requires about 4 to 5 hours to grow. After removing the cone of fritted powder, a sharp blow on the stem will usually split the boule into roughly symmetrical halves, the flat faces of which are essentially plane. These half boules are in a convenient form for the subsequent operation of cutting into jewels. The general shape of the boule is a right cylinder, but occasionally a tendency to a hexagonal cross-section is noticed.

Pure alumina powder is generally used, giving a "white" sapphire for instrument jewels and other industrial purposes, but coloured stones, rubies and blue sapphires may be made by incorporating the appropriate substances with the alumina.

The boules are cut up into jewels and then polished in much the same way as already described for natural stones.

Jewel Screws.

When assembling the instrument, the jewel screw is adjusted in the bridge-piece to such a distance that it will hold the jewel spindle truly and without appreciable end-shake, for any undue amount of slackness of the movement in its jewels will shorten their life and lead to rapid deterioration of the pivots.

With instruments having a horizontal axis this adjustment requires very special attention, for a small amount of end-play must necessarily be allowed on account of thermal expansion. It is here that the inserted pivot and brass staff has some advantage, since the spindle and jewel bracket are then of the same material and expand together ; but the compensation cannot be perfect, owing to the enormous difference in heat capacity of the two parts. Again, where individual pivots are used, as in moving-coil instruments, both expansion and warping of the coil will give trouble, and this is an argument for the employment of a continuous spindle through the coil.

On account of these difficulties usually far too much end-play is permitted, and the effect is then one of a badly-designed roller bearing, often with very variable friction, and a good parallel pivot and ring stone would give better results. For this reason, therefore, the spring supported jewel sometimes used with inwardly projecting pivots in moving-coil instruments is to be preferred, since in this case the jewels are mounted to slide in a central hole in the core, and are urged forward by a spring between them. Expansions can then only have the effect of varying pivot pressure, and the correct adjustment can be made with great delicacy. A modification of this scheme has been employed in which the two jewels are carried in the ends of a single staff which is supported by springs in the centre iron core. It is claimed that this construction gives greater protection to the jewel under severe shock.

To facilitate adjustment and reduce the risk of accidental fracture, the jewel screw should be of ample dimensions, and should be threaded with a screw of fine pitch so as to provide a slow and delicate motion. After adjustment the jewels must be locked in position by a locknut running on the thread, which is eventually tightened against the cheeks of the bridge-piece (Fig. 2.1*a*). Not infrequently the ad-

justment is made on one jewel only, the second or back jewel being set in a hole in a small metal disc which is screwed permanently into place (Fig. 2.1*b*) in the back support. The final adjustment and locking of the jewels demands considerable skill and patience on the part of the operator, for if it is not skilfully carried out the spindle will be left with too much end-play, or on the other hand, the jewels may be so tight as to compress and destroy the pivots or fracture themselves. The Weston Instrument Company have reduced the difficulty of the locking operation by the employment of the split locknut shown in Fig. 2.13. After adjustment this nut is screwed

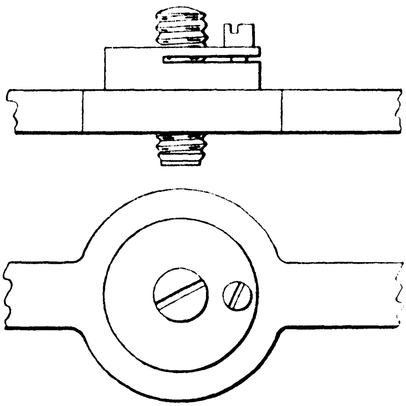


FIG. 2.13.—Weston split lock nut.

into contact with the support, and when in position is locked by tightening the cap screw, which has the effect of causing the nut to grip the thread of the screw without in any way affecting its position. Another method, found in some continental instruments where a bracket support is used, is to split the ends of the bracket with a saw-cut right through the screwed hole and then tighten in the screw by means of a transverse, pinching screw.

The life of the jewel bearing is a very variable quantity, a great deal depending on the type of pivot, the hardness of the materials, the adjustment, the amount of mechanical vibration and the original cleanliness in assembly. If the stone has been properly selected and carefully polished, and is of the correct form and hardness in a stationary instrument, the life is practically indefinitely long. The authors have examined pivots and jewels of instruments

after twenty years of service, and found them in perfect condition; on the other hand, they have found bearings useless in a few months in some other instruments. The examination of one switchboard instrument, for instance, revealed the fact that no jewels are employed at all, the bearing being simply a pivot bearing in a polished cup turned in the end of a phosphor bronze spindle. Such a bearing has nothing to recommend it, and necessarily has a short efficient life, since the friction increases very rapidly with age.

In this connection the photo-micrographs shown in Fig. 2.14 are of interest: they show a pivot and jewel from a mechanism actuating

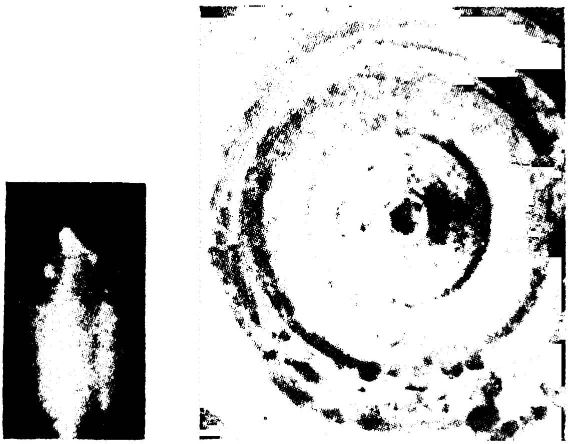


FIG. 2.14.—Photo-micrograph of pivot and jewel which has survived more than 200×10^6 oscillations.

the seconds contact of an electrically maintained astronomical clock. It is estimated that this bearing has survived more than 200×10^6 oscillations produced by an applied load of approximately one gramme. The corresponding front bearing failed through fracture of the pivot and complete destruction of the front jewel.

A not infrequent fault is that of the two pivots or the two jewels not being truly on the axis of rotation: in this case if the movement is not set very slack in its jewels it will ride up the jewel cups and jam; so it is left slack, with the result that vibration soon leads to a fractured jewel (like that shown as E in Fig. 2.15), or to broken or deformed pivots and a large friction error. Another fruitful source of trouble is dust, which may in part be due to the case not being dust-proof, to a want of cleanliness during assembly, and to

mutual abrasion of pivot and jewel due to imperfect workmanship in one or both. The solid matter collects in the cup and grinds the pivot, which is usually the chief sufferer; but at the same time the perfect polish of the jewel is impaired. Once started the evil is cumulative, and eventually leads to a large friction error.

Continual and small impact will also shorten the life of the bearing, and for this reason the spring jewel should be used where the movement is heavy and vibration is present. Such bearings are illustrated

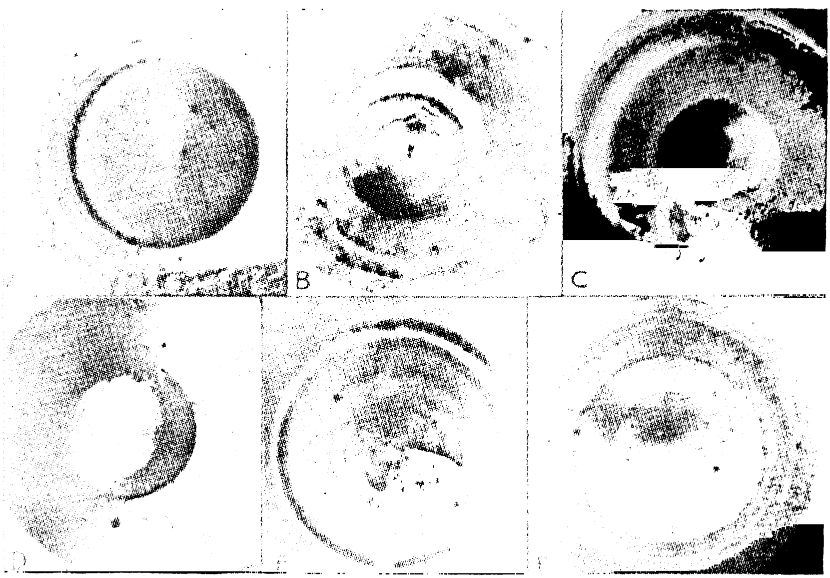


FIG. 2.15.—Photo-micrographs of jewel surfaces from indicating instruments.

in Part II, but they necessarily increase the depth of the instrument in most cases. It is probable that the majority of failures are due primarily to impact from an imperfectly adjusted pivot staff rather than from actual wear.

The life of a sapphire jewel may run into millions of revolutions if it is correctly assembled and is not subjected to any serious impact forces. In Fig. 2.15 are shown some photo-micrographs of jewel surfaces taken from commercial instruments.

The frictional load on the bearings of an instrument may be reduced to quite a negligible quantity by magnetic suspension, but hitherto such methods of support have been practically confined to

supply meters, and therefore discussed in the Chapter on Electric Supply Meters in Part II.

The Optic Axial Angle and Lubrication

Investigations by Shotter have brought to light the fact that the angle between the direction of the applied load and the optic axis of a sapphire jewel has a very important influence on the life of a bearing. The sapphire jewel has a crystal structure, and is characterized by a series of parallel cleavage planes, the optic axis of the

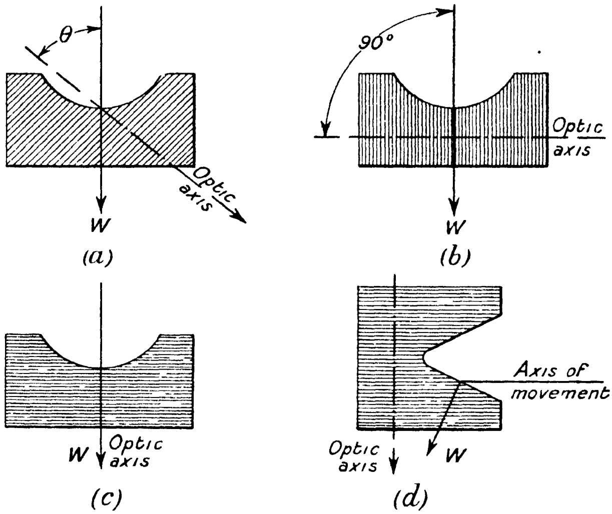


FIG. 2.16.—Diagrams showing optic axis and cleavage planes of jewels.

jewel being at right angles to these planes. In Fig. 2.16 the shading lines indicate the cleavage planes. Consider first the case of a watt-hour meter or an indicating instrument used with a vertical spindle. Generally the conditions will be as shown in Fig. 2.17*a*, in which there is an angle θ between the direction of the applied load W and the optic axis. Shotter has found that very prolonged life can be obtained if this angle θ is 90° as in (*b*), and a very short life if θ is 0° as in (*c*). For instruments with vertical spindles, then, the jewels should be chosen to have an optical axial angle of as nearly 90° as possible. Some tolerance is permissible, and a range from 85–90° should be satisfactory.

This angle can be determined roughly by examining a jewel with a microscope, the jewel being placed between crossed Nicols. Distinctive patterns are obtained for various angles, and a sketch of these patterns obtained with spherical cup jewels is given in Fig. 2.17.

The case of the instrument with a horizontal spindle and cone jewels is rather more difficult. This is illustrated in Fig. 2.16*d*. Owing to the necessity of leaving a slight amount of end-play in the movement assembly the pivot will rest on the side of the cone, and

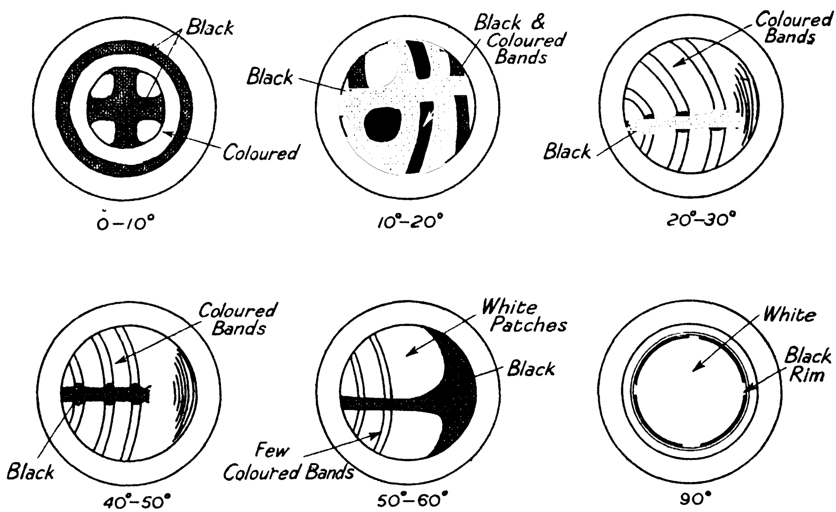


FIG. 2.17.—Optic axis patterns of special cup jewels.

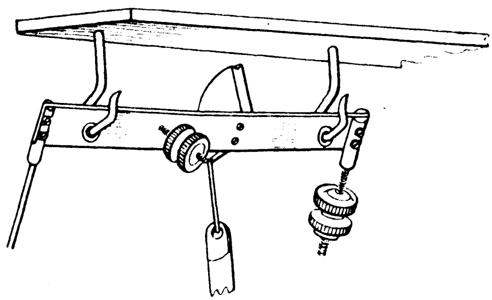
the direction of the applied load will be at right angles to the surface of the cone at the point of contact. Theoretically, to obtain the maximum life from the bearing the optic axis should be at right angles to this direction of load. It is quite evident, however, that if the jewel is rotated this condition is lost, and all jewels are rotated during the adjustment of the jewel screws. It is a practical impossibility therefore in this case to control the optic axial angle.

Shotter has also found that lubrication of the pivot with the right kind of oil has a marked effect in lengthening the life of a bearing. The Pennsylvanian oils are best for this purpose, and with this oil even a meter jewel with a 0° optic axial angle can be given a long life.

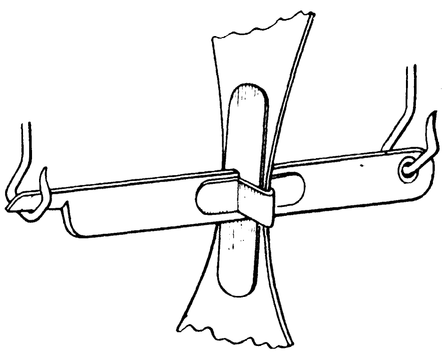
SUSPENSIONS

Knife Edge Suspensions

The second method (*b*) of suspension has many advantages, but its application is, of course, limited to vertical-scale instruments. When carefully designed and accurately made it is very satisfactory,



(*a*)



(*b*)

FIG. 2.18*a*, *b*.—Forms of knife-edge suspensions.

but comparatively insignificant errors in workmanship will lead to very imperfect results.

This form of suspension was adopted in several of the Kelvin instruments, where the simple construction shown in Fig. 2.18 was employed. The knife edges are formed in a thin metal bar, and the

planes on which they rest are in the form of sharply bent hooks. In order to allow for longitudinal expansion and yet keep the movement in the same relative position, the front knife edge in some cases is formed by cutting away the bottom of the bar, leaving a comparatively sharp, straight edge in the axis of rotation ; the other edge takes the form of a circular hole drilled through the bar and countersunk on each side, the extreme top edge of the hole being in the same straight line or axis as the front edge. Thus the movement is prevented from shifting either by expansion or mechanical disturbance, and yet is free to expand longitudinally (Fig. 2.22*b*).

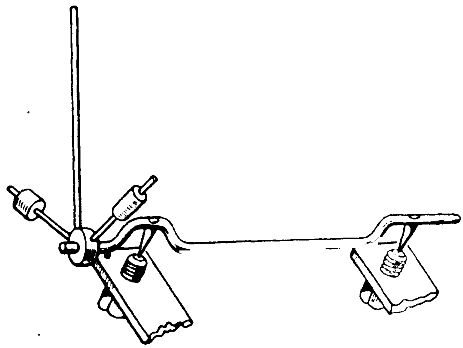


FIG. 2.19.—Modified suspension with vertical needle-points.

It is essential in carrying out this scheme that the knife edges and the planes on which they rest shall all be in the same straight line, laterally and vertically, and that the surfaces must all be truly worked and hardened, or else the movement will take up different positions and its indications be seriously affected at each deflection. A modification of this type of suspension consists of two vertical pivots bearing in jewel cups ; the movement thus hangs from the cups (Fig. 2.19). The same careful adjustment is necessary here as with knife edges, but much greater care is necessary in setting the pivots so that they bear properly in the jewels and do not tend to ride up. Even under the best circumstances thermal expansions invariably lead to trouble, as they cannot be sufficiently accurately compensated, and, moreover, the pivot is less robust than the knife edge, and therefore such a method of support is inadmissible where there is any mechanical vibration.

Filar Suspensions

The application of this type of suspension to commercial instruments is comparatively limited, since its employment necessarily involves accurate levelling, and the axis of the instrument must be vertical. It must also be adequately protected against mechanical shock. Where, however, the deflecting forces are small and the mass to be supported large, the method can be employed to advantage, as, for instance, in the case of low-reading electrostatic instruments; friction can thus be entirely eliminated, and, if a metallic suspension is used, it can form the connection between the moving system and the terminal.

In practice a fine metallic wire or strip is generally employed, of some hard metal or alloy like hard-drawn silver, platinum, phosphor bronze or platinum iridium and tungsten. Round wires, 0·025 to 0·06 mm. in diameter, are often used, or thin strip rolled or drawn from wires of suitable gauge may be employed. A round wire of phosphor bronze, 0·06 mm. in diameter, will support about 250 gm. before it breaks, and has a torsion constant of 5 dyne-cm. per radian per cm. length. A strip of the same material 0·12 mm. by 0·008 mm. will support between 50 and 60 gm. before it breaks, and has a torsion constant of 0·062 dyne-cm. per radian per cm. A phosphor bronze wire 0·108 mm. in diameter and a strip 0·3 by 0·05 mm. have approximately the same torsion constant, viz. 55–60 dyne-cm. per radian per cm.; the breaking load of the former is between 500 and 600 gm., and of the latter, 1,500 gm.

It is possible to obtain ductile tungsten in the form of wire 0·02 mm. in diameter. The material appears to have excellent qualities for the purpose of suspension and is very strong. It is, however, not very satisfactory to solder, and can therefore only be used where clamping into position is permissible.

In all cases the suspension material should be carefully annealed, since the elasticity of the suspension is of the greatest importance, and when soldering the wire to its attachments, the greatest care should be taken that it does not become unduly heated beyond the point where it leaves the support, as this may impair its elastic quality at the most critical position in the suspension. It is therefore usual to employ solders of low melting-point for the purpose. Alloys containing lead, tin, bismuth and sometimes cadmium are employed, the proportions being adjusted to give the required melting-point. The wire must leave the support cleanly

and axially, and a good workman can often effect this by putting a slight tension on the thread just at the moment when the solder is solidifying. In delicate, low-reading instruments the viscous yield of the solder may lead to some uncertainty of zero, and this can be partially overcome by using clamps over the soldered joint which grip the wire where it leaves the support, thus exactly defining the working length of the suspension.

To guard against fracture from accidental shock, which may easily happen when large and massive movements have to be

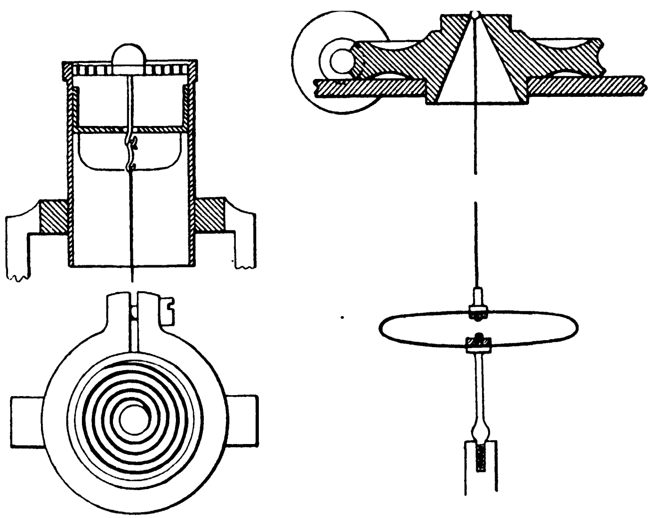


FIG. 2.20.—Methods of protecting filar suspensions against mechanical shock.

suspended, it is usual to terminate the suspension at one end on a spring which, by reason of its resilience, takes up the disturbance. In Fig. 2.20 are shown methods of carrying out this scheme in practice, and in those cases in which the movement is strung between a top and bottom strip the spring has the additional advantage of keeping the suspensions taut under a gentle tension. With long suspensions temperature expansions may become troublesome, particularly when the clearance between moving and fixed systems is very small, as in some delicate electrostatic instruments, where the elongation of the suspension will cause a change in constant. For instance, in the case of the electrostatic watt-meter, Miles Walker has shown that if d is the displacement of the movement due to expansion

from its position of vertical symmetry, and t is the clearance, the rate of change of the constant is proportional to $4d/t^3$. A further discussion of metallic and other filar suspensions will be found in a subsequent chapter on galvanometers.

SYSTEMS OF CONTROL

In practically all forms of commercial instruments the restoring couple is applied in one of two ways,* viz. :—

(*a*) By means of a coiled spring or torsion of a wire.

(*b*) By gravity acting on suitably placed weights on the movement.

The contrast of the two types of control is shown in Fig. 2.21. The spring or filar control gives a torque which is nearly proportional

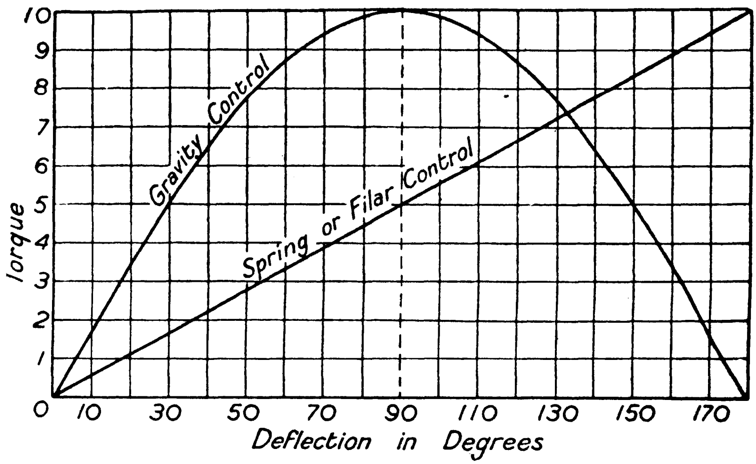


FIG. 2.21.—Curve showing comparison of systems of control.

to the deflection over the whole working range, while gravity control is proportional to the sine of the deflection, and reaches a maximum at 90°, falling away to zero as the deflection nears 180°.

Springs

In the first method (*a*) the deflecting forces, in moving the pointer over its scale, distort a spring or twist a wire, and on the removal

* Magnetic control has been employed only in a few special cases, and has approximately the same laws as gravity control, with the advantage of independence of position.

of these forces the spring returns to its initial form and thus brings the indicator back to its original position. The spring may be either flat like the hairspring of a watch, or in the form of a long helix. As the first type reduces the necessary depth of the case to a minimum it is most generally adopted in commercial instruments.

In order that the action shall follow a proportional law the number of convolutions must be large, or, in other words, the deformation per unit length of the spring material must be small, while the area of cross-section should be so chosen that with a reasonable length it will give the required restoring torque.

The long helical spring is almost universally employed in suspended torsion instruments such as Siemens Dynamometers and Standard Wattmeters, since they have the advantage of being more truly proportional and regular in their behaviour.

Torque in Cylindrical Springs

The theory of cylindrical springs is more easily worked out and will be dealt with first. If T be the total torque applied to the axis in dyne-centimetres, then it is evident that the tension (or compression) at every point of the spring $F = T/a$, where a is the radius of curvature, and that this produces a bending moment M at the perpendicular point of the spring equal to Fa, so that $M = \frac{T}{a} . a = T$. It is evident from symmetry that this is the same at all points of the spring. If we suppose that the axial pitch of the spring is small compared with its diameter, the obliquity may be neglected, and the effect of the torque is simply to alter the curvature of the spring. Fig. 2.22 shows a segment of a spring, the full lines repre-

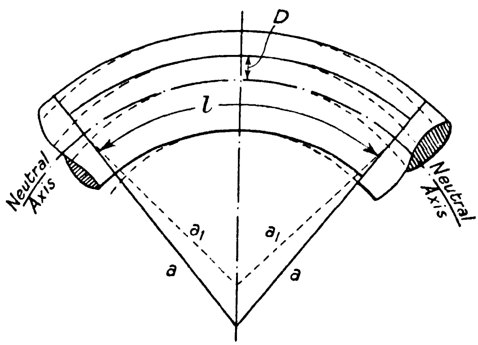


FIG. 2.22.—Segment of spring to illustrate theory of bending.

senting it in its unstrained state, and the dotted lines in its strained state. If the spring is twisted so as to tighten it the result, as seen above, is to apply a bending movement $M = T$ at every part, tending to curve it more than before. This means that the inner part of the material is compressed and the outer extended, and somewhere between these two there is a "neutral axis," where it is neither compressed nor extended.

Let a be the radius from the centre of the spring to this neutral axis. Then if we consider a zone at a distance D from the neutral axis it is evident that its length is $\frac{a+D}{a} l = \left(1 + \frac{D}{a}\right) l$. Now suppose that the twist is applied and that the spring is curved to a radius a_1, the zone will now have a length $\left(1 + \frac{D}{a_1}\right) l$, or it will have increased in length by the amount $\left(1 + \frac{D}{a_1}\right) l - \left(1 + \frac{D}{a}\right) l = D\left(\frac{1}{a_1} - \frac{1}{a}\right) l$, hence the strain or fractional increase in length is $D\left(\frac{1}{a_1} - \frac{1}{a}\right) l \div l = D\left(\frac{1}{a_1} - \frac{1}{a}\right)$, i.e $D \times$ change in curvature. But by Hook's Law stress $S = E \times$ strain $= ED\left(\frac{1}{a_1} - \frac{1}{a}\right)$ where E is the coefficient of elasticity of the material. Hence the whole force or tension over the area of section of the spring $F = \int S dA = E\left(\frac{1}{a_1} - \frac{1}{a}\right) \int D dA$. In light springs such as are used in instruments the tension is negligibly small compared with forces of bending, and we may therefore take $\int D.dA$ as zero. This means that the moment of the area about the neutral axis is zero, or that the neutral axis is at the mass centre of the area.

Next, if S is the stress and $S.dA$ the force on the element of section dA, then $S.dA \times D$ is the moment of this force, and the total moment $= \int S\, D\, dA$. This must balance the bending moment M.

Hence

$$M = \int S\, D\, dA = E\left(\frac{1}{a_1} - \frac{1}{a}\right) \int D^2 dA.$$

But $\int D^2 dA = I$, the moment of inertia of the section about its mass centre, so that

$$M = EI\left(\frac{1}{a_1} - \frac{1}{a}\right) \text{ and } \frac{1}{a_1} - \frac{1}{a} = \frac{M}{EI} = \frac{T}{EI}.$$

Suppose that the spring has initially a total angular length γ radians, so that its actual length $l = a\gamma$ and $\gamma = l/a$, then after twisting it will have a total angle $\gamma + \beta = l/a_1$. Hence $\beta = \frac{l}{a_1} - \frac{l}{a} = l\left(\frac{1}{a_1} - \frac{1}{a}\right) = \frac{Tl}{EI}$ radians, and the torsion constant of the spring $k = \frac{T}{\beta} = \frac{EI}{l}$. If the spring is made of flat strip of breadth b and thickness x, the moment of inertia $I = \frac{1}{12} bx^3$ and $k = \frac{bx^3E}{12l}$.

Spiral Springs

To calculate accurately the torque of a spiral spring is a difficult matter, as the curvature varies continuously along its length. For an ordinary spiral (the Archimedean spiral) the form is represented by the expression $r = a + \frac{D}{2\pi}\beta$, where a is the initial radius, D the distance between the turns. For our purpose, however, a simple approximation may be made by considering the spring as made up of successive circles. For each of these the expression $\beta = \frac{T}{EI} . l$ holds, hence for the whole spring $\beta = \frac{T}{EI} . l$ where β is the total angular deflection and l the total length of the spring. If a_i is the inner radius, and a_o its outer radius, γ being its total angular length, then it is evident that $l = \frac{1}{2}(a_i + a_o)\gamma$ approximately, so that

$$\beta = \frac{(a_i + a_o)\gamma}{2\,EI} . T \text{ and } k = \frac{T}{\beta} = \frac{2\,EI}{(a_i + a_o)\gamma}.$$

It will therefore be seen from the above that for the same section and length of strip the torsion constants of a cylindrical, and of a spiral spring are approximately the same.

Conditions for Perfect Elasticity

In order that the spring shall have no permanent set, the maximum stress S_m due to bending should not exceed a certain value.
But

$$S_m = E\,D_{max}\left\{\frac{1}{a_1} - \frac{1}{a}\right\} = \frac{E\beta}{l}\,D_{max},$$

from which

$$\frac{l}{D_{max}} = \frac{E}{S_m}\,\beta$$

If x is the thickness of the flat strip, or the diameter of the round wire used for the spring, $D_{max} = x/2$ and

$$\frac{l}{x} = \frac{E}{S_m}\cdot\frac{\beta}{2}$$

For phosphor bronze, which is commonly used for electrical instrument springs, E is about 12×10^8 gm. per sq. cm., and S has been given as 6×10^5 gm. per sq. cm., from which $E/S = 2{,}000$ and $l/x = 1{,}000\beta$. If $\beta = \pi/2$, as in a large number of deflectional instruments, l/x is approximately 1,500, or the length of the spring should be at least 1,500 times its radial thickness. In good instruments the ratio l/x is more nearly 3,000, so that S_m is only about 3×10^5 gm. per sq. cm.

As an example, suppose we require a spring for a soft-iron instrument having a torque of 0·1 gm.-cm., or say 100 dyne-cm., for a deflection of 90°, we then have

$$k = \frac{T}{\beta} = \frac{EI}{l}$$

from which

$$x^3 = \frac{12\,l\,T}{b\,E\,\beta}$$

Also

$$x = \frac{2\,l\,S_m}{E\beta} \text{ from above}$$

Hence, dividing, we get

$$x^2 = \frac{6T}{bS_m}$$

It is desirable both for lateral stability and for conductivity (when necessary) for b to be large compared with x. For a spring

of this kind b may conveniently be 1 mm., or 0·1 cm., and if we take S_m as 3×10^5 gm., or 3×10^8 dynes per sq., cm.,

$$x = \sqrt{\frac{6T}{bS_m}} = \sqrt{\frac{6 \times 100}{0{\cdot}1 \times 3 \times 10^8}} = 0{\cdot}0045 \text{ cm.}, = 1{\cdot}8 \text{ mils nearly}$$

and

$$l = \frac{b\,E\,\beta\,x^3}{12\,T} = \frac{0{\cdot}1 \times 12 \times 10^{11} \times 0{\cdot}0045^3 \times \frac{\pi}{2}}{12 \times 100} = 14{\cdot}3 \text{ cm.}$$

If we made the inner radius of the spring 2 mm., and the outer 7 mm., the mean radius is 0·45 cm., and the mean circumference 2·8 cm., from which the spring would have 5 turns with a pitch of 1·0 mm. between the spirals. For moving-coil instruments, where high sensitivity is desirable, the breadth of the spring should be 2 to 2·5 mm. The spring material is usually non-magnetic, and in most cases necessarily so. Phosphor bronze, hard rolled silver or copper, platinum silver, platinum iridium and German silver have been employed, but phosphor bronze is by far the most general. Use is also being made in some modern instruments of a copper tin alloy and beryllium copper. The so-called Paillard's alloy, which consists of a large proportion of palladium, with varying quantities of copper, nickel, gold, silver, platinum, and iron, has been employed with success by watchmakers and others for non-magnetic hair springs.

Flat spring material is usually rolled and drawn from wire, of such diameter that it will give the required breadth ; the process of drawing and rolling gives the material great hardness. Long helical springs are usually constructed from round wire by winding tightly and uniformly on a mandrel of suitable diameter, and where tempering is resorted to, this is best done while the material is still on the mandrel. The electrical conductivity should be as high as possible where the spring is used to conduct the current through the coil of the instrument, particularly in the case of D.C. milli-voltmeters, where the amount of dead resistance which they offer is serious, and the changes in elasticity, due to temperature change through ohmic heating, lead to troublesome zero changes. This may be compensated by employing two similar springs wound in opposite directions (see Fig. 2.23), but it is better to reduce as far as possible the causes of temperature change by employing silver or gold ligaments, exerting little or negligible control on the movement for leading the current in and out of the coil ; thus the current heating

of the spring is eliminated, and it is left with only the duty of providing the controlling force. The elasticity of the average phosphor bronze spring used in commercial instruments has a temperature coefficient of about $-0{\cdot}04\%$ per $^\circ$ C. (Janus gives the value $-0{\cdot}038\%$). For German silver, which is sometimes employed for helical springs in torsion instruments, the value is about $0{\cdot}03\%$ per $^\circ$ C.

The elastic fatigue exhibited by some springs is very difficult to eliminate, and is usually detected by the tendency of the pointer to creep up to too high a reading when the instrument is deflected

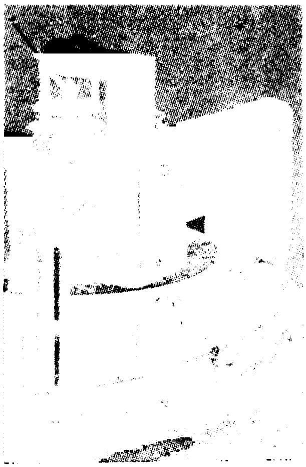


FIG. 2.23.—Double spring arrangement applied to an induction instrument.

for a considerable time, and to show too high a zero on switching off; resetting the zero will not compensate for this, for on further rest the spring will regain its normal form and the zero is therefore indefinite. Proper initial treatment of the material so as to bring it into the required state of permanent elasticity is the only remedy; this may be accomplished by annealing and subsequently artificially ageing the spring by repeated deflection and release on a suitable clockwork mechanism, at first with gentle heating, which is gradually reduced as permanency is approached. The spring constant may be measured in several ways: we may, for instance, attach the inner end to a freely pivoted axle, which carries at right angles to its axis a lever arm arranged steelyard fashion, so that a jockey weight can be set at any desired distance from the axis of rotation;

this lever when in its position of equilibrium is over a fixed index. The outer end of the spring is attached to a pin, suitably placed in a graduated disc, capable of hand rotation, concentrically with the axis of the balance lever, and the arc through which the outer end of the spring must be turned in order to balance the leverage of the balance weight on the pivoted lever is read off on the scale engraved on the outer edge of the disc.

Then if l is the length of the balance arm from the axis of rotation to the centre of mass of the jockey weight, w the weight of the jockey, and β the angular displacement of the outer end of the spring required to restore equilibrium, the torque in gm.-cm. per unit angle or torsion constant of the spring is

$$k = \frac{wl}{\beta}$$

A second method consists of attaching the inner end of the spring to the axle of a delicately pivoted small disc flywheel and the outer end to a fixed post. The disc is then given a twist of about a quarter revolution, and then allowed to oscillate freely, the periodic time being observed. Then if t is the time of a complete oscillation,

$$t = 2\pi\sqrt{\frac{\text{angular displacement}}{\text{angular acceleration}}} = 2\pi\sqrt{\frac{w}{T/I}}$$

If w is the mass of the disc and a its radius, then $I = wa^2/2$. In other cases the torque may be determined directly with great accuracy by electrical measurements, as will be explained later.

The manufacture of spiral springs may be carried out in several ways, which are mostly based on watchmakers' practice in the construction of hair springs. Fundamentally this consists of winding the flat strip into a shallow box whose depth is the same as the breadth of the spring material, and whose diameter is such that it will just contain the tightly coiled strip. The box is provided with a set of tangential openings in its sides, through which the strips are fed, and the central winder, with a corresponding set of slits for the reception of the inner ends, is arranged to turn in bearings formed in the bottom and cover of the box. Two, three, four or even more strips, each from a separate opening in the side of the box, are wound in at once, the number depending on the desired pitch of the spiral, until the box is quite filled. Steel springs are heated and quenched before being removed, and phosphor bronze or hard

alloy springs are tempered by heating in a non-oxidizing atmosphere such as coal gas, or in a bath of hot oil. Various forms of machine to facilitate the manufacture based upon this principle have been designed.

The attachment of the spring to the rotating element and the fixed tongue must be made in such a manner that when deflected the spring is coiled symmetrically. For good proportionality, therefore, it is advisable to avoid bending the spring material, and to make the cross-arm or collet in the rotating spindle of such dimensions that the inner end of the spring leaves its point of attachment there smoothly and tangentially, and similarly the outer end should meet the fixed tongue without requiring to be deformed. The serious bending of the ends so often found in spring-controlled instruments is usually responsible for the irregularity of the scale which frequently occurs in such instruments. As much care is required in soldering to the attachments as is required in the case of filar suspensions to avoid over-heating.

With long helical springs it is, of course, essential that when fixed in position they should be truly axial and that their ends leave the points of attachment tangentially, otherwise the law, with various positions of the torsion head or movement, may be seriously altered.

Gravity Control

The second method of control, by gravity, has been widely used, since it materially cheapens construction, and does not demand such high skill in workmanship and adjustment.

In this method weights are so disposed about the axis of rotation that, while they normally balance the mass of the movement, the deflecting force has to lift them against the action of gravity. It is therefore obvious that an evenly divided scale cannot be expected under ordinary circumstances, since the lifted weight will exert a torque proportional to its mass and to the sine of the angle through which it is deflected (Fig. 2.21).

The weights are usually arranged in two sets to facilitate adjustments, one serving for zero setting, the other so placed that its movement in or out from the axis of rotation alters the sensibility without modifying the zero. This arrangement is shown in Fig. 2.24, where the zero weights are lettered Z, and the sensibility weights S. The adjustments are effected by mounting the weights as little screwed nuts running on a finely threaded cross-arm, so

that they may be screwed in or out to the required position. Usually each weight is in two parts, so as to act as lock-nuts one on the other when the final position is attained, or the nut may be provided with a long screwed sleeve, slotted along its length, which grips the thread on the cross-arm securely. In poorly constructed instruments locking is obtained by dropping shellac varnish on the screw thread where it enters the nut, or by soldering the outer end to the arm. The correct proportions of the control weights cannot be settled by any general rule, for we have to remember that the life of the bearing is dependent on the load it carries; if, therefore, we endeavour to reduce the load to a minimum by using a small weight

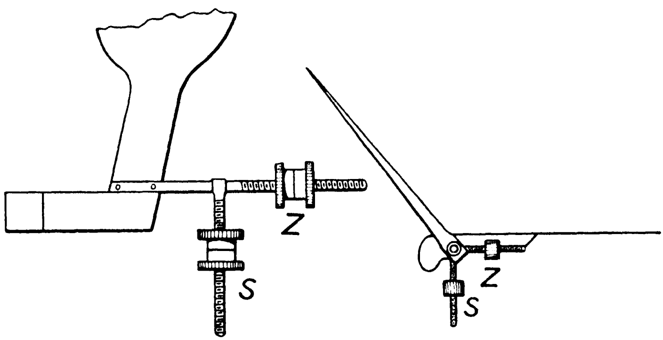


FIG. 2.24.—Arrangement of weights for gravity-controlled instruments.

at a considerable radius, we are confronted with the trouble that the inertia, which is proportional to the square of the radius, will be unduly increased, and thus demand heavier damping or a reduction in torque. A compromise has therefore to be made between mass and radius.

Probably the most satisfactory procedure is to start the design from the electrical requirements and settle the available torque and weight of the essential parts. The pivots can then be proportioned, as indicated in the early part of this chapter, and then the control so arranged as to have the minimum movement of inertia allowable to give a small enough friction error.

Balancing

It is essential that the mechanical balance of the system should be as perfect as possible, so that the centre of gravity of the movement

is always in the axis of rotation for all positions of the instrument, and when this is achieved the indications of spring-controlled movements will be independent of position, and the wear on the bearings will be uniform and symmetrical. In the cheaper types of gravity-controlled instrument this is approximately attained by suitably disposing the zero and sensibility so that they also compensate for the mass of the pointer, damping vane, etc. Where finer balance is desired the axis carries two screwed cross-arms with threaded nuts arranged at right angles to one another, as in Fig.

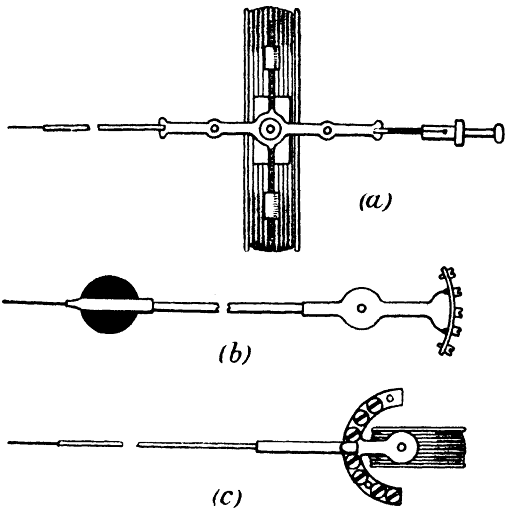


FIG. 2.25.—Methods of balancing movements.

2.25*a*. A modification of this, employed in cheaper instruments, consists of replacing the screwed nuts with wire which is tightly coiled round the arms, sufficient being wound on to secure balance, and after the adjustment is completed the spiral is often fixed by means of a drop of shellac varnish.

Another scheme is to prolong the pointer axis on the other side of the pivot staff, and provide it with a little metal arc into which small screws may be inserted and the required balance secured in the same way as a watchmaker adjusts the balance of a watch (Fig. 2.25*b* and *c*). The barbarous method adopted by some instrument makers of adjusting and balancing by means of masses

of half-cold solder more or less attached to the arms and weights of the moving system cannot be too strongly condemned as inaccurate, inefficient and unsightly.

The V type of balance is shown in Fig. 2.26*a*, and is an excellent and simple device which has been employed in some of the best

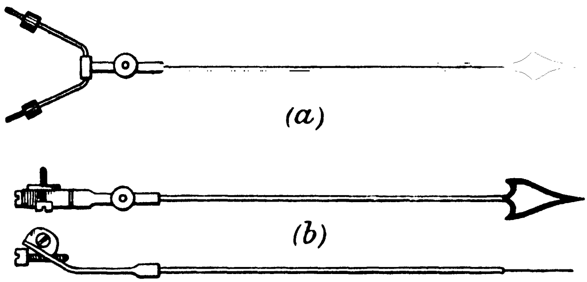


FIG. 2.26.—Two simple methods of balancing used in Continental instruments.

continental instruments, while Fig. 2.26*b* is a simple form of balancing which was fitted to some of the A.E.G. instruments.

As far as possible the movement should be so designed that the addition of balancing weight is reduced to a minimum, as this adds both to the load on the pivots and to the inertia of the system.

FIGURE OF MERIT

When an instrument has been designed, the designer wishes to have some criterion by which he can judge whether his design will be satisfactory in service. Some "figure of merit" or means of judgment therefore appears to be necessary. The main question, to which the designer requires an answer, is whether the friction between pivots and jewels will introduce any appreciable error in the instrument. This is determined by the relation between the working torque and the friction torque. Since the latter is determined partly by the weight of the movement, a figure which has been used very extensively is the ratio (torque for a given deflection)/(weight of the moving system) or its reciprocal.

This has been a matter of considerable discussion, and the question has been raised as to whether the ratio torque/weight should be used or the ratio torque/(weight)^b where b is some power, usually fractional. In his paper Stott states that—

(1) b should be 4/3 if the pivot is subject to elastic deformation only.

(3) b should be 3/2 if the pivot is permanently deformed

(3) b should be 1 if the pivot is worn and is acting as a flat surface.

It is clear that pivots should be designed to work within the elastic limit, and we shall accordingly confine our consideration of this problem to pivots which are subject to elastic deformation only.

Consider first the instrument which is used with the scale horizontal so that the axis of the movement is vertical. The weight of the movement is carried on one pivot, which should rest in the centre of the jewel. Then the torque due to friction is from equations (2) and (10)

$$C_f = \frac{(3/16)^{\frac{1}{3}}\ \pi\ \mu\ W^{\frac{4}{3}}\ (\psi_1 + \psi_2)^{\frac{1}{3}}}{\left(\frac{1}{r_1} - \frac{1}{r_2}\right)^{\frac{1}{3}}} \text{ dyne-cm.}$$

where W is the weight of the movement expressed in dynes.

Now the amount of error due to friction which can be allowed is determined by the required accuracy of the instrument.

$$\text{Suppose } \frac{\text{Deflection}}{\text{Friction error}} = m$$

then deflection $=$ Torque/spring torque per radian.
friction error $=$ Friction torque/spring torque per radian.

$$\text{and therefore } m = \frac{\text{Torque}}{\text{Friction torque}}$$

$$\text{Consequently } m = \frac{T}{\dfrac{(3/16)^{\frac{1}{3}}\ \pi\ \mu\ W^{\frac{4}{3}}\ (\psi_1 + \psi_2)^{\frac{1}{3}}}{\left(\frac{1}{r_1} - \frac{1}{r_2}\right)^{\frac{1}{3}}}}$$

where T = torque in dyne-cm.
W = weight of moving system expressed in dynes.

From this

$$\frac{T}{W^{\frac{4}{3}}} = \frac{(3/16)^{\frac{1}{3}}\ \pi\ \mu\ (\psi_1 + \psi_2)^{\frac{1}{3}}\ m}{\left(\frac{1}{r_1} - \frac{1}{r_2}\right)^{\frac{1}{3}}}$$

The right-hand side of this expression contains only quantities which depend on the physical properties and dimensions of the

instrument materials and on the permissible friction error. It therefore appears that the ratio $T/W^{\frac{4}{3}}$ is a satisfactory criterion for comparing instruments having the same jewel and pivot radii.

The above expression is in absolute units, and if the weight W is expressed in grammes, r_1 and r_2 in thousandths of an inch, $(\psi_1 + \psi_2)$ is given the value $0{\cdot}1601 \times 10^{-11}$ and μ is assumed to be $0{\cdot}1$, then

$$\frac{T}{W^{\frac{4}{3}}} \fallingdotseq 5{\cdot}25 \times 10^{-3} \left\{\frac{r_1 r_2}{r_2 - r_1}\right\}^{\frac{1}{3}} m$$

An even better figure of merit would be

$$\frac{T}{W^{\frac{4}{3}}} \left\{\frac{r_2 - r_1}{r_1 r_2}\right\}^{\frac{1}{3}} \cdot \frac{1}{m}$$

This would enable instruments of all sizes to be compared, and should be at least $5{\cdot}25 \times 10^{-3}$.

As an example, consider an instrument in which $r_1 = 1{\cdot}5$, $r_2 = 3{\cdot}15$, and the error due to friction must not exceed $0{\cdot}05\%$, as may quite well be demanded in a portable instrument. Then

$$\frac{T}{W^{\frac{4}{3}}} = 5{\cdot}25 \times 10^{-3} \left\{\frac{1{\cdot}5 \times 3{\cdot}15}{3{\cdot}15 - 1{\cdot}5}\right\}^{\frac{1}{3}} . \ 2{,}000 = 14{\cdot}9$$

The ratio T/W is more easily calculated, and a curve giving the relation between T/W and $T/W^{\frac{4}{3}}$ is given in Fig. 2.27.

The case of an instrument with a horizontal axis is rather different. Here the pivots do not rest vertically in the jewels, but, due to the necessary end shake, will rest on the side of the cone, as in Fig. 2.28. If it is assumed that the end shake is such that the pivot is resting on the side of the jewel, then the actual pressure on the jewel is greater than the weight carried by the pivot. Suppose one pivot carries a weight xW, and the other a weight $(1 - x)W$, both expressed in dynes, then for the pivot carrying xW the force acting on the jewel is

$$\frac{xW}{\cos\theta}$$

where 2θ is the angle of the jewel. The friction force opposing the motion will therefore be $\mu xW/\cos\theta$, and the radius at which this force acts is $r_1 \cos\theta$. Then the torque due to friction is

$$C_f = \frac{\mu xW}{\cos\theta} \cdot r_1 \cos\theta = \mu xWr_1$$

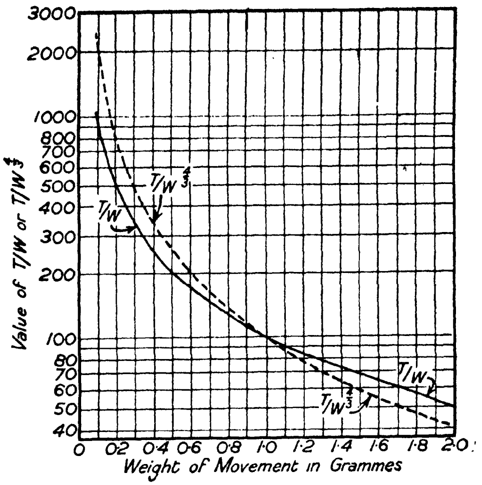


FIG. 2.27.—Variation of torque/weight ratio with movement weight for a torque of 100 dyne-cm.

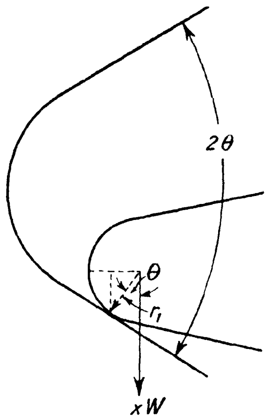


FIG. 2.28.—Position of pivot in instrument with horizontal axis.

Similarly the friction torque due to the pivot carrying a load $(1 - x)\ W$ is

$$\mu(1 - x)Wr_1$$

So the total friction torque is $\mu xWr_1 + \mu(1 - x)Wr_1 = \mu Wr_1$

Therefore

$$m = \frac{\text{Torque}}{\text{Friction torque}} = \frac{T}{\mu Wr_1} \text{ or } \frac{T}{W} = \mu r_1 m$$

For this type of instrument, therefore, the correct ratio to take is torque/weight. The conditions are actually more complicated than the simple consideration above, since the pivot has a tendency to roll round the jewel. This effect is, however, neglected here.

Putting the last expression in practical units we have

$$\frac{T}{W} = 0{\cdot}249\, r_1\, m$$

where again T is in dyne-centimetres, W in grammes, and r_1 in thousandths of an inch. It is usual to allow somewhat larger errors due to friction in this type of instrument, so suppose this error to be 0·2%, i.e. $m = 500$, then with a pivot radius of 1·5 we have

$$\frac{T}{W} = 0{\cdot}249 \times 1{\cdot}5 \times 500 = 187$$

On comparing the two cases, it is immediately evident that there is a marked difference in the values of these "figures of merit." If we take the weight of the movement as 1 gm., then in the case of the vertical axis the torque/weight ratio must be at least 14·9, while with the horizontal axis it must be at least 187. This large difference is borne out in practice, as it is found that quite satisfactory instruments having a vertical axis can be made with torque/weight ratio as low as 15, while to obtain a satisfactory instrument with a horizontal axis this ratio must be 150 or more.

DAMPING

This subject is fully dealt with in Chapter 3, and is introduced here only on account of the mechanical construction involved. In order that the moving system shall take up its position of equilibrium under the deflecting forces without oscillation or overshooting, some form of damping must be provided. If this is of such a kind that the pointer reaches its position of rest without any overshoot in a minimum of time, the instrument is variously stated to be "aperiodic," "critically damped" or "dead beat." If the damping is carried too far the pointer will slowly creep up to its position, and its indications will be indefinite. It is usual in commercial instruments to carry the damping to a point just short of the critical condition, it being claimed that this gives the required sharpness of reading on variable loads. Methods of accurately determining the degree of damping are given in Chapter 3.

There are three methods in general use for damping instruments viz. :—

Air Friction.
Fluid Friction.
Eddy Currents.

(a) *Air Friction Damping.*—In the first method it is usual to attach to some part of the moving system a light vane or vanes, which have considerable area, and damp the motion by reason of the air they displace. Fundamentally such a system may be varied in three ways :—

(*a*) By having a vane moving in free air.
(*b*) By having a vane moving at high velocity.
(*c*) By enclosing a vane in a box or chamber.

Firstly, the vane merely acts as a fan in free air. In order that such a scheme shall be effective it is necessary for the vane to present a large area, and as a consequence the mass and moment of inertia of the movement is increased. As a method of damping such a system has nothing to recommend it, and our own experiments on free air damping have shown that it is practically impossible to deduce a law from observations on such a simple system moving at the velocities such as are used in commercial instruments. In an early form of instrument the pointer itself did duty as a damper, and was for this purpose bent up sharply at right-angles along its length; and although it swung in the somewhat confined space between the glass and the dial-plate, the records in Table VI show it to be very inefficient. In another form, a large aluminium vane was carried on a separate stem parallel to and behind the pointer, the dial plate being interposed between them when the movement was in position. The result was an exceptionally heavy movement with a poor damping factor. Few manufacturers seem to realize how much the damping may be increased by roughing, ridging and ribbing the surface of the vanes. One of the most efficient materials for damping vanes is the wing of an insect (such, for instance, as a dragon-fly) ; the structure is light and rigid, and probably slightly pervious to air, and the multitude of minute hairs on its surface grip the air and increase the retarding effect. MM. Favé and Carpentier, in 1904, showed that it was possible to obtain a high damping coefficient with very little increase in weight or inertia by using a

structure consisting of a number of very light arms attached to the axis of rotation. Each of these arms carried a number of transverse hairs or glass fibres 0·01 cm. diameter, and even when the distance between individual fibres was as great as 100 times their diameter the drag was perceptible. Indeed, they showed that unless they were kept well apart the shielding of one by the other reduced the efficiency. They give the relation between the diameter d of the fibre and the coefficient k of resistance in dynes per square centimetre of transverse section for a velocity of 1 cm., per second as

$$(k - 0{\cdot}00135) \times (d - 0{\cdot}00283) = 0{\cdot}007765.$$

Thus, such a structure has a similar effect to that of the dragon-fly wing already mentioned, and were it not for the costliness of manufacture, such dampers would be of the greatest utility.

In the same way, the turning up of the edges of the vane increases the efficiency of the damper by controlling the freedom of leakage of the air round them.

The second method, of increasing the damping by increasing the speed with which the vane moves, is obviously difficult to carry out effectively, for although the air friction increases rapidly with the speed, this is more than counterbalanced by increase of inertia and other troubles which arise when such a scheme is attempted.

The third method, of complete enclosure, is probably the best of all, since it reduces the size of the vanes and permits of a fair amount of control by adjusting the clearance between the edges of the vane and the chamber in which it swings. In order to obtain a high damping factor by this method three things are of importance. Firstly, the clearance must be very *small and uniform* around the vanes. Secondly, the chambers must be highly finished in their interiors. Thirdly, the vanes must be enclosed as completely as possible so that the useless leakage to the outer air is reduced to a minimum. Fig. 2.29 shows the various methods of enclosure, and other instances are seen in Figs. 7.6—7.15. The use of two vanes moving in sector-shaped chambers has the advantage of allowing greater symmetry and better balance, and usually it is possible to enclose them much more completely. As in the case of free-air damping, advantage is taken of the increase of friction by turning up the edges of the vanes. Table VI, however, shows how rarely such systems of air damping are highly efficient, and there is probably no part of instrument production which makes such demands on

skilful workmanship as the making and adjustment of a good enclosed air damper.

It is partly for this reason that the piston damper, originally introduced by Messrs. Siemens & Halske, has been so widely adopted by other manufacturers, since it is possible by employing them to obtain a fairly high damping factor without unduly great refinement of workmanship.

In principle the scheme is a modification of the enclosed air damper, and its effectiveness is dependent on the clearance between the little piston head and the cylinder in which it moves. The piston must be supported on its comparatively long arm in such a way that shock will not throw it out of alignment with its cylinder;

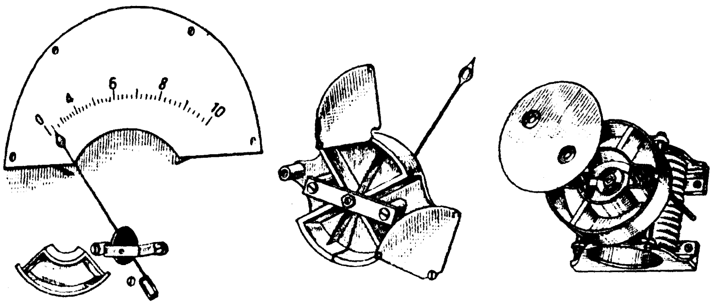


FIG. 2.29.—Examples of enclosed damping vanes.

the arm is therefore required to be stiff and light, and the piston head must be rigidly supported at right-angles to it. Aluminium wire is unsuited mechanically for the purpose, and if this material is used it should be in the form of flat punchings from sheet metal or small diameter tube, but probably the best material is a light spring wire of brass or phosphor bronze. The forms and method of attachment are shown in Fig. 2.30. Under any circumstances the piston damper will add considerably to the inertia of the system, and therefore the curved arm is usually bent to a radius that will permit it just comfortably to enter the cylinder, and it is then carried sharply upward behind the piston head and secured at its centre.

To facilitate the adjustment of the clearance the cylinder is usually constructed in two halves, the top portion being removed during adjustment, and is finally screwed to the lower section when these

have been made, each half being provided with flanges for this purpose. Sometimes a rectangular piston is employed, and in such cases it is sufficient to make the chamber in which it moves with a flat removable cover. As in the case of enclosed dampers, interior finish is of great importance in obtaining a high damping factor. Fig. 2.30 shows typical examples of the employment of these devices.

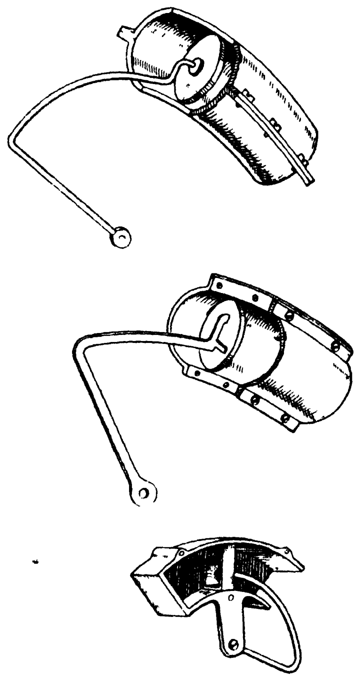


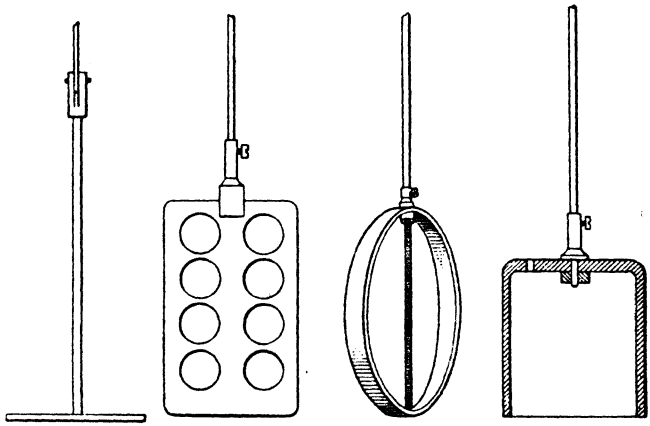
FIG. 2.30.—Examples of piston air dampers.

The attachment of the vane to the arm in all forms of air damper is of great importance, for the movement may be subject to mechanical shock during transportation, or by overload, or reverse deflection, and this will tend to produce want of true alignment with the enclosing chamber, either through distortion or loosening and detachment of the vane. Good mechanical attachment is, therefore, essential, and where possible it is best to construct the vane and supporting arm out of a single sheet pressing, without any mechanical

joint at all. In a few instances the arm is riveted to the side of the vane, but this is expensive to carry out satisfactorily.

The usual method is to "stitch" the arm through suitable slits in the sides of the vane so that it is gripped firmly. The objectionable practice of cementing the vane to the arm with shellac varnish should be entirely condemned.

(b) *Fluid Damping.*—Fluid dampers are mostly employed in instruments with a vertical axis, although in one or two cases they have been fitted to vertical scale instruments. They are also em-



(a) (b) (c) (d)

FIG. 2.31.—Forms of fluid damping vanes.

ployed extensively in chart-reading instruments, which necessarily have rather heavy movements. There are many serious objections to the employment of this system of damping in commercial instruments, which then become unportable; the damping may change considerably with temperature, creeping cannot be prevented, the oil soon destroys the good appearance of the instrument, and most oils tend to overdamping. The practice in applying the method consists of providing some form of vane attached to the axis of rotation and working in a dash-pot containing the damping fluid. Fig. 2.31 shows the various forms of vane employed.

In all cases it is important that the whole of the damper is completely submerged beneath the surface of the damping fluid, and that the portion of its support which penetrates the surface should present as small an area to the fluid as possible. In this way

creeping and the varying effects of surface tension may, in a great measure, be eliminated.

In some of the older types of instrument two dampers, symmetrically placed, were employed, and their buoyancy relieved the load upon the bottom pivot. In some high-voltage instruments a similar advantage is obtained by immersing the whole movement in oil; it thus becomes its own damper, and the oil at the same time improves the insulation of the system and reduces the tendency to brush discharge.

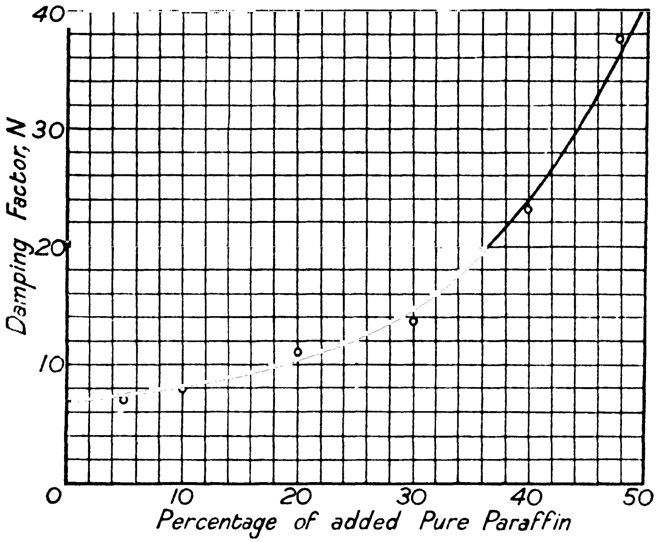


FIG. 2.32.—Curve showing variation of damping factor for mixtures of kerosene and pure paraffin.

The choice of a suitable fluid should depend on its possessing the following characteristics: The loss from evaporation should be small; it should be constant in its chemical and physical properties; it should not attack metals; change of temperature should have but little effect on its viscosity; and it should preferably be a good insulator.

Mineral oils fulfil these requirements fairly well, and the ease with which the viscosity may be adjusted by mixing different grades is in their favour, together with the fact that they are all good insulators. The effect of mixing pure paraffin with kerosene is shown

in Fig. 2.32; the rapid rise in the damping factor as 50% of pure paraffin is approached is well shown.

Creeping is practically impossible to eliminate entirely, and is a very great drawback to the employment of fluid damping, and soon leads to the destruction of the good appearance of the instrument. Overlapping or touching surfaces, grooves and recesses should be

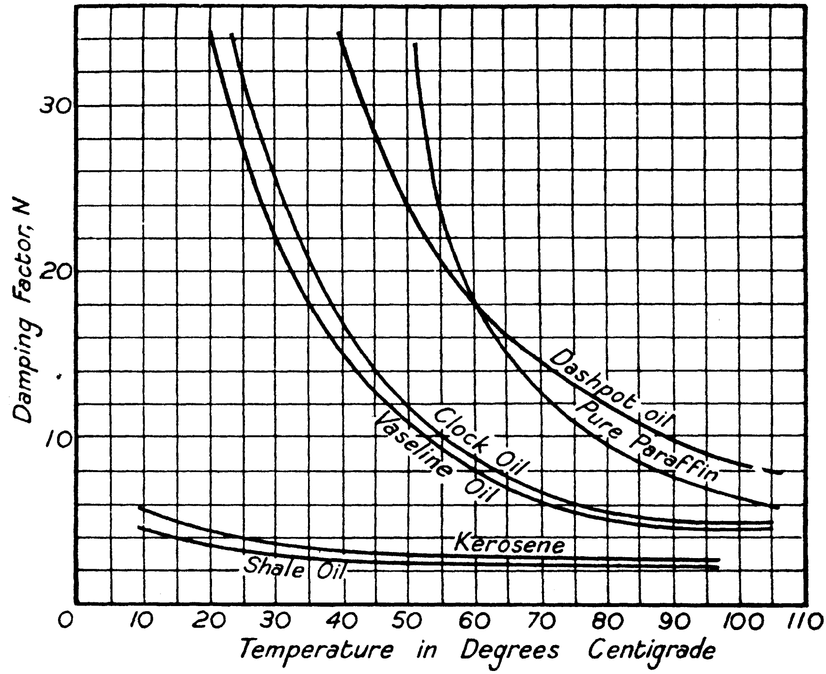


FIG. 2.33.—Curves showing change of damping factor with temperature for various oils.

avoided in such parts as are liable to become wetted by the fluid, which should preferably be contained in a glass vessel deep enough to allow of a fairly long unwetted surface above it. If a metal container is employed it is usually found better to dome over the open end, leaving an aperture just sufficient for the damper to pass in, and to nickel-plate heavily and *polish* the outer surface. The change of damping with temperature is shown for a few oils in Fig. 2.33. The more fluid oils over the ordinary range of temperature show a fall in the damping factor which practically follows a straight-line law, and the change with temperature, although quite marked,

is not serious. With more viscous oils, however, the change is much more serious. It would seem, therefore, from our results that it is better to increase the size of the damper and use a fluid of small viscosity, rather than employ a more viscid fluid and a small damper in cases where there is a liability to moderate changes in temperature and a high damping factor is desired.

(c) *Eddy-Current Damping.*—Fundamentally, magnetic damping depends on the reaction of the induced currents set up in a moving conductor with the flux producing them, the torque so produced tending to oppose the motion. The principle is applied in two ways in commercial instruments. In the first a symmetrically arranged

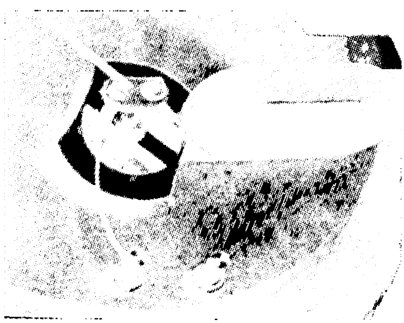


FIG 2 34—Eddy current damper of hot wire meter

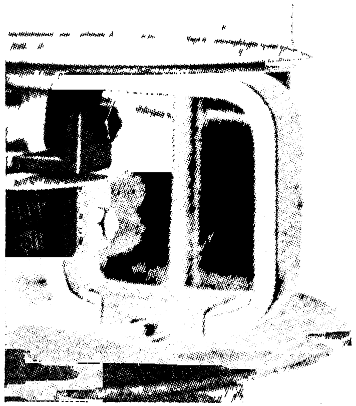


FIG 2 35—Eddy current damper of induction ammeter

disc or sector of good electrical conductivity—such, for instance, as copper or aluminium—is attached to the axis of rotation, so that as the movement is deflected the disc moves between the poles of a suitably placed permanent magnet or magnets (Figs. 2.34, 2.35). In some types of alternating current instruments the disc forms the whole movement, the deflecting force being produced by an alternating current magnet acting on one side of the disc, whilst a permanent magnet acts at the other end of the diameter (Figs. 2.23 and 2.35).

The correct proportions of the disc and magnet are dealt with in Chapter 3, so that it is only necessary here to deal with the details generally. The disc is usually of aluminium, since for equal conductivity there is a considerable saving in weight, but it is

necessary that the metal is of good conductivity, as otherwise the duplication of magnets will seriously increase the expense.

Discs and sectors are usually die-pressed to give them rigidity and perfect flatness, and they must then be mounted on the axis in such a way that there is no possibility of their eventually getting out of truth, as the gap of the magnet should be as small as possible. It is usual, therefore, to secure the disc to a central sleeve and boss to which the spindle is fitted. The magnets should preferably be provided with concentrated poles acting at the radius at which they give maximum torque. It is surprising how little this point seems to have been appreciated, for in some instruments broad adjustable polar extensions are provided, and in others a number of magnets acting at the extreme edge of the disc are used. We have found that in the first instance a concentration of the pole, and in the second instance a removal of some and readjustment of the remaining magnets, has led to a better damping factor. The authors have found that soft steel pole-pieces, arranged so that the flux passed through the disc in radially distributed tufts, will give excellent damping where a high damping factor is necessary. In some instruments, instead of a disc a cup or bell of metal is employed, and the magnets may then be arranged so that the sides of the cylinder pass through the gap, or the magnetic circuit may be completed through a steel core mounted concentrically within the cylinder. The flux of each magnet thus crosses two gaps, but with the vertical arrangement these can be made quite short with perfect safety, and if the field is kept well inside the edge, the resistance of the return path of the eddy currents can be kept low, and good damping attained with a comparatively thin cylinder of small radius and, therefore, small inertia. When, however, magnetic damping is applied to such instruments as ironless dynamometers, the arrangements require special consideration, as the presence of permanent magnets may seriously alter the working fields; and although this may be allowed for in the initial calibration, any subsequent change in the magnets will cause error, and permanent magnets, unless very satisfactorily shielded, may be weakened even by feeble alternating fields. The latest forms of magnet alloys show a great improvement in this respect.

The second method of applying the principle of eddy-current damping is found in most moving-coil instruments. The coil is wound on a light conducting former or frame of copper, or more

usually aluminium, which is traversed by eddy currents as it is deflected in the magnetic field, and thus provides the damping torque.

In high-resistance instruments, such as voltmeters, practically all the damping is due to the former, but with milli-voltmeters, working across a shunt as an ammeter, a fair component of the damping is contributed by the winding itself, since it is a closed circuit of comparatively low resistance, and the former may therefore be of much lighter construction. It should be pressed or moulded from flat sheet metal or from seamless rectangular tube, so as to have no joints or seams, as these may interpose sufficient resistance in the circuit of the induced currents to reduce the damping very considerably. Cases have been known where a single well-made joint in a former has reduced the damping constant to one-fifth of that obtained with a similar jointless former. Also, with the intense magnetic fields which can now be obtained, it is essential that the material used for the former be free from iron, as otherwise distortion of scale shape and possible instability may result.

The operation of pressing-up invariably leads to an increase in the specific resistance of the material, which must be allowed for when calculating the dimensions. The pivots may be inserted in separate mountings and cemented to the winding, or secured by clipping the mounts over the edge of the form ; in some cases they are bound in with silk. The objections to independent pivots have already been mentioned. Usually good alignment can be secured with a well-constructed assembly jig, but warping is always liable to occur.

In some cases the pivot or jewel mount is directly riveted to the former. If they project inwards there is but little objection to this, but where they pass through the winding space the coil turns must be splayed round them, and this usually detracts from the neat appearance of the instrument, and demands much more care in winding and balancing. In some D.C. instruments and A.C. dynamometers metallic formers are not employed. The coil is then wound on a temporary former, treated with a hard-setting cement and baked ; the former is then removed before the final mounting.

Such practice demands particular care in the selection of a suitable cement, for unless it has a high softening point, and is quite rigid at ordinary temperatures, the coil may warp, due to the natural twist in the wire. The covering used for the wire is also of importance.

Many of the older instruments used silk-covered wire, but in modern instruments the majority of coils are wound with wire covered with enamel, or one of the new plastic materials, such as nylon. When using enamel-covered wire particular attention must be paid to the cement. The general tendency is to use a synthetic resin varnish, and some forms of enamel are attacked by this and may be dissolved leaving the wire bare. Fortunately in many instruments in which formerless coils are employed there is no objection to the use of a through spindle to carry the pivots, and this is, therefore, frequently used. In the case of power factor indicators and instruments where the moving system consists of two or more coils arranged with their planes at definite angles to one another, the construction is much more difficult. The Weston Instrument Company solved this difficulty in a very elegant manner by winding the coils so that the layers of each interlace at the points of intersection, thus securing a light, rigid and perfectly symmetrical system of coils, each having equal magnetic moment.

Another and far less perfect method is to mount upon the spindle a light spherical body-former of moulded insulation, cork or wood, and wind the coils on this. Or again, the coils may be wound separately to a form so that one will slip inside the other; this, however, is a clumsy expedient, and the coils are difficult to secure in their proper position.

AUXILIARY COMPONENTS

Resistances

Series resistances are generally employed in the voltage circuits of instruments such as voltmeters, wattmeters, power factor indicators, frequency meters, etc. In many cases these are arranged on bobbins within the instrument, frequently beneath the dial plate. This practice has some objections, since it often leads to an increase of the internal temperature of the instrument, due to the power expended in the coils, and the heat so generated may cause inaccuracies by increasing the resistance of the copper-wound coils within the instrument, or by causing undue expansion of the various parts, and other effects which vary with the temperature. On the other hand, the resistances may be located in a recessed part of the base, or in a separate ventilated compartment beneath the instrument.

Where circular bobbins are employed the following considerations must be taken into account :—

(*a*) Since the P.D. across the resistance may be comparatively high, the insulation must be maintained at a high value.

(*b*) It is usually better and more economical to construct each spool as a self-contained unit adjusted to correct value, and hence it should be possible to terminate the coil on suitable terminal plates or tags carried on the bobbin.

(*c*) The bobbin should be capable of efficiently radiating the heat generated.

(*d*) The material of the spool should be indestructible, and unaffected by moderately high temperatures.

Porcelain is often employed because of its indestructible and high insulation qualities. It, however, suffers from the defect that the bobbins are heavy, somewhat irregular in dimensions, the heat conduction is poor, and there is no method of satisfactorily terminating the coil. Not infrequently the wires are simply brought through holes left in the end flange and fixed with cement. The loose ends of coils terminated in this fashion are very liable to break off short, partly because of the frequent bending which unavoidably occurs, and partly because of the action of the varnish cement used for fixing them.

Plain boxwood is often employed, and is fairly satisfactory if thoroughly impregnated and the maximum temperature is not allowed to run too high. The coil can be terminated in tags secured to the upper flange. The material is, however, by no means fireproof, and is a very poor heat conductor.

Ebonite suffers from similar defects, and in addition is liable to give off sulphurous fumes at moderate temperatures, which attack the fine wire wound upon it, causing corrosion and eventual breakdown.

Vulcanized fibre is sometimes employed and is fairly satisfactory (at low temperatures and where insulation is not important), provided the bobbin is constructed in a single piece either by turning or moulding it. Made-up bobbins in which the end flanges are pressed on the central tube are unsatisfactory, as sooner or later the flanges become loose. The material, unless well cured, is liable to warp and deform, and at high P.D.'s the material is a poor insulator, and, like the previous materials, is a poor heat conductor.

Wide use is made nowadays of bobbins moulded from one of the synthetic resin compounds. Bobbins of this type are rigid, good insulators, and are not affected by moderately high temperatures. Also the ends of the coils can be attached to terminals or tags fixed to the end flanges. Care must be exercised in choosing the synthetic resin material to be used, and in deciding the dimensions of the bobbin, in order to eliminate warping with time.

A number of bobbins are also made from thermo-plastic moulding materials such as cellulose acetate by injection mouldings. These, while being good insulators, are not so rigid as the synthetic resin bobbins, and will not withstand even moderately high temperatures. They are, however, very cheap to produce when large quantities are required, and are quite satisfactory if used under the right conditions.

The plain metal tube bobbin is cheap, efficient, and simple to construct. A thin brass tube from 2 to 2·5 cm. in diameter is served on the outside with a thin layer of insulation, preferably a thin tube of reconstructed mica, although sometimes impregnated paper, leatheroid, silk tape or empire cloth is employed for this purpose. The wire is wound over this, the depth of winding being kept as small as possible, and the whole unit is then varnished and baked. The bobbin is held in position by a pin which passes through central holes in a cross-bar at each end of the metal tube. Such coils are light and very durable, but on account of the necessity of keeping the depth of the winding small they are somewhat wasteful of space. Where the resistance is located in the back of the instrument deep bobbins cannot be employed, and under these circumstances the wire is wound bifilarly either on an open rectangular frame or on a thin slab or card made of some kind of insulating material. Resistances constructed in this way have very many advantages. They have excellent cooling properties, and their inductance and capacity can be made quite small (see Chapter 8). The coil can be terminated on metal tags secured by riveting them through the insulating support. It is, however, important that the material on which the wire is wound should have a thermal expansion less than that of the wire wound on it, as otherwise the insulator may seriously strain the winding at the upper temperatures ; also the edges of the support must be smooth and well rounded, as it is usually found that breakdown occurs at the edges where the wire is sharply bent over.

Micanite and some of the synthetic resin compositions answer well for the purpose, while ebonite and fibre are quite unsatisfactory for the reasons given above, and because they warp and deform badly in flat unsupported sheets.

Each unit can be adjusted to a standard value, and the required resistance is then made up by assembling a series of units on four pins which pass through holes in the corner of the insulator, each unit being kept from contact with the one below by interposing fibre washers between them.

A somewhat similar system sometimes employed is to wind the wire bifilarly in notches in the edges of an insulating slab, each notch accommodating a separate unit, but the system has nothing to recommend it over the method described above.

After winding, all resistances should be annealed and otherwise aged before final adjustment, as some materials show considerable unsteadiness for some time after winding, due principally to the necessary tension under which they must be wound.

In many modern instruments, use is made of high stability carbon resistors, of the type illustrated in Fig. 2.36. The pyrolytic carbon

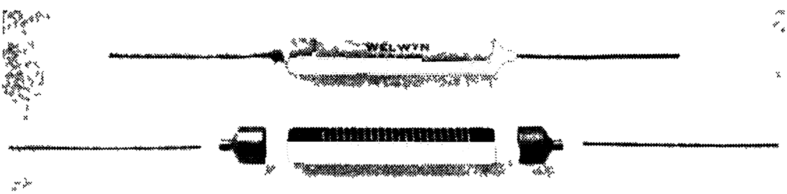


FIG. 2.36.—Example of high stability carbon resistor suitable for instrument use.

resistor, or so-called cracked high stability carbon resistor, was a German development and was introduced into Great Britain in the early 1940's.

The resistor consists of a porcelain rod upon which is deposited a hard, firmly adherent, lustrous coat of crystalline carbon. This layer is obtained by raising the rod to a temperature of approximately 1,000° C., and injecting hydro-carbon into the space surrounding the rod. It is found that decomposition of the hdrocarbon takes place only on the surface of the porcelain and provided that the deposition is not too rapid, the carbon is deposited in a regular crystalline structure, which provides a very stable electrical resistive material.

The required range of resistance values is obtained by grinding a helix into the surface, as can be seen in Fig. 2.36 lower view, the gain in resistance depending upon the increase in length and reduction in width of the track. The protection of the resistors is very important, since carbon appears to be strongly affected by the presence of moisture and full stability cannot be attained unless the carbon is completely protected against the access of moisture. Several finishes have been used with varying degrees of success; among the more important are silicone, bitumen, certain phenol-formaldehyde lacquers, distrene and polyvinyl chloride sleeves. The ideal protection is sealing in glass, but the expense involved and the fragility of the product has not made such protection universally popular.

These resistors are normally supplied in tolerances of 1% or wider, although, in many instances, closer tolerances than 1% have been used with success. Stability under normal working conditions can be as good as 0·2% for the lowest resistance values and of the order of 1% for the higher values. In the presence, however, of high humidity and high temperature, such as exist in the tropics, the stability of such resistors is impaired and variations to three times the figures, given above, can be expected.

Reactances or Choking Coils

In several alternating current instruments, like phase indicators, synchroscopes, and induction wattmeters, it is often necessary to produce a large phase difference in one of the circuits, and for this purpose a reactance or choking coil is put in series with the coil through which it is desired to have a lagging current. Or again, in some types of frequency meter a reactance is employed to control the current in such a way that it varies with the frequency of the supply.

Such coils usually take the form of a laminated magnetic circuit on which the coil is wound, and in order to reduce the flux, and thus bring down the iron losses, it is usual to arrange a short airgap in the path of the flux.

The behaviour of such a coil will be understood by reference to the vector diagram (Fig. 2.37). If V is the applied P.D., and I the current, then I is the resultant of the core loss current I_c in phase with V_1 and I_m, the magnetizing current lagging 90° behind V_1. In the same way V is the resultant of V_1, the reactive voltage set

up in the windings, and the ohmic drop IR, which is in phase with I, while the angle of lag φ is the angle included between V and I. It is at once obvious from a consideration of the diagram that φ is dependent both on the IR drop and the magnitude of I_c. It is therefore impossible for φ to attain 90°, and in order that it shall be as great as possible, both the resistance and the iron losses must be reduced to their lowest values. On the score of economy of copper and space it becomes necessary to employ an iron circuit, and the material employed for this should have high permeability

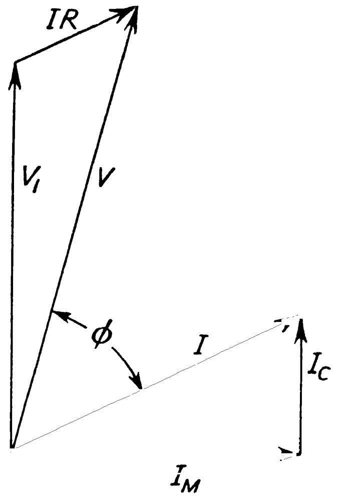


FIG. 2.37.—Vector diagram for iron-cored reactance.

and low specific loss, and for this reason the silicon iron alloys like "Stalloy" or the nickel-iron alloys such as "Mumetal" are to be preferred.

It will usually be found best to work at induction densities near the maximum permeability of the iron, but in all cases the best condition is that which gives equal iron and copper losses, and high flux densities lead to distortion of the current wave shape.

In order to design a reactance of this type we have

$$V = 4{\cdot}44\ \Phi N f \times 10^{-8}$$

where V is the voltage, Φ the maximum flux, N the turns on the coil, and f the frequency in cycles per second. Hence

$$\Phi = \frac{V \times 10^8}{Nf \times 4{\cdot}44}$$

The magneto-motive force in the circuit is

$$\sqrt{2}\,\frac{4\pi}{10}\,IN = \Phi \times \text{reluctance},$$

and if A is the area of cross-section of the airgap, which for simplicity we assume is the same as the iron cross-section, we have approximately

$$1{\cdot}775\,IN = \frac{\Phi}{A}\left(\frac{l_i}{\mu} + l_g\right)$$

where l_i is the length of the iron circuit, and l_g the gap length. Substituting for Φ the value obtained above, we have

$$I = 0{\cdot}127 \times 10^8 \frac{V}{N^2 f}\left(\frac{l_i}{A\mu} + \frac{l_g}{A}\right)$$

and

$$N = 3{,}560\sqrt{\frac{V}{If}\left(\frac{l_i}{A\mu} + \frac{l_g}{A}\right)}$$

which gives the turns required when the other quantities are fixed. In designing such coils a certain amount of judgment has always to be exercised in choosing the right proportions of airgap and iron circuit. Usually the external dimensions are fixed by the available space. As a guide to the proportions of the gap it may be taken that the volume of the gap space Al_g may be expressed in the form

$$Al_g = K\frac{VI}{f}$$

where K is a constant dependent on the flux density and has the following values :—

B	6,000	7,000	8,500	11,000
K	1	0·75	0·5	0·3

these values covering the range of flux densities employed. Thus a choice of l_g or A is possible when the volt-amperes and frequency are fixed. And since usually about 90% of the ampere turns are required for the gap, a long gap will be extravagant in copper, while, on the other hand, too short a gap will require an unduly large iron circuit. In most cases the manufacturer standardizes a definite form of iron lamination, and the volume of the iron is varied by adjusting the number of these assembled together.

The curves in Fig. 2.38 illustrate the case for a coil of 5 volt-ampere capacity (50 volts and 0·1 ampere) at 50 cycles, the windings

in each case being arranged to fill the central opening of the punching. The best gap in this case is 5·5 mm., and the power factor is 0·15. This is lower than is usually obtained, due to the employment of a special iron and very thin laminations (0·3 mm.), and in practice it is found that the iron loss is usually higher than the estimated value: usually power factors between 0·2 and 0·3 are as good as can be expected. These remarks apply to reactances made with

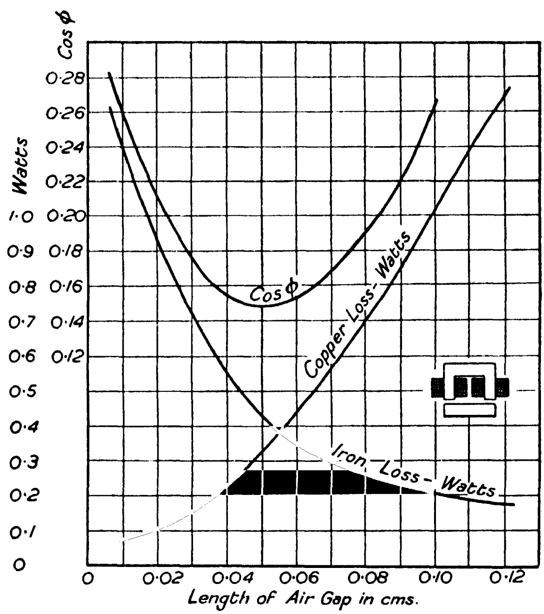


FIG. 2.38.—Curves for iron-cored reactance.

silicon iron, and somewhat better results can be obtained by the use of nickel-iron alloys.

Fig. 2.39 shows the methods employed in building up the core and arranging the coils. The method of building from rectangular strips with interleaved joints is not very satisfactory on account of the possible variation in reluctance of the built-up joints. A better method is to use a complete punching, as shown in *b* (Fig. 2.39), and complete the circuit through the double airgap between the ends and the yoke. With this arrangement former-wound coils can be slipped on the limbs and the gaps set by inserting gauged slips of insulation or press-board between the surfaces of core and yoke.

The coils should be disposed, as far as possible, uniformly round the entire magnetic circuit in order to reduce stray field and prevent spreading of the gap flux, and for this purpose the Weston Instrument Company arrange the airgap within the magnetizing coils, thus rendering the gap reluctance constant and the stray field a minimum.

The supporting frame must be designed to have sufficient rigidity to clamp the laminations firmly and prevent vibration of the stampings, which, if permitted, would make the instrument noisy

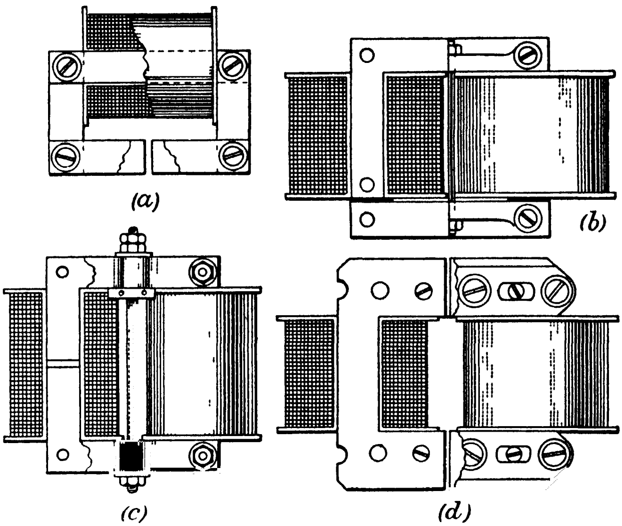


FIG. 2 39—Types of iron-cored reactance

and render the reactance variable. It is also important that the framework should not form any complete conducting circuit linking with the alternating flux, as eddy currents would be produced in these circuits which would diminish the inductance and phase angle as well as produce wasteful heating. In some cases it is found advisable to construct the frame of metal having a high specific resistance, but such a refinement is seldom necessary with good design of magnetic circuit, and choking coils, consisting of a coil with a straight central iron core, are rarely employed in instrument work, since they necessarily create large external fields which may not only affect the instrument itself, but also cause errors in neighbouring instruments. Moreover, owing to the indefiniteness of the return

path of the flux it is not possible to calculate with any degree of exactitude a suitable winding without employing an empirical formula.

One of the best of these was given by Evershed in 1893, which may be written in the following form :—

$$N = 4{,}750\sqrt{\frac{V}{If}}\left(0{\cdot}563\,\frac{l}{A_1} + \frac{0{\cdot}3174}{\sqrt{A}}\right)$$

This is similar in form to the one employed for the nearly closed magnetic circuit, but here l is the length of coil not occupied by the core when the latter is partly withdrawn, and A_1 is the area of the central layer of the coil in a plane at right-angles to the axis. If the core is symmetrical, and therefore entirely fills the central aperture of the coil, the term $0{\cdot}563\, l/A_1$ is zero, and for a long range the core should project at each end to an extent of at least 25% of the length of the bobbin. A in the above expression is the external surface of the projecting ends, and if the core is unsymmetrical, the average of the two projecting ends should be taken, and with a core just as long as the coil, so that its ends are flush with the end cheek, A is the cross-sectional area of the coil. If the coil is partly withdrawn A is half the sum of the superficial area of the projecting end and A_1, and when the core is entirely withdrawn $A = A_1$.

The above formula must, however, be employed with considerable reserve, since much depends on relative dimensions of core and coil. With about average proportions a coil having no iron core gives results about 4 or 5% low ; with a symmetrical core entirely filling the coil and projecting beyond its ends the results are about as much high ; but with an unsymmetrical core the errors are much higher, reaching 30% for a core inserted half-way through the coil, which, according to our tests, is the position of maximum error.

Condensers or Capacitors *

Condensers are often employed in alternating current instruments, particularly where it is necessary to produce a large phase displacement, in which case a condenser is sometimes preferable to a reactance, usually giving a considerably greater phase angle for a much smaller expenditure of power.

*The word capacitor is now being more generally used in place of the older word "condenser."

In the case of a perfect condenser the current will lead by 90° on the applied voltage, and its magnitude for sine waves will be given by $I = VC2\pi f \times 10^{-6}$, where V is the applied voltage, C the capacity in microfarads, and f the frequency of the supply.

If, however, as is usually the case, a solid dielectric is employed to separate the condenser plates, energy will be absorbed and converted into heat, due to dielectric hysteresis, etc., and there will be an energy component of the current which will reduce the phase angle between the current and P.D., and it is usual to designate the power factor of the condenser by sin φ, where φ is the angle by which the current and voltage depart from true quadrature.

With a good condenser the angle φ is usually quite small, but in poorly constructed condensers, particularly those with a paraffin paper dielectric, the phase angle may be very appreciably increased and when this is the case, the condenser is usually very sensitive to change of frequency.

The various methods of calculating the dimensions of the plates and thickness of the dielectric are given in Chapter 4 ; it is, however, important to remember that there is a correction necessary for the spreading at the edge of the plates which in some cases is quite appreciable. This correction has been given by G. Kirschoff for circular plates, and for very thin plates it may be written in the form

$$C = \frac{\pi a^2}{4\pi t}\left(1 + \frac{t}{\pi a}\log_e \frac{16\pi a}{t}\right)$$

where a is the radius of the electrode plates, t the thickness of the dielectric. With square plates there is a further spreading at the corners, which is not taken into account in the above. Undoubtedly the best and most reliable dielectric is mica, its high specific inductive capacity and electric strength being excellent for the construction of condensers, and great constancy of behaviour results. The variations with both temperature and frequency are small, and the angle of departure from strict quadrature is also very small, even with comparatively poor condensers.

The cost of manufacture is, however, high, and it is difficult to obtain thin uniform sheets of clear ruby mica larger that 100 sq. cm. without unduly increasing the cost. The usual working thickness is 0·038 to 0·05 mm., and electrodes of tinfoil are to be preferred to silvering the mica, which has several unsatisfactory features. It is usual to lay the condenser with pure filtered shellac varnish.

In this connection it is interesting to note that Dr. Muirhead has stated that mica-shellac condensers have a positive temperature coefficient, while mica-paraffin have a negative one, and hence a combination of the two can be arranged which give a condenser independent of temperature. The frequency error, as already mentioned, is small, even in poor condensers, where it is seldom greater than 0·2% between 50 and 1,000 cycles per second. The electrodes should be brought into contact as intimately as possible with the dielectric, and every care should be taken to exclude air-bubbles and particles of foreign matter: desiccation should be slowly carried out at a temperature of 100° C., and the condenser firmly clamped before cooling.

By far the greater number of condensers in commercial use have a paraffin paper dielectric, because of their cheapness and comparative ease of construction. As far as performance is concerned, they cannot be compared with a good mica condenser. They usually have a much larger and positive temperature coefficient, and there is a marked change in capacity with frequency, the apparent capacity falling as the temperature rises. In good condensers the departure from quadrature is small, but in many cases an angle of several degrees occurs, and two apparently similar condensers may have widely differing phase angles.

The early condensers of this type were built up with tinfoil and impregnated paper, and the assembled condenser hot-pressed, the greatest attention being paid to scrupulous cleanliness and the exclusion of moisture.

A great advance was made by the introduction of the Mansbridge system of building this type of condenser from foiled paper strips interlaced with plain paper, which was rolled into a cylinder, impregnated and finally pressed flat. Such condensers will operate fairly satisfactorily on commercial supply voltages and are very compact, and they have the further advantage that because of the thin metal foil, electrode faults are self-healing, the metal around the puncture being volatilized and clearing the fault.

For very high voltages glass is employed when the desired capacity is small. It cannot be obtained in very thin sheets, and in this form its extreme brittleness is a serious drawback. In practice the plates must be chemically cleaned and silver deposited on the surface; this deposit is then protected either by coating with copper electrolytically or otherwise by giving a coat of varnish. By this means

perfect contact between electrode and dielectric is secured. The well-known Moscicki condensers, used on high-voltage supply systems, are of this type, and to avoid the edge stresses the glass is thickened in the region of the edges of the electrode.

From what has been said above it is obvious that by the use of a condenser of suitable capacity a considerably greater phase displacement can be obtained with a much smaller expenditure of energy than can be obtained by the use of a reactance. On the other hand, it must be remembered that the terminals are really only a small fraction of a millimetre apart, and the effect of wave form may be serious, for although in any particular case the capacity in circuit may be so small that the conditions of the circuit are far removed from the point of resonance with the fundamental, the capacity for resonance with the third harmonic is only one-ninth of that required to produce resonance with the fundamental, or for the fifth, only one twenty-fifth, and correspondingly smaller for the higher harmonics, and any harmonic in resonance will give rise to currents which are limited only by the ohmic resistance of the circuit.

Internal Connections

The internal connections of the instrument are of considerable importance, and the methods employed are often somewhat slovenly and inefficient. In general, loose wires should not be permitted, and it is better to run all leads and support them so that no vibration or shock can move or displace any part of them. The lead wire should be sufficiently stiff to be self-supporting, and should, as far as possible, be carried in short, straight lengths, the bends being made sharply at turns and corners. As little as possible crossing of leads should be permitted, and joints should be soldered whenever possible. Enamel wire is frequently used for the purpose of making such connections, and is neat and effective, but in some cases bare or lightly insulated wire is used. With such material, where the wires are likely to come into contact, cross one another, or pass through holes in the metal framework or the case, they should be protected by slipping over them a tube of insulating material. Vulcanized rubber tube is often used for this purpose, but this material has nothing to recommend it, and is better avoided, as it hardens and breaks with time and is liable to cause corrosion. It should therefore on no account be used to protect

fine wires. A better scheme is to employ fine braided cotton tube, but with this there is always the difficulty of finishing the ends neatly, and fraying is sometimes prevented by waxing and binding the cut ends. Considerable use is now made of a sleeving of fine braided tube impregnated with flexible insulating varnish, and also of insulating tube made from various plastic materials. There is no difficulty with the ends, which are simply cut off clean with a sharp knife, and these materials therefore make a neat and durable job.

The heavy-current connections are preferably made with bare copper strip bent to the required shape. The go and return lead can often be assembled together with an insulating strip between them, and the pair bound at intervals with cord, thus minmizing stray fields in the instrument itself. The connection to the lugs and coils is made by soldering or by drilling, and clamping the strap with nuts and washers. Flexible cable, cotton braided, is sometimes employed, but there is little or no advantage in the use of such material, as the ends must be sweated into thimbles and the braiding bound and waxed down to the thimble shank if a neat appearance is desired, and any attempt to solder the flexible directly to the lugs invariably leads to an unsightliness which no amount of subsequent painting will ever disguise.

POINTERS AND SCALES

Pointers naturally fall into two classes, viz. those for reading at a considerable distance, and, secondly those intended for precision work at short range.

In the first class boldness of outline is, of course, essential, but this must be combined with lightness and rigidity, and at the same time the whole structure must have a natural period of vibration well outside the range of any impulses likely to be communicated to it, as otherwise it will get into resonant vibration and its indications become indefinite. This is of particular importance with alternating current instruments, and especially those intended for use on the higher ranges of frequency.

In the effort to secure boldness and rigidity the pointer is often made unduly heavy. In instruments with the restricted sector-shaped observation window the head only of the pointer is visible, and there is no object, therefore, in making the supporting structure a broad, heavy punching of sheet aluminium as is frequently the

case. On the other hand, where the open glazed dial is used, the whole length of the pointer should present a bold, clear outline.

The truss pointer of the Weston Instrument Company is an excellent example of good design, and is free from the objections mentioned above, combining lightness with great rigidity. Its general construction is shown in Fig. 2.40. The triangular framework is made from thin aluminium tube, and carries a bold indicating head at its apex, punched from very thin aluminium foil and stiffened by means of a ridge raised round its entire periphery. The side tubes of the triangle are tied across at the base by a tubular cross-stay, and are then again brought in towards one another, and are cranked down to the correct load and secured to the collet of the balance fitting on the axis of the movement, which carries the counterbalance arm, with a long sleeve weight running on a thread cut in it, having 272 threads to the inch. It will be seen, therefore, that the

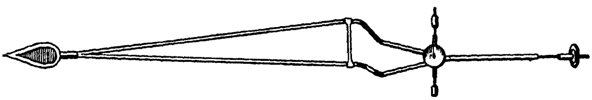


FIG. 2 40.—Weston truss pointer.

whole pointer structure is two triangles base to base, a form of girder having great rigidity for a small amount of inertia.

Where instruments are intended to work normally over a limited part of their scale a large circular hole is often punched through the head, and to the observer the white spot so produced is visible at considerable range ; a little black disc, slightly larger in diameter than the hole, is mounted on a separate arm capable of hand adjustment by means of a milled head on the case of the instrument, and this can be set to a position indicating the normal working value. Thus, when the pointer reaches this position the hole and disc coincide and the white spot disappears, and any deviations are readily detected by the reappearance of the white spot.

The employment of aluminium tube has many advantages, since it gives a light and rigid pointer, but usually it does not allow of sufficient breadth to make a suitable pointer for long-range reading. In some instruments, however, particularly miniature quick-deflection ones, such tube pointers are employed, and carry near their ends a blackened disc or diamond of considerable size compared with the pointer tube. This disc is punched from thin aluminium

sheet, and attached to the pointer by suitable clamping means or cement. For medium-size instruments with open fronts, considerable use is made of pointers stamped from aluminium sheet, and ribbed to give additional strength.

Tubular pointers of fine gauge can be employed for pointers of the second class, where the end can be flattened into a fine knife edge, and this is almost universal practice in precision instruments. Observations are made by bringing the eye over the knife edge to such a position that it appears to have minimum breadth, or parallax is eliminated by the use of a strip of mirror inserted beneath an opening in the scale plate below the pointer, so that when the pointer and its image coincide the eye is in the correct position for observation. Silvered glass mirror has been largely employed for this purpose, and gives the best results, but use is also made of metal with highly-polished nickel or chrome-plated surfaces. Care must be taken to see that these metal mirrors are perfectly flat, to avoid distorted images.

Another method of avoiding parallax is to bring the tip of the pointer to the same level as that of the scale, by mounting the latter on a raised platform, the divisions being carried right up to the inner edge along which the pointer swings. The coincidence of the pointer can then be accurately determined (Fig. 2.41). In some of Hartmann and Braun's instruments a fine stretched wire replaced the usual knife edge and the pointer carried a small translucent screen. The image of this screen in the parallax mirror below gave a bright background against which the wire and its image were easily distinguished.

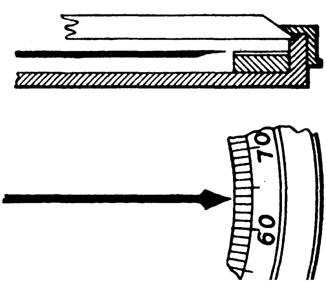


FIG. 2.41.—Kelvin antiparallax device.

The scale is now almost universally a metal plate carrying an enamel or cardboard surface upon which the instrument scale is marked. In early types a silvered dial was employed upon which the scale was engraved, but this has proved both unsatisfactory and expensive; the scale is only clearly visible with a careful arrangement of lighting, on account of the sheen and glare from the metal surface, and the contrast between the divisions and the polished metal is not sufficiently great, while in time the silvering is liable to tarnish and become streaky. The

white enamel dial with painted scale is an improvement on the silvering, and quite usual practice is to use a dial plate finished with a matt white enamel and to show the scale in indian ink. In some cases the scales and the necessary legends are entirely written by hand, while in others special scale-drawing machines are employed. Free expansion must be provided for in mounting the scale in its position in the instrument by allowing a slight amount of freedom between the plate and its supports, as otherwise buckling may occur and cause the pointer to foul.

For this reason the fan-shaped scale plate supported from a central tongue, as employed by the Weston Instrument Company and subsequently by several other makers, has the advantage of allowing perfect freedom for expansion, and hence there is little or no deformation. Certainly it is better to support the scale plate from a single point, rather than risk the tendency to buckle that may occur when supported between two or more rigid supports.

In most instruments it is usual to limit the travel of the pointer at the extreme ends of the scale by buffer or stop-pins. In some cases these are simply pins inserted in the scale plate and covered with rubber tubing, but a much better construction is to employ light, curved spring wires with upturned ends against which the pointer strikes, these ends being insulated by means of sleeves of rubber, glass or plastic material. They should possess great resilience to be effective, for if they are too rigid the pointer may strike them very sharply on momentary overload or reversal, and although the movement may not be otherwise damaged, the impact may bend the pointer or seriously displace the delicate knife-edge tip.

INSTRUMENT CASES

The case of an instrument should be constructed so as to be dust- and moisture-proof. At the same time accessibility for final adjustment should be considered, and will prove economical.

The round metal case is used quite generally for switchboard instruments, moulded bakelite cases for miniature instruments, and square hardwood or bakelite moulded cases for portable instruments. With round cases the base usually carries the movement, and a round cover, in which is the observation window, fits over it. In many instruments the base and cover are of cast-iron or pressed steel, advantage being taken of this to screen the movement from

the disturbing influence of external magnetic fields (see Chapter 7), or sometimes the cover is a light brass spinning, a non-ferrous die-casting or of moulded material.

However constructed, the cover should be provided with an ample flange where it meets the base-plate, and a gasket inserted in the joint to make it dust- and waterproof. Leakage of dust into the instrument is, however, most frequently traced to the imperfect contact made between the glass of the observation window and the cover itself. The practice of bedding this in with putty or plaster-of-paris can only be condemned, for, although when first finished the joint may be satisfactory, these materials harden and shrink with time, and vibration, may then cause them to drop away from the glass and metal surfaces. A gap then occurs through which dust can enter, and the particles of cement which have fallen inside may do serious damage to the movement. Cleating the glass to a properly-cut, felted-cloth, or velvet joint is much more satisfactory if well carried out, and this, incidentally, minimizes the liability to fracture.

The risk of a fractured observation window in portable instruments and testing sets may be greatly reduced by using one of the various types of toughened glass, now so generally employed in motor-car fitting. The size of the observation window is also of considerable importance, and the visibility of the pointer when observed at some distance will depend both on the illumination of the scale and also upon the length of pointer uncovered. Such instruments, therefore, are better provided with a completely glazed front so that the maximum amount of light can fall on the dial-plate, and the whole pointer is then visible. In the effort to save switchboard space, where a large number of similar instruments are to be installed, the edgewise type of instrument has been developed. In this the scale is in the form of a comparatively narrow band, bent to an arc, along which the pointer moves, the head being bent over sharply at right angles so as to be visible on the surface of the scale behind the curved glass window. Undoubtedly such a construction saves space (although the front projection is increased, unless the instruments are inset in the board), but precise observations with such instruments, particularly at the ends of the scale, is difficult, and in many cases impossible, on account of the large parallax errors which occur, and, moreover, the reflections from the bent glass window are liable to become very troublesome. Edgewise instruments with a flat

scale have also been tried. A device, which is also used for recorders (see Part II), is employed in which the centre about which the pointer rotates is provided with a glass roller moving between sharp-edge parallel guides: the axis of rotation of the movement is linked to this pointer, and on deflection shifts the pointer axis backwards or forwards between its guides, and its indicating end thus traverses a straight line.

In many of the round-type instruments the back of the base-plate is recessed to accommodate the series resistance or the ammeter shunt, the outer flange being notched out to allow of free circulation of air around the resistance, which is usually wound in a rectangular framework and secured to the back. Although from the point of view of heating this practice has some advantages, it suffers from the very serious disadvantage that the resistance rapidly becomes covered with a heavy coating of dust, which is carried up and deposited from the stream of air which the heated resistance promotes, and it then becomes difficult to maintain the insulation of the coil satisfactorily.

Illuminated dials are often used on instruments to be read at considerable range, or in cases where the general illumination is not good. This type of instrument is usually of sector shape, the upper part of the sector being a narrow framework, glazed on the front with the ordinary observation window, and on the back with an opal or milk glass upon the inner face of which the instrument scale is painted. In the narrow space between these two glasses the head of the pointer moves, and behind the opal scale plate electric lamps and reflectors are arranged so as to secure a uniform and brilliant illumination of the scale as seen from the front of the instrument.

Where instruments are intended for use in damp situations it is usual to specify watertight cases. The construction of a really good watertight joint demands considerable care, and the chief points of weakness are in the packing of the observation window and the lugs. Usually the surfaces brought into contact are made broad, machined, and brought together with a rubber packing ring between them, and then secured by screws. The best joint is made by distributing the pressure uniformly round the flange, and hence it is better practice to employ several small screws at close intervals round the joint than a few large screws widely spaced. The perfect watertight joint is probably that shown in Fig. 2.42, where the two machined surfaces are provided, one with a recess, and the

other with a rib or ridge coinciding with it. A rubber ring of circular section is inserted in the recess before the surfaces are screwed up, and the compression of this makes a thoroughly sound job. But such a joint is probably better than is required in most commercial instruments, unless they are required for complete immersion.

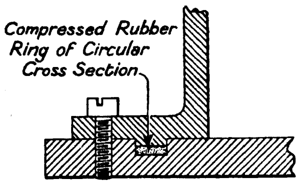


FIG. 2.42.—Method of making watertight joint.

The lugs should also be brought through glands, and the chief difficulty here is to secure insulation as well as watertightness. A satisfactory form of gland is shown in Fig. 2.43, but usually it is better to take a twin or concentric cable right into the instrument through a central single gland of this type.

Much time and trouble in assembly may be saved by giving the entire inside of the case and base a coating of dull enamel of light

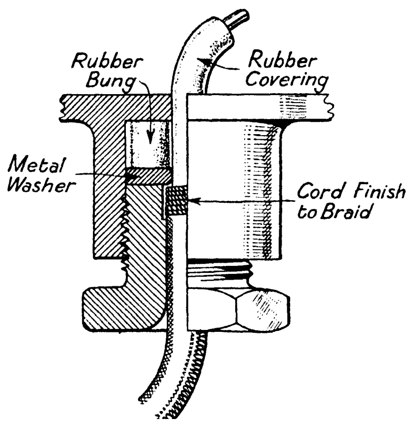


FIG. 2.43.—Watertight gland for leading-in wire.

colour (preferably matt white), as then dust, minute hairs, fibres, fluff, and other foreign matter, which are always liable to creep in on assembly, may be easily detected and removed by the workman; and, moreover, it provides more illumination, and forms an excellent background for the final adjustments, besides giving the apparatus a well-finished appearance.

CHAPTER 3

CONDITIONS OF RAPID INDICATION

NEXT in importance to accuracy of principle in an instrument is promptitude of indication. In fact, on all but the most steady sources of supply, high accuracy is thrown away if the instrument does not register quickly. The ideal instrument should instantly indicate its reading when connection is made, and follow any variations without lag or oscillation. Unfortunately it is impossible to realize this ideal completely, owing to the inertia of the moving system, which causes it to lag behind the variations and to oscillate, unless suitable damping is provided. The general conditions for securing promptitude of reading apply to all instruments, and may be dealt with once for all. This matter is of great practical importance, and is presented here in a form which has been found most useful to designers of instruments.

Steady Torque applied to an Instrument

One important case is that in which the torque applied to an instrument is constant in value, i.e. it does not change as the movement deflects. This applies particularly to moving coil instruments of the usual type of uniform airgap, but does not necessarily apply to electro-magnetic, or electrostatic instruments, or to moving coil instruments with non-uniform air-gap, in all of which the torque applied to the movement may vary with its angular position. This also means that the currents flowing through the working parts of the instrument reach their final values immediately, i.e. the circuits in which the instrument is connected contain only resistance, and there are no reactive elements to retard or in any way modify the instantaneous growth of the current. If the applied torque is not constant, then the time-deflection characteristic is modified, and such conditions are dealt with later.

It is therefore assumed that a constant torque is suddenly applied to an instrument with its movement at rest in the zero position. The movement possesses inertia, the motion is damped, and is subject to a control such as that of a spring. The applied torque has, therefore, to supply the following components :—

(1) The torque required to accelerate the movement from rest, i.e. the torque required to overcome the inertia of the

movement. This is $Kd^2\theta/dt^2$, where K is the moment of inertia of the complete movement, and θ is the angular deflection in radians at any time t seconds after the application of the torque.

(2) The torque required to overcome the damping torque. This is $D.d\theta/dt$, where D is known as the damping coefficient. It is assumed that the damping torque is proportional to the angular velocity of the movement. This is strictly true when magnetic damping is used and has been found by experiment to be reasonably true when air-damping is used, for the comparatively low velocities encountered in electrical instruments.

(3) The torque required to overcome the control torque. This is $s\theta$, where s is the control torque in dyne-cm. per radian.

The equation giving the deflection θ radians at t seconds is thus

$$K\frac{d^2\theta}{dt^2} + D.\frac{d\theta}{dt} + s\theta = T \quad . \quad . \quad . \quad (1)$$

where T is the applied torque.

The solution of this equation will give the required information as to the manner in which the deflection varies with the time.

It is clear that when the instrument has come to rest and taken up its final steady deflection Θ, the whole of the applied torque T is taken up in overcoming the control torque. So $T = s\Theta$ and equation (1) can be rewritten

$$K\frac{d^2\theta}{dt^2} + D.\frac{d\theta}{dt} + s\theta = s\Theta \quad . \quad . \quad . \quad (2)$$

Notes on the General Solution

Since a constant torque is suddenly applied, a solution to equation (2) can be obtained by Heaviside's Operational Calculus. Writing p for the operator d/dt and solving for θ gives as the operational solution

$$\theta = \frac{s\Theta/K}{p^2 + pD/K + S/K} \quad . \quad . \quad . \quad (3)$$

Any solution of this equation involves a knowledge of the roots of the equation

$$p^2 + p.\frac{D}{K} + \frac{s}{K} = 0 \quad . \quad . \quad . \quad (4)$$

These roots are $p = -\frac{D}{2K} \pm \sqrt{\left(\frac{D}{2K}\right)^2 - \frac{s}{K}}$ . . . (5)

Now these roots involve the quantities, K the moment of inertia of the moving system and D the damping coefficient, both of which vary considerably from instrument to instrument and are not easy to determine experimentally. They can be replaced by other characteristic quantities which are more easily ascertained and are more easily handled in design.

The natural period of vibration of the undamped moving system is

$$T_\lambda = 2\pi\sqrt{\frac{s}{K}} \qquad . \quad . \quad . \quad . \quad (6)$$

and so

$$\sqrt{\frac{s}{K}} = \frac{2\pi}{T_\lambda} \qquad . \quad . \quad . \quad . \quad (7)$$

Again the roots of equation (4) are equal, if $(D/2K)^2 = s/K$. This is the case of critical damping, when the motion is just non-oscillatory. Suppose the value of the damping coefficient for this condition is D_0, then

$$\left(\frac{D_0}{2K}\right)^2 = \frac{s}{K} = \frac{4\pi^2}{T_\lambda^2} \qquad . \quad . \quad . \quad (8)$$

Let us now define a " damping ratio " n equal to D/D_0, so that $D = nD_0$. Then

$$\left(\frac{D}{2K}\right)^2 = n^2\left(\frac{D_0}{2K}\right)^2 = \frac{4\pi^2 n^2}{T_\lambda^2} \qquad . \quad . \quad (9)$$

The roots of equation (4) as given by (5) can now be rewritten

$$p = \frac{2\pi}{T_\lambda}\left\{-n \pm \sqrt{n^2 - 1}\right\} \qquad . \quad . \quad . \quad (10)$$

If the damping is less than that required for critical damping, then n is less than unity, and the roots are complex, leading to an oscillatory solution. If n is equal to 1 the motion is critically damped, while if n is greater than 1 the motion is overdamped.

The Underdamped Condition

When the damping ratio n is less than unity, the roots of equation (4) become

$$p = \frac{2\pi}{T_\lambda}\left\{-n \pm j\sqrt{1 - n^2}\right\} \qquad . \quad . \quad (11)$$

where $j = \sqrt{-1}$. Since various functions of n enter into the solution it is convenient to write $n = \cos\varphi$, then

$$p = \frac{2\pi}{T_\lambda}\left\{-\cos\varphi + j\sin\varphi\right\} \qquad . \quad . \quad (12)$$

The solution in this case is given by

$$\theta = \Theta\left[1 - e^{-\frac{2\pi\cos\varphi\, t}{T\lambda}}\left\{\cot\varphi \sin(2\,\pi\sin\varphi)\frac{t}{T_\lambda} + \cos\left(2\pi\sin\varphi\frac{t}{T_\lambda}\right)\right\}\right] \quad . \quad . \quad . \quad (13)$$

Now let $t/T_\lambda = \lambda$, so that λ is a measure of time expressed as a fraction of the natural period of oscillation of the instrument. Also let $\theta/\Theta = y$, so that y is a measure of the deflection expressed as a fraction of the final steady deflection. Then equation (13) becomes

$$y = 1 - e^{-2\pi\lambda\cos\varphi}\{\cot\varphi\sin(2\pi\sin\varphi)\lambda + \cos(2\pi\sin\varphi)\lambda\} \quad (14)$$

Now it follows from equation (14) that the actual period of oscillation with a damping ratio $n = \cos\varphi$, which is known as the damped period of oscillation, T_0, is

$$T_0 = T_\lambda/\sin\varphi \quad . \quad . \quad . \quad (15)$$

If the time, instead of being expressed as a fraction of the natural period is expressed as a fraction q of the damped period, T_0, then

$$q = \lambda\sin\varphi \quad . \quad . \quad . \quad . \quad (16)$$

and equation (14) becomes

$$y = 1 - e^{-2\pi q\cot\varphi}\{\cot\varphi\sin 2\pi q + \cos 2\pi q\} \quad (17)$$

It will be noted that this equation is dimensionless, and is applicable to all instruments meeting the specified conditions. This equation is in a convenient form for plotting curves of the performance and typical curves for values of $\cos\varphi$ of 0·2, 0·5 and 0·9 are given in Fig. 3.1. The ordinates y of these curves are measures of the deflection expressed as a percentage of the final steady value, and the abscissae q are measures of the time expressed as a fraction of the damped period.

The Critically Damped Condition

When the damping ratio is unity, the motion is critically damped and the two roots of equations (4) are equal and become

$$p = \frac{2\pi}{T\lambda} \quad . \quad . \quad . \quad . \quad (18)$$

In this case the solution is

$$\theta = \Theta\left[1 - e^{-\frac{2\pi t}{T}}\left\{2\pi\frac{t}{T_\lambda} + 1\right\}\right] \quad . \quad . \quad (19)$$

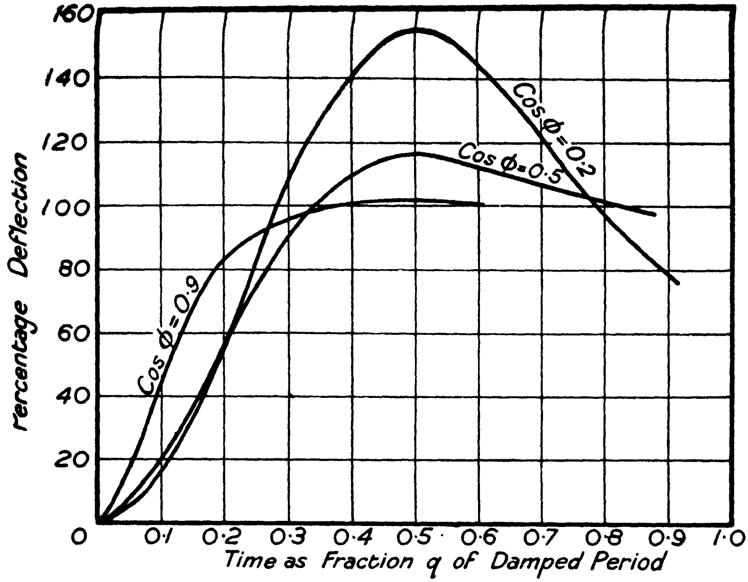


FIG. 3.1.—Underdamped motion.

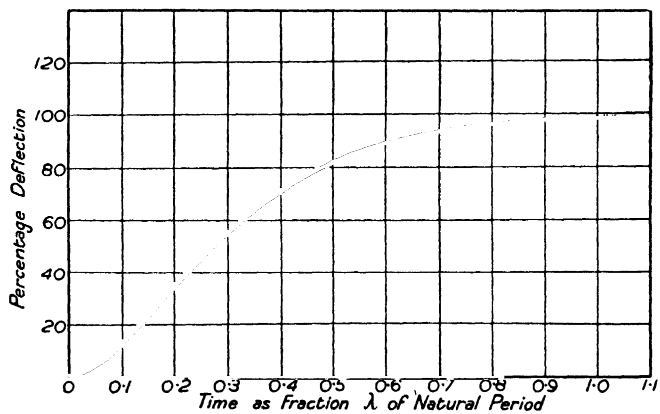


FIG. 3.2.—Critically damped motion.

or using the same symbols as before

$$y = 1 - e^{-2\pi\lambda}(2\pi\lambda + 1) \quad . \quad . \quad . \quad (20)$$

There is no damped period in this case and the natural period is used as a measure of the time. A curve showing the relation between y and λ is given in Fig. 3.2 for the critically damped case.

The Overdamped Condition

When the motion is overdamped, the motion is non-oscillatory and the roots of equation (4) are real. In this case it is convenient to write $n = \cosh \varphi$ and the solution becomes

$$\theta = \Theta\left[1 - e^{-\frac{2\pi \cosh \varphi . t}{T_\lambda}}\left\{\coth \varphi \sinh (2\pi \sinh \varphi)\frac{t}{T_\lambda} + \cosh (2\pi \sinh \varphi)\frac{t}{T_\lambda}\right\}\right] \quad . \quad . \quad . \quad (21)$$

As before writing $\theta/\Theta = y$ and $t/T_\lambda = \lambda$ this becomes

$$y = 1 - e^{-2\pi\lambda \cosh \varphi}\{\coth \varphi \sinh (2\pi\lambda \sinh \varphi) + \cosh (2\pi\lambda \sinh \varphi)\} \quad (22)$$

Since the motion is non-oscillatory it is not possible to define a "damped period" in the same manner as in the underdamped case. It is, however, convenient to define an imaginary period T_0 such that $T_0 \sinh \varphi = T_\lambda$. If then $t/T_0 = q$, it follows that $\lambda \sinh \varphi = q$ and the solution can be written

$$y = 1 - e^{-2\pi q \coth \varphi}\{\coth \varphi \sinh 2\pi q + \cosh 2\pi q\} \qquad (23)$$

In this way the solution is similar to equation (17) for the underdamped case. Curves showing the relationship between y and q for values of $n = \cosh \varphi$ of 1·2, 1·5 and 2·0 are given in Fig. 3.3.

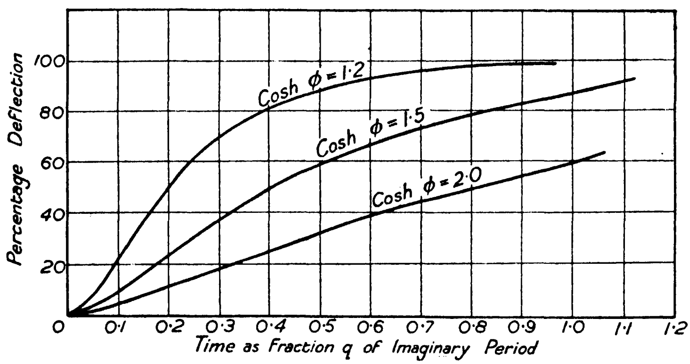


FIG. 3·3.—Overdamped motion.

The Natural Period as a Common Factor

While the damped or imaginary periods are convenient for illustrating the underdamped or overdamped motions individually, they are not so convenient when it is desirable to obtain a complete picture of the motion under any condition of damping. It is not

possible, for instance, to include the case of critical damping, since the damped period in this case is infinite. The natural period is therefore taken as a common factor, since this depends only on the moment of inertia and spring control of the instrument and is independent of the damping.

From curves similar to those of Figs. 3.1, 3.2 and 3.3, it is possible to read off the time, expressed as a fraction of the damped, natural or imaginary periods, at which the deflection reaches 10%, 20%, 30%, etc., of the final steady value. These times can be converted into fractions of the natural period, λ, since

$\lambda = q/\sin\varphi$ for damping ratio less than unity.
$\lambda = q/\sinh\varphi$ for damping ratios greater than unity.

Curves of this type are plotted in Fig. 3.4 for various percentage deflections. The oscillatory nature of the deflection for values of the damping ratio less than unity is indicated by the looped curves. From these curves it is possible to trace the deflection of the pointer for any value of the damping ratio between 0·1 and 10, and also to determine the time, in terms of the natural period, required to reach any stated percentage of the final steady deflection.

Response Times and Response Curves

An important time for any instrument is that which can loosely be described as the time taken to come to rest. Theoretically, according to equations (17), (20) and (23) the movement does not come to rest until an infinite time has elapsed. Actually the friction between the jewels and pivots does bring it to rest in a reasonable time. The response time of an instrument can be defined as the time which elapses from the moment of switching on until the pointer reaches the final reading within the stated accuracy of the instrument and after that time remains within that accuracy. For example, if an instrument has a stated accuracy of $\pm$ 1%, then the response time would be that time required for the pointer to reach and stay within $\pm$ 1% of the final value.

For instruments which have no overswing, i.e. critically or over-damped instruments, the damping ratio being equal to or greater than unity, the response time is evidently the same as the time taken by the pointer to travel from its starting point to a point within the accuracy limit, and the response curves for such instruments are the same as the deflection curves, Fig. 3.4. Thus the

response curve for an instrument having an accuracy of $\pm$ 1% and a damping ratio equal to or greater than unity is the same as the curve marked 99% in Fig. 3.4.

In the case of an underdamped instrument the pointer may overswing several times before it stays within the desired accuracy limit, and the response curve for such instruments does not there-

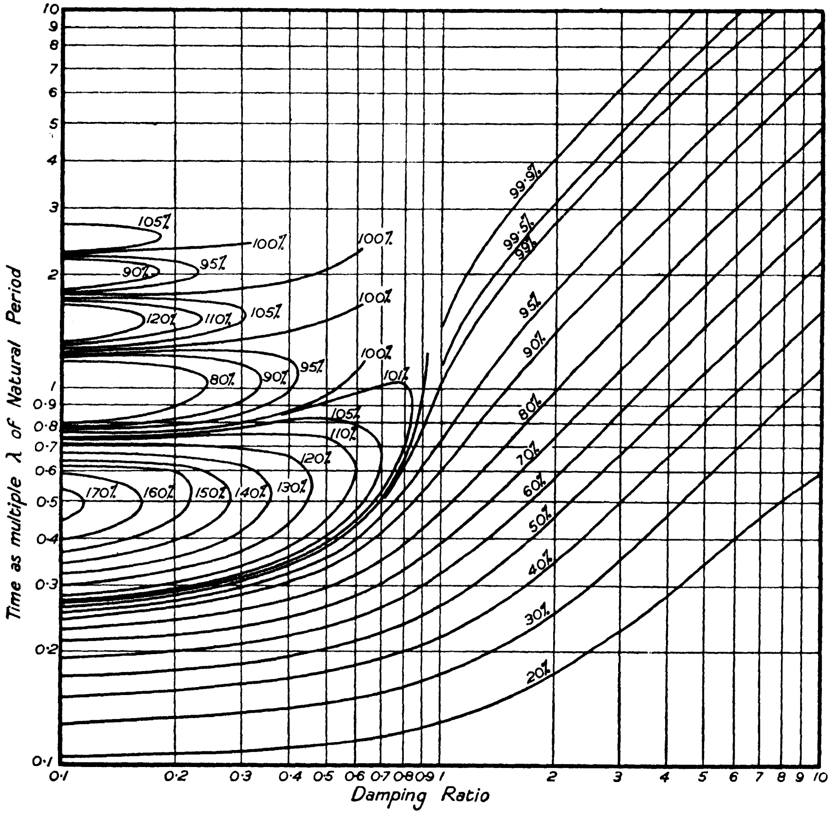


FIG. 3.4.—Time/deflection curves.

fore correspond exactly with any one deflection curve. The response curves for these instruments may be calculated as follows :—

For an underdamped motion such as that shown in Fig. 3.1, it can be shown from equation (17) that the peaks have the values

$$y_{\max} = 1 \pm e^{-2\pi m \cot \varphi}$$

where $m = 0, \frac{1}{2}, 1\frac{1}{2}, \ldots$ etc., damped periods, the plus sign

is used for $m = \frac{1}{2}, 1\frac{1}{2}, 2\frac{1}{2} \ldots$ etc., and the negative sign when $m = 0, 1, 2 \ldots$ etc. The equations of the envelopes to this curve may therefore be written

$$y = 1 \pm \exp(-2\pi \cot \varphi)$$

The response time may be taken to a sufficient degree of accuracy as given by the intersection of the envelopes with the limit lines. The difference between the final steady reading and the envelopes is $\exp(-2\pi q \cot \phi)$. If the accuracy limit is μ expressed as a fraction then

$$\mu = \exp(-2\pi q \cot \varphi),$$

from which
$$q = \frac{1}{2\pi \cot \varphi} \log_e \frac{1}{\mu}$$

and since

$$q = \lambda \sin \varphi$$

$$\lambda = \frac{1}{2\pi \cos \varphi} \log_e \frac{1}{\mu} \quad . \quad . \quad . \quad (24)$$

This gives the response time up to a certain value of the damping ratio. Now consider that value of the damping ratio where the overswing is such that the curve just reaches the upper limit line. This is illustrated in Fig. 3.5. An application of equation (24) to

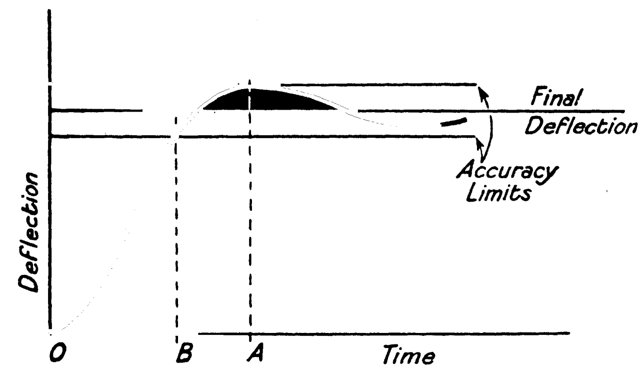


FIG. 3.5.—Effect of damping ratio on response time.

this case would give the response time as OA. It is clear from Fig. 3.5, however, that in this case the deflection lies within the accuracy limits at all times greater than OB. At the value of damping ratio which corresponds to this condition there is a discontinuity in the response curves, the time dropping from the value given by

equation (24) to the normal deflection curve corresponding to n.

Complete response curves for values of damping ratio from 0·1 to 10 are plotted in Fig. 3.6 for various values of μ.

The natural period of the instrument which is used as a measure of the time depends on the spring control and the moment of inertia. In designing an instrument these two quantities may be determined by considerations other than the speed of response, although it is

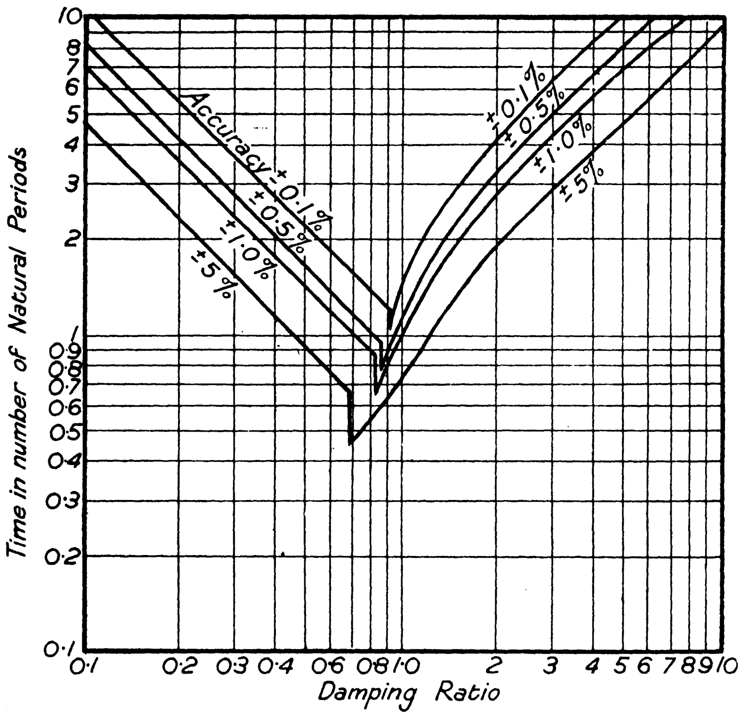


FIG. 3.6.—Response curves.

obvious that for a rapid response a low natural period is necessary. For a given natural period it is evident from the response curves of Fig. 3.6 that there is one value of damping ratio for any given accuracy which gives the most rapid response. These values are given in Table III.

For instruments having an accuracy better than $\pm 1\%$ the damping required for the quickest response is slightly less than critical damping.

TABLE III.

μ as percentage.	Damping ratio.
0·1	0·910
0·2	0·893
0·5	0·860
1·0	0·826
2·0	0·780
5·0	0·689
10·0	0·591
20·0	0·454
50·0	0·214

Use of Curves

In some cases it is not the response time as defined in the previous section which is specified, as it is sometimes stated that an instrument must reach a certain percentage of its final deflection in a certain time, and that the overswing must not exceed a certain percentage. From the overswing the damping ratio may be calculated as described in the next section.

Assuming that the natural period is known, as it would be if the design of an instrument were under consideration, the time required to reach any specified percentage of the final deflection can be obtained from Fig. 3.4.

If, on the other hand, the accuracy and the response time are given and the natural period is again known, it is possible from the response curves in Fig. 3.6 to determine the damping ratio which is required.

Relation between Damping Factor and Damping Ratio

In the case of an underdamped motion, equation (17), the motion is of an oscillatory nature, as shown in Fig. 3.7. There are a number of maxima and minima, and the quantity known as the damping factor F is defined as

$$F = \frac{\Theta}{\theta_1 - \Theta} = \frac{\theta_1 - \Theta}{\Theta - \theta_2} = \frac{\Theta - \theta_2}{\theta_3 - \Theta} \text{ etc.} \qquad (25)$$

The damping factor so defined is a quantity which can readily be determined by observation, except in those cases when the damp-

ing ratio is close to unity, i.e. critical damping. From this damping factor the damping ratio can be determined as follows :—

The maxima and minima of equation (17) will occur at those values of q for which $dy/dq = 0$. Differentiating equation (17) and equating

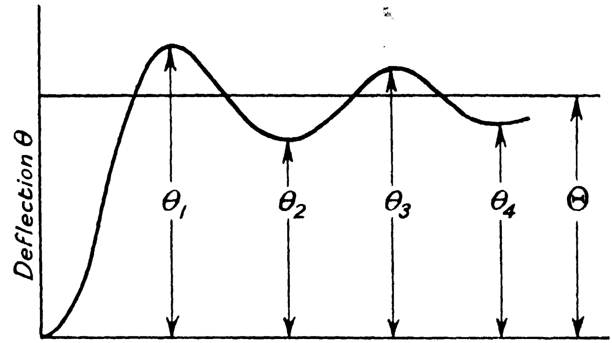


FIG. 3.7.—Typical oscillatory deflection.

to zero gives $\sin 2\pi q = 0$ from which $2\pi q = 0$ or $m\pi$, where m is any integer. The successive maxima and minima then become

$$\begin{aligned} \theta_1 &= 1 + \exp.\,(-\ \pi \cot \varphi) \\ \theta_2 &= 1 - \exp.\,(-\,2\pi \cot \varphi) \\ \theta_3 &= 1 + \exp.\,(-\,3\pi \cot \varphi) \\ &\text{etc.} \end{aligned}$$

Substituting these values in equation (25) gives for the damping factor

$$F = \exp.\,(\pi \cot \varphi) \quad . \quad . \quad . \quad . \quad (26)$$

Hence

$$\cot \varphi = \frac{1}{\pi} \log_{\varepsilon} F \quad . \quad . \quad . \quad . \quad (27)$$

from which the damping ratio ($n = \cos \varphi$) can be calculated for any given value of F. A curve showing the relationship between damping factor and damping ratio is given in Fig. 3.8.

PRACTICAL DETERMINATION OF TIME-DEFLECTION CURVES

In order to check the accuracy of a design which has taken into account speed of response and damping it is necessary to be able to obtain a curve showing how the pointer deflection varies with time.

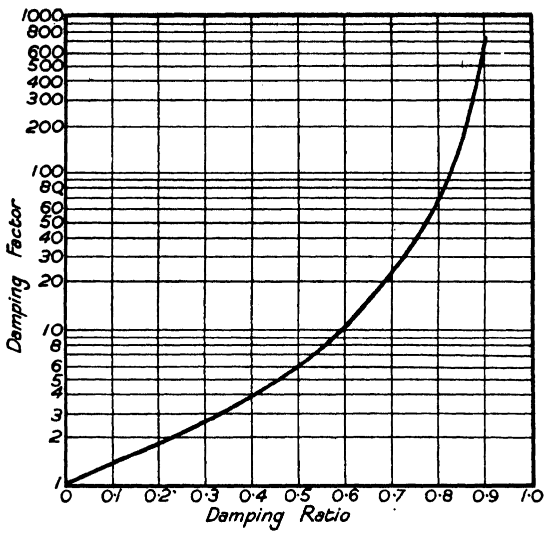


FIG. 3.8.—Relation between damping factor and ratio.

This involves the use of a combined timing and viewing mechanism so that the position of the pointer can be observed at any given time after the instrument is switched on. Descriptions are given here of two forms of device, the first consisting of a resistance-capacity circuit for timing and a discharge lamp for viewing, the other being a mechanical device in which the timing is obtained by the aid of a synchronous motor and the viewing is done through a shutter tripped by the motor at the desired time.

The Resistance Capacity System

If a capacity is charged at a constant rate, the potential difference between its terminals will increase in proportion to the rate of flow and time, i.e.

$$Q = It = CV \text{ or } V = \frac{C}{It}$$

where Q = quantity in coulombs.
I = current in amperes.
t = time in seconds.
C = capacity in farads.
V = potential difference between the terminals.

If the capacity C is fixed and the potential difference V between the terminals is constant, then the time, t secs., taken to charge the capacity to this potential difference will be inversely proportional to the current I. This is used in the device of which a diagram of connections is given in Fig. 3.9.

The capacity C_1 is charged linearly through the pentode valve V_1, thus carrying the cathode of valve V_2 negative, relative to its anode. The control grid of V_2, however, is already negative with regard to its anode, because of the voltage drop in R_1. This will drive the suppressor grid of V_3 negative, thus reducing the anode current of V_3 and driving the anode of V_3 and hence the control

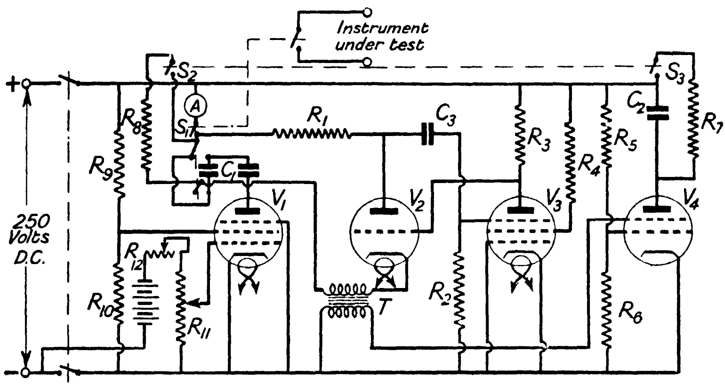


FIG. 3.9.—Resistance-capacitance timing and viewing circuit.

grid of V_2 positive. This immediately increases the anode current of V_2 and hence the action becomes cumulative, the capacity C_1 being discharged rapidly through V_2.

The discharge current pulse is passed through the primary of the transformer T, thus inducing a voltage in the secondary, which is in the circuit of the second grid of the Strobotron lamp V_3. This lamp has its electrode potentials set such that the impulse voltage in the second grid is sufficient to cause the lamp to fire, and this provides the high intensity flash of light required for observation purposes. The firing of the Strobotron, however, also charges the capacity C_2, and this reduces the anode volts to a value sufficient to extinguish the lamp. Thus a single flash of light is produced.

An ammeter in the anode circuit of the pentode valve V_1 will

read inversely proportional to the time lag required. The battery B and the potentiometer P serve to vary the bias of the pentode control grid and thereby vary the anode current. Calibration is effected by setting the controls so that the time lag is sufficiently long to be measured accurately on a stop-watch, and then relying on the inverse relationship between current and time.

The switch S_1 consisting of two ganged switches is used to control simultaneously the apparatus described and the instrument under observation. It is necessary to ensure that the capacities C_1 and C_2 are discharged before taking an observation, and the ganged switches with the resistances R_7 and R_8 are provided for this purpose.

The pentode valves V_1 and V_3 are used on that part of their characteristic curves where the anode current is practically unaffected by variations in the supply voltage.

The Synchronous Motor-shutter Device

The principle of operation of the synchronous motor-operated shutter device is shown diagrammatically in Fig. 3.10. A synchronous motor M of the type used in electric clocks drives an arm A through a train of gears so that A makes one revolution in two

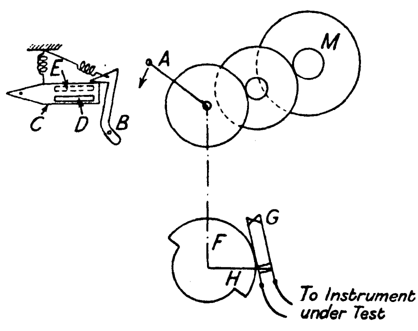


FIG. 3.10.—Principle of timing and viewing mechanism.

seconds. At one point in its revolution the arm A hits the spring-urged pawl B and releases the spring-driven shutter C. This shutter has a slot D, and when the shutter C is released this slot flies past a corresponding slot E in a fixed plate, allowing momentary observation of an instrument through the two slots. Attached to the

spindle carrying the arm A is a cam F, and loosely mounted on the spindle is an arm H carrying contacts G operated by the cam F. The arm H can be moved round so that the contacts G are closed any desired time, within the range of the apparatus, before the arm A releases the shutter C. The contacts G close the instrument circuit.

The cam F is so shaped that the contacts G are closed for one-half of a revolution of the arm A and open for the other half, thus closing the instrument circuit for half a revolution and opening it for half. This is done so that the instrument can come to rest after switching on or switching off before the next switching off or on operation. The action is continuous, and if the shutter C is reset by hand after every operation the apparatus can be held in front of the instrument under observation, and the instrument viewed through the slots D and E until the exact position of the pointer for the given time setting is determined.

The arm H carries a pointer moving over a scale 180° long, scaled 0–1 second, and is fitted with a vernier so that the time may be determined accurately. The cam F is adjusted so that with the arm H at zero time, the contacts close at the same instant that the shutter C is operated.

The manner in which the apparatus is used is as follows: The instrument is connected up in the circuit in which it is to be tested, the contacts G being included in the instrument circuit. The apparatus is held so that the scale of the instrument can be seen through the slots D and E, the time as given by the pointer on the arm H set to various values and the position of the instrument pointer observed.

This apparatus can only be used for these instruments which come to rest completely in one second. For instruments which take a longer time than this the resistance capacity system is used.

Typical Deflection Curves

A set of typical deflection-time curves obtained with a small moving coil instrument in which the moving coil was frameless are given in Fig. 3.11. Since the coil had no metal frame or former, the whole of the damping was provided by currents flowing in the main circuit, and the curves show how this damping is affected by the resistance of this circuit. The circuit diagram of the instru-

ment is given in Fig. 3.12, and Table IV gives the values of the circuit resistances for the various curves.

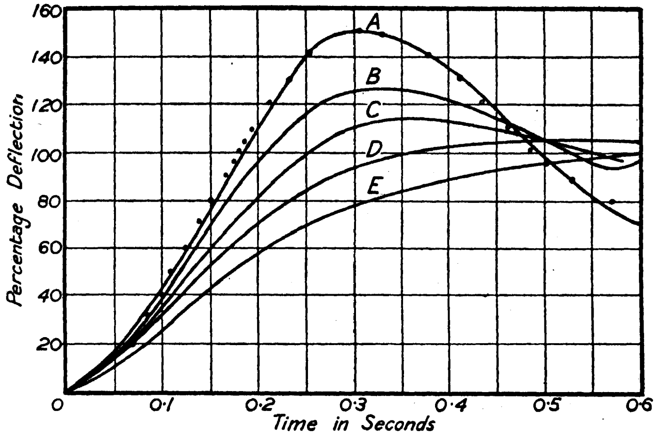


FIG. 3.11.—Typical deflection curves.

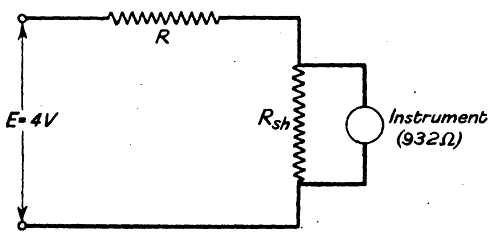


FIG. 3.12.—Circuit for curves of Fig. 3.11.

TABLE IV.

Curve No.	R.	S.	Damping factor.	Damping ratio.
A	84,400 Ω	15,000 Ω	1·97	0·210
B	80,000	6,080	3·88	0·394
C	76,000	3,986	7·56	0·542
D	68,500	2,540	33·3	0·743
E	66,000	1,498	—	1·08

From curves such as these it is easy to determine whether an instrument has the required degree of overswing and time of response.

Correlation of Practical Curve and Theory

It is a simple matter with the aid of Figs. 3.4 and 3.8, to correlate any practical curve with the corresponding theoretical one. It is first necessary to find the damping ratio. The damping factor can be determined from the overswing, and the values for the curves of Fig. 3.11 are given in Table IV. The corresponding values of damping ratio are then found with the aid of the curve in Fig. 3·8. The manner in which the theoretical curve is checked against the practical curve is best illustrated by taking curve *A* in Fig. 3.11 as an example. The first maximum occurs at a time equal to half the damped period T_0. So from the curve $T_0/2 = 0{\cdot}314$ sec., and $T_0 = 0{\cdot}628$ sec. Since the damping ratio $\cos\varphi = 0{\cdot}210$ and the natural period $T_\lambda = T_0 \sin\varphi$, then

$$T_\lambda = T_0 \sin(\cos^{-1} 0{\cdot}210) = 0{\cdot}628 \times 0{\cdot}9777 = 0{\cdot}613 \text{ sec.}$$

On referring now to the curves in Fig. 3.4 the times required to reach any given percentage deflection can be read off for a damping ratio of 0·21 and converted into actual seconds since $T\lambda = 0{\cdot}613$ sec. The figures are given in Table V.

TABLE V.

% defln.	λ.	Seconds.	% defln.	λ.	Seconds.
20	0·109	0·069	150	0·494	0·303
30	0·134	0·085	150	0·540	0·331
40	0·159	0·100	140	0·620	0·380
50	0·180	0·110	130	0·675	0·414
60	0·202	0·124	120	0·710	0·436
70	0·224	0·137	110	0·760	0·466
80	0·241	0·148	105	0·775	0·475
90	0·267	0·164	100	0·795	0·488
95	0·282	0·173	95	0·832	0·510
100	0·291	0·179	90	0·860	0·527
105	0·302	0·185	80	0·930	0·571
110	0·316	0·194			
120	0·340	0·208			
130	0·374	0·230			
140	0·412	0·253			

The theoretical figures from Table V are shown in Fig. 3.11, and it will be seen that there is quite good agreement between the practical and theoretical curves.

Practical Determination of the Damping Ratio

As a practical check on a design it is necessary to be able to determine the damping ratio from the deflection curve. When there is a reasonable overswing as in the curves *A–D* of Fig. 3.11 this presents no difficulty. All that is necessary is to determine the damping factor from the value of the first overswing, and then read off the damping ratio from the curve of Fig. 3.8.

In cases where the overswing is not measurable or there is no overswing at all this method cannot be used, and another method

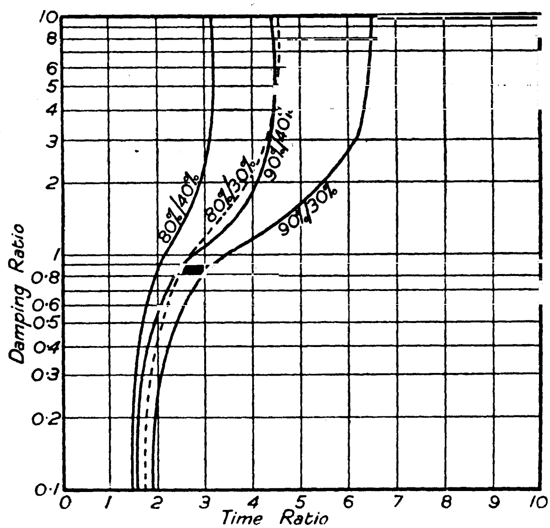


FIG. 3.13.—Time ratio curves for determining damping ratio.

is necessary. If the time taken to reach, say, 90% of the steady deflection is divided by the time taken to reach, say, 30%, the number so obtained varies with the damping ratio. These times, in terms of the natural period, can be taken from Fig. 3.4, and the ratio of the times for any two values of percentage deflection can be calculated and plotted against the damping ratio. Curves of this type are given in Fig. 3.13 for four values of time ratio, and these can be used to determine the damping ratio for any instrument.

This can be explained by reference to curve *E* of Fig. 3.11, where there is no overswing. From this curve :—

Time required to reach 90% of the steady deflection = 0·427 sec.
,, ,, ,, 80% ,, ,, ,, = 0·324 ,,
,, ,, ,, 40% ,, ,, ,, = 0·144 ,,
,, ,, ,, 30% ,, ,, ,, = 0·112 ,,

$$\text{So ratio } \frac{\text{time required to reach } 90\%}{\text{,, \quad ,, \quad ,, \quad } 30\%} = \frac{0{\cdot}427}{0{\cdot}122} = 3{\cdot}82$$

$$\frac{\text{time required to reach } 90\%}{\text{,, \quad ,, \quad ,, \quad } 40\%} = \frac{0{\cdot}427}{0{\cdot}144} = 2{\cdot}97$$

$$\frac{\text{time required to reach } 80\%}{\text{,, \quad ,, \quad ,, \quad } 40\%} = \frac{0{\cdot}324}{0{\cdot}144} = 2{\cdot}25$$

$$\frac{\text{time required to reach } 80\%}{\text{,, \quad ,, \quad ,, \quad } 30\%} = \frac{0{\cdot}324}{0{\cdot}112} = 2{\cdot}89$$

Referring to the curves in Fig. 3.13, the values of the damping ratio given by the figures are 1·08, 1·07, 1·08 and 1·10, an average value of 1·08.

EFFECT OF THE FORM OF SCALE ON PERIODIC TIME AND DAMPING

The preceding discussion applies to an instrument in which the torque is constant, and the scale shape follows the normal law for the type of instrument concerned. There are, however, a number of cases in which the scale shape is deliberately distorted from the normal law, such distorted shapes being of greater value than the normal for certain applications. For such instruments the time of oscillation and the damping will vary with the deflection, as shown by the curves of Fig. 3.14, which are the time-deflection charac-

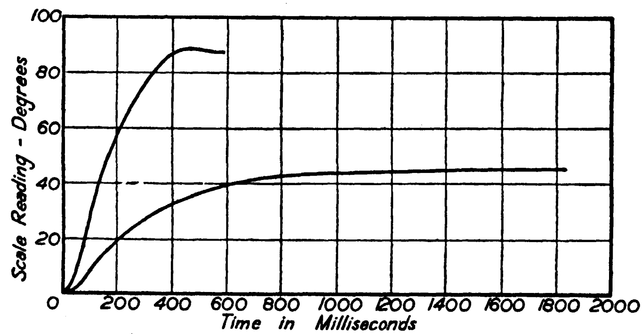


FIG. 3.14.—Response curves for M.C. instrument with shaped pole-pieces.

teristics of a moving coil instrument with shaped pole pieces, having a scale approximately to a logarithmic law.

If T is the deflecting and T^1 the external controlling torques on the movement, $(T^1 - T)$ must always be zero when the pointer is at rest. But T^1 varies with the deflection θ, and T varies with the current or P.D., and may also vary with constant current or P.D. if the pointer is moved. Since $T^1 = T$ when the instrument is normally deflected, the applied control

$$S = \frac{dT^1}{d\theta} = \frac{dT}{d\theta} = \frac{\partial T}{\partial\theta} + \frac{\partial T}{\partial i} \cdot \frac{di}{d\theta}$$

where $\partial T/\partial\theta$ is the rate of change of T when i is constant; $\partial T/\partial i$ the rate of change of T with i when θ is constant; and $1/(di/d\theta)$ the rate of change of θ with i. This latter $= \delta^1$, so that $\delta^1 = d\theta/di$, and is the deflection for unit change of current or the size of the scale divisions.

Hence

$$S = \frac{dT^1}{d\theta} = \frac{\partial T}{\partial\theta} + \frac{1}{\delta^1} \cdot \frac{T}{di}$$

and if $\partial T/\partial\theta = O\ S = (1/\delta)(\partial T/\partial i)$, where δ is the size of the scale divisions in this case.

Now if the current is maintained constant and the pointer is displaced forward mechanically, there will be a restoring control tendency to bring it back to its steady position $S^1 = dT^1/d\theta = \partial T/\partial\theta$ (i const.). In this case $dT^1/d\theta$ must be equal to the original value of $dT/d\theta = \partial T/\partial\theta + (1/\delta^1)(\partial T/\partial i)$ and $\partial T/\partial\theta$ (i const.) $= \partial T/\partial\theta$ so that $S^1 = (dT^1/d\theta) - (\partial T/\partial\theta) = (1/\delta^1)(\partial T/\partial i)$.

Consequently $S^1/s = \delta/\delta^1$, and since the periodic time for the applied control S is $T_\lambda = 2\pi\sqrt{K/S}$, the periodic time when deflected will be $T_\lambda{}^1 = 2\pi\sqrt{K/S^1}$ and hence $S^1/S = \delta/\delta^1 = T_\lambda{}^2/(T_\lambda{}^1)_2$.

If $\partial T/\partial\theta = 0$, the instrument has its natural linear or square law scale, and in this case $S^1 = S$, and $T_\lambda{}^1 = T_\lambda$; but if the form of scale is modified, S^1 and $T_\lambda{}^1$ will differ from S and T_λ, and will generally vary from point to point of the scale. Since for critical damping the damping coefficient $D = 2\sqrt{KS}$, it must be changed to $2\sqrt{KS^1}$, but if S^1 varies along the scale the instrument will be underdamped when S^1 is higher, and over-damped when it is lower, than the value for which the damping was adjusted.

For instruments which have a natural linear or proportional scale, such as permanent magnet moving coil instruments, dynamometer wattmeters and heterostatic electrometers, T is proportional to i or V, and $\partial T/\partial i$ is constant and equal to T/i; so that with spring control, for which $T^1 = s\theta$, $\delta = d\theta/di = \theta/i$. But for instruments which have a natural square law scale, such as soft iron, series dynamometer, or idiostatic voltmeters $T \propto i^2$, in which case $\partial T/\partial i = 2T/i$ and $\delta = d\theta/di = 2\theta/i$. Hence if we take any point P on a calibration curve such as that of Fig. 3.15, and draw straight lines

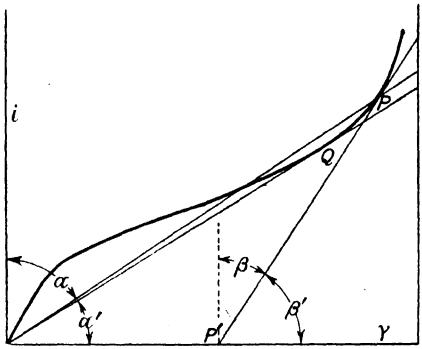


FIG. 3.15.—Relation between effective and spring control.

to the origin and tangential to the curve, $\tan\alpha = \theta/i$ and $\tan\beta = d\theta/di$, so that $S^1/S = \delta/\delta^1 = (\theta/i)(d\theta/di) = \tan\alpha/\tan\beta = \tan\beta^1/\tan\alpha^1$ for linear law instruments; and $(2\theta/i)(d\theta/di) = 2\tan\beta^1/\tan\alpha^1$ for square law instruments.

Hence the only way of securing uniform periodic time and damping over the whole range of an instrument the scale of which is distorted from its natural form is to make S^1 constant, or $\partial T/\partial i$ proportional to δ at every part and to make S vary accordingly. The most practicable method of effecting this in general is by employing magnetic control, which can be made to vary in any desired manner.

ALTERNATING TORQUE APPLIED TO A MOVING COIL INSTRUMENT

The moving coil instrument is, of course, fundamentally a direct current instrument, but there are occasions when an alternating current is passed through such an instrument, and it is necessary to know how it responds. The Duddell oscillograph and the vibra-

tion galvanometer are examples of moving coil instruments which are required to respond to an alternating current. On the other hand, there is the rectifier-operated moving coil instrument used for alternating current measurements. In this the current passing through the instrument can be considered as consisting of a direct current with an alternating current of complex wave-form superimposed on it, and the instrument is required not to respond to the alternating current.

Since the instantaneous value of the torque developed is proportional to the instantaneous current, an alternating current of the form $I \sin \omega t$, where $\omega = 2\pi f$ (the frequency of the alternating current), will cause an alternating torque in phase with the current to be developed. Hence the torque may be written as $T \sin \omega t$ and the equation of motion becomes

$$K \frac{d^2\theta}{dt^2} + D \cdot \frac{d\theta}{dt} + s\theta = T \sin \omega t \quad . \quad . \quad (28)$$

the solution of this equation can be shown to be

$$\frac{\theta}{\Theta} = \frac{1}{[(1 - f^2T_\lambda{}^2)^2 + 4\cos^2 \varphi f^2T_\lambda{}^2]} [(1 - f^2T\lambda^2) \sin \omega t + 2\cos\varphi \cdot fT_\lambda \cos \omega t] \quad (29)$$

In this expression Θ represents the steady deflection which would have resulted from an application of a constant torque T, and the other symbols have the same meaning as before.

It is clear that the moving coil vibrates at the same frequency as the current through the coil, the amplitude of the oscillation depending on the relation between the frequency of the current and the natural time period of the instrument. The oscillation is also out of phase with the current to an extent dependent on the damping ratio.

The maximum value or amplitude of the oscillation given by equation (29) is

$$\theta_{max} = \frac{\Theta}{[(1 - f^2T_\lambda{}^2)^2 + 4\cos^2 \varphi \cdot f^2T_\lambda{}^2]^{\frac{1}{2}}} \quad . \quad (30)$$

For those instruments which are required to respond to the alternating current, i.e. θ_{max} is to be as high as possible, equation (30) shows that fT_λ should be made as nearly equal to unity as possible and the damping should be reduced to a minimum. On the other hand, for instruments which are not required to respond, θ_{max} must

Table VI.

Type.	Maker.	Instrument.	System of damping.	Control (dyne-cm. per radian).	Moment of inertia (gm-cm.^{2}).	Periodic time.	Damping ratio.	Damping coeff. (dyne-cm. per radian per sec.).	Friction angle (degrees).
Moving coil.	Siemens & Halske	Ammeter	Magnetic	296·7	9·1	1·1	0·486	50·6	0·17
	Abrahamson	,,	,,	208	5·27	1	1	66·2	1·08
	Reiniger, Gibbert & Schall	,,	,,	371	4·28	0·675	1	79·9	0·27
	Keiser & Schmidt	,,	,,	398	6·14	0·78	0·538	53·1	0·81
	Everett, Edgcumbe & Co., Ltd.	,,	,,	323	7·7	0·97	0·506	50·4	0·25
	Weston Inst. Co.	Voltmeter	,,	317	1·63	0·45	1	45·45	0·17
	Nadir	,,	,,	196·9	4·99	1	1	62·7	0·9
	Hartmann & Braun	Ammeter	,,	195	2·35	0·69	0·607	26	0·1
	Nalder Bros. & Thompson	,,	,,	194	3·3	0·82	0·34	17·2	0·96
	Crompton	,,	,,	990	15·68	0·8	0·145	36·2	0·7
	,,	Voltmeter	,,	441	2·02	0·425	0·510	28·7	0·17
	Compagnie Compteur	Ammeter	,,	529	6·55	0·7	1·0	118	0·17
	Record Elec. Co.	,,	,,	384	10	1	0·311	38·6	0·6
	,, ,,	Voltmeter	,,	622·5	3·94	0·5	0·57	56·5	0·354
	,, ,,	Ammeter (Circscale)	,,	167	11·65	1·8	0·59	52	1·2
	White Elec. Co.	Voltmeter	,,	221	4·24	1·1	0·160	7·8	0·4
	Crompton	,,	,,	863	4·34	0·66	0·2	20·6	0·81
	Elliott Bros.	,, (illuminated dial)	,,	470	32·4	1·8	0·382	93·8	..
Dynamometer.	Weston Inst. Co.	Voltmeter	Air vanes in box	129·1	2·885	1	0·359	13·78	..
	Elliott Bros.	Ammeter	Magnetic	445	52·7	2·16	0·085	26	0·11
	Everett, Edgcumbe & Co.	Wattmeter	Air piston	368·3	20·25	1·47	0·2	34·45	0·7
	Nalder Bros. & Thompson	Ironclad wattmeter	Magnetic	795	11·65	0·76	0·095	18·25	0·7
	Reiniger, Gibbert & Schall	Wattmeter	Air vanes in box	615	7·21	0·68	0·234	31·25	0·1

TABLE VI—*continued.*

Type.	Maker.	Instrument.	System of damping.	Control (dyne-cm. per radian).	Moment of inertia (gm.-cm.^{2}).	Periodic time.	Damping ratio.	Damping coeff. (dyne-cm. per radian per sec.).	Friction angle (degrees)
Dynamometer.	Weston Elec. Co.	Double wattmeter	Air vanes in box	248·5	6·3	1	0·574	45·5	..
	Siemens & Halske	Wattmeter	Air piston	659	11·49	0·8	0·17	29·5	0·2
	Weston Elec. Co.	,,	Air vanes in box	103	3·55	1·16	0·8	30·6	..
	Siemens & Halske	,,	Air piston	256·5	3·9	1	0·5	31·9	0·1
	Weston Inst. Co.	Voltmeter	Air vane in box	75·5	1·445	1·1	0·433	9·05	..
	,, ,,	Wattmeter	Ditto	103	1·554	1	0·552	13·9	..
	Cambridge Inst. Co.	Irwin Ammeter	Enclosed coil	23·2	3·53	2·6	0·11	1·99	2·3
	,,	Irwin Voltmeter	Ditto	17·5	1·955	2·1	0·08	0·937	..
	,, ,,	Irwin Wattmeter	,,	38·25	2·8	1·7	0·12	2·48	..
Soft iron.	Lund und See Kabel Werke	Ammeter	Free air vane.	10·65	1·371	2·28	0·129	0·985	0·56
	Reiniger, Gibbert & Schall	,,	Air vane in box	76·95	1·89	0·986	0·165	3·98	0·4
	Hartmann & Braun	,,	Ditto	42	1·8	1·3	0·194	3·38	0·13
	Abrahamson	Voltmeter	None	65·3	1·65	1·0	0·025	0·52	0·6
	Nalder Bros. & Thompson	,,	Free air pointer	82	4·07	1·4	0·049	1·645	0·5
	G.E.C. (Stanley)	Ammeter	Air piston	127·1	3·88	1·1	0·11	4·9	0·4
	Evershed & Vignoles	Voltmeter	Air vane in trough	383·5	15·68	1·27	0·56	81·25	0·21
	,, ,,	Ammeter	None	104·9	1·395	0·72	0·075	1·812	1·15
	Weston Inst. Co.	Voltmeter	Air vanes in box	62·25	1·007	0·8	1	15·85	0·118
	British Thomson-Houston	,,	Air piston	255·5	4·52	0·83	0·347	19·62	0·66
	Nalder Bros. & Thompson	Ammeter	Ditto	81·5	1·75	0·92	0·36	8·64	0·91

TABLE VI—*continued.*

Type.	Maker.	Instrument.	System of damping.	Control (dyne-cm. per radian).	Moment of inertia (grm.-cm.^{2}).	Periodic time.	Damping ratio.	Damping coeff. (dyne-cm. per radian per sec.).	Frictic angle (degree
Soft iron.	Weston Inst. Co.	Ammeter	Air vane in box	75	2·3	1·1	0·8	21	0·2
	Keiser & Schmidt	,,	Air vanes in box	79·1	3·23	1·27	0·137	4·425	0·5
	Siemens & Halske	Voltmeter	Air piston	95·4	2·235	0·96	0·253	7·38	0·28
	Everett, Edgcumbe & Co.	Ammeter	Ditto	238	13·57	1·5	0·332	37·75	1·66
	Nalder Bros. & Thompson	Voltmeter	,,	92·1	2·56	1·1	0·5	15·4	1
	Record Electrical Co.	Ammeter	Air vane in box	80	2·02	1·1	0·232	5·9	0·57
	,, ,,	Voltmeter	Ditto	70·5	2·86	1·33	0·296	8·38	0·9
	,, ,,	,,	,,	89·3	4·06	1·34	0·276	10·65	..
	White Elec. Inst. Co.	Ammeter	Piston	77·8	3·48	1·4	0·319	10·51	0·75
	Crompton	,,	Vane in box	61·7	3·02	1·4	0·125	3·4	0·1
	,,	Voltmeter	Piston	318·5	8·075	1	0·13	13·17	0·2
Electrostatic.	Robt. W. Paul	Ayrton Mather voltmeter	None	10·4	9·345	6	0·129	2·56	0·05
	,,	Ditto	,,	5·25	6·07	6·75	0·259	2·92	..
	Kelvin & Jas. White	Multicellular voltmeter	Oil	3·215	12·4	12·35	1	12·62	..
	Everett, Edgcumbe & Co.	E.S. leakage indicator	None	9·30	1·605	2·6	0·25	1·94	..
	Nalder Bros. & Thompson	Ayrton Mather voltmeter	Magnetic	288·5	16·45	1·5	0·33	45·5	..
Induction.	Westinghouse	Ammeter	,,	640	213·6	3·63	0·387	286	1·42
	A.E.G.	Wattmeter	,,	630	51·7	1·8	0·685	248·5	0·6
	,,	,,	,,	902·5	42·5	1·36	0·388	152·5	0·4
	Compagnie Compteur	Double wattmeter	,,	3950	590	2·45	0·201	616	0·13
	Siemens & Halske	Ammeter	,,	1560	101	1·6	1	795	0·12

be as small as possible and the product $fT\lambda$ should be as high as possible.

Effect of Pivot Friction

Pivot friction has been ignored in the preceding analyses, and with a well designed and well made instrument this is justifiable. It must be realized, however, that we rely on the very small pivot friction to bring the movement to rest, and if this friction is too high it will produce an error, which can be measured in degrees, on the steady readings.

PRACTICAL CONSIDERATIONS

The above analysis requires to be supplemented by consideration of the actual numerical values of the four quantities, the moment of inertia K, the control S, the damping ratio n, and the torque or angle of friction θ_F, in various types of instrument. Table VI gives results we have obtained for a fair range of types. and the results are summarized in Table VII.

TABLE VII.—*Approximate Values of Mechanical Constants of Indicating Instruments.*

Type of instrument.	Moment of inertia (K gm-cm.^{2}).	Spring constant (S. gm.-cm. per radian).	Periodic time ($T\lambda$ sec.).	Angle of friction (θ_F degrees).	Damping coeff. (D. dyne-cm. per radian per sec.).
Permanent magnet moving coil	1·6–9·0	0·2–1·0	0·4–1·1	0·1–0·9	0·17–80
Induction	40–220	0·6–0·9	1·3–3·6	0·4–1·4	150–300
Dynamometer	3–50	0·1–0·8	0·7–2·0	0·1–0·7	14–45
Soft iron	1·3–16	0·01–0·4	0·7–2·5	0·1–1·0	0·5–81
Electrostatic	1·6–16	0·003–0·3	1·5–12	0·05	2·0–45

The figures given in Tables VI and VII refer to fairly large instruments, and mention must be made here of the miniature range of instruments, particularly of the permanent magnet moving coil class, which have become very popular of recent years. These instruments have scale lengths varying from 1 to 3 inches, and in these the moment of inertia may be as low as 0·5 gm.-cm.^{2}, the spring constant as low as 30 dyne-cm. per radian and the periodic time as low as 0·1 second.

AIR DAMPING

Air damping may be of two kinds: (*a*) Free-air damping, or (*b*) closed chamber or piston damping. Examples of both have been given in Chapter 2.

The experimental determination of the laws in either case is very difficult—in the former since draughts and changes in the atmospheric condition may cause extremely discrepant results, and in the latter because of the mechanical difficulty of producing a

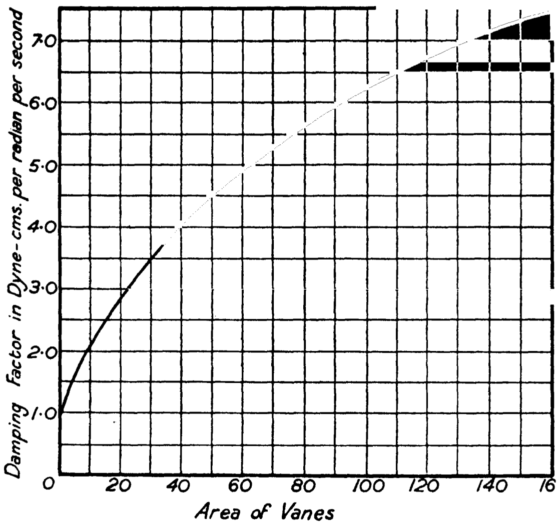


FIG. 3.16.—Curves for free-air damping with vane areas 5 cms. deep and varying width.

number of different chambers and vanes of various sizes and small, accurately known clearances.

As mentioned above, there are so many influences tending to change the conditions with free-air damping that it is practically impossible under ordinary laboratory conditions to obtain results of general utility, and although it is possible to obtain good curves, such as that shown in Fig. 3.16, for a given set of conditions, a new set of observations may differ very considerably when the experiment is repeated on some subsequent occasion under apparently similar conditions.

The movement must be screened to eliminate draughts, and the screening has to be very perfect if it is to be at all effective, and the proximity of the walls of the enclosure modify the air currents

very seriously if the enclosure is not large ; whilst if it be enlarged sufficiently to eliminate the effect of the walls, persistent air currents probably caused by convection, etc., give rise to erroneous values and the large enclosure renders accurate observations difficult to make. The atmospheric conditions, such as barometric pressure, humidity, etc., also have their influence, and although it would be possible by the use of more elaborate methods to overcome these difficulties, we have not considered it necessary to carry the investigation further, since the use of free-air dampers in commercial instruments has now been practically abandoned.

Closed Chamber Damping

For the purpose of determining the actual value of the damping in various cases a model was made as shown in Fig. 3.17. It consisted of a delicately pivoted vertical spindle mounted between

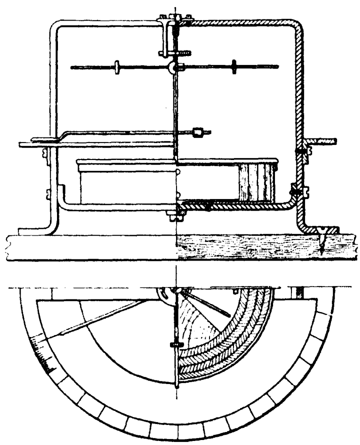


FIG. 3.17.—Model for investigating laws of closed chamber damping.

jewels and carrying a light pointer over a scale of degree. The control was provided by a spiral spring of phosphor-bronze, and its value was determined both by weighting the pointer and by means of the little torque tester described in Chapter 7 after each setting of the instrument. The vanes were simply plane rectangles of aluminium, 0·3 mm. thick and approximately 2 cm. broad, and were fastened to the spindle in such a manner that while rigidly held in position, a small adjustment of the clearance in the damping-box was possible. This latter was a circular metal box with cover in

two halves, fitting tightly over the top, the only opening to the damping chamber being an annular space 0·3 mm. wide round the central spindle. The diameter of the damping chamber could be varied at will by the insertion of turned rings, and two sector-shaped solid pieces were inserted with each ring to limit each chamber to a sector of about 80° angular breadth.

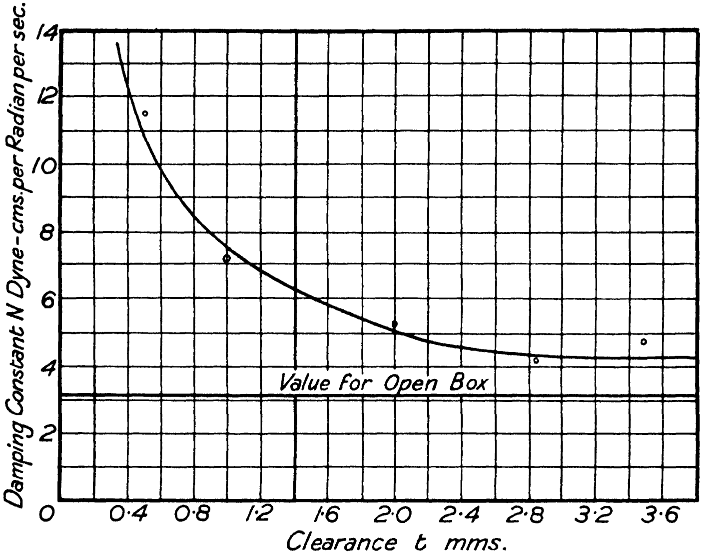


FIG. 3.18.—Experimental curve for closed chamber damping, using plain vanes and varying clearance.

Vane setting was done by means of specially made gauge pieces. The periodic time was never less than 1 second or more than 2 seconds, and practically the whole of the damping was due to the vanes. In order to minimize the effect of pivot friction, which was small, an electric bell mechanism was arranged to vibrate the apparatus gently during the observations.

The procedure consisted in first mounting the vane symmetrically, and then carefully observing the undamped periodic time from which the moment of inertia of the system was obtainable. The box was then closed and the pointer deflected, the damping being obtained by the ratio of successive swings. The final results are plotted in Figs. 3.18 and 3.19, which correspond closely with the formula

$$D = \left(\frac{0 \cdot 345}{t} + 0 \cdot 235\right) ba^2 \text{ dyne cm. per radian per sec.,}$$

where b and a are the width and radius of the vanes respectively in cm. and t the clearance in millimetres. Over the ordinary practical limits (between 0·5 cm. and 4 cm.) the damping was found to be proportional to the width of the vane, but below 0·5 cm. the damping increased more rapidly, and above 4 cm. less rapidly than

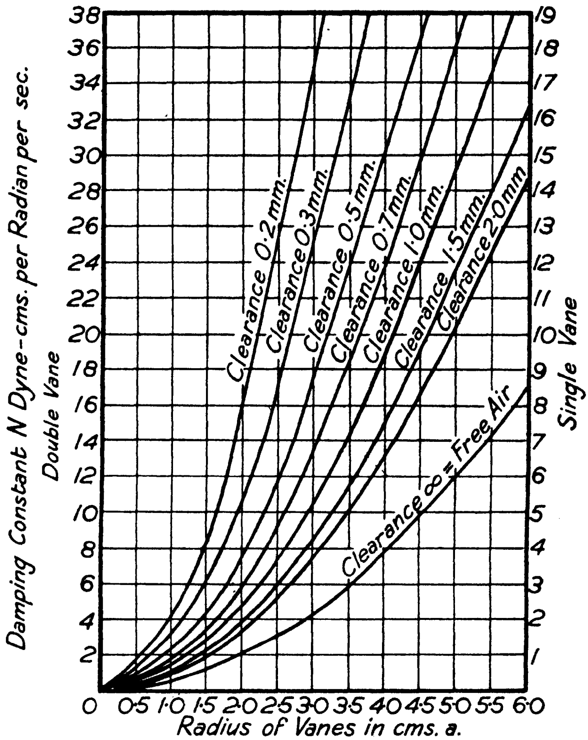


FIG. 3.19.—Curves for closed chamber damping with plain vanes.

the width. The above formula is for a double vane; for a single one the value of D would of course be halved.

If we consider the damping as being due to the difference of pressure Δp on the two sides of the damping vanes, it is evident that the torque $T_k = 2\Delta p \times ba \times \frac{a}{2} = \Delta p \; ba^2$ and $D = \frac{T_k}{\omega} = \frac{\Delta p}{\omega} ba^2$,

from which it appears that $\frac{\Delta p}{\omega}$ or the difference of pressure between

the two points of the damping chamber for unit angular velocity = $\frac{0{\cdot}345}{t} + 0{\cdot}235$ dynes per sq. cm., and is nearly independent of the size of the vane.

The curves in Fig. 3.19 make it quite easy to determine the dimensions of an air damper for any required system. Experiments made with plain paddle-shaped vanes of the Weston type in annular box gave results 10 per cent. higher than the above for the same clearances, while turning up the edges of the vanes into tray form, as is common in many instruments, appeared to increase the damping constant by about 30 per cent.

Piston Damping

Experiments were also made with a piston damper, and are shown in Fig. 3.20, the diameter of the cylinder being 1·85 cm., and the radius of action from the axis of rotation to the centre of

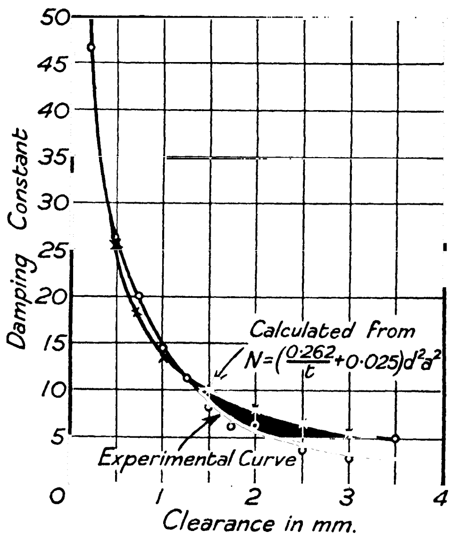


FIG. 3.20.—Curves for piston damper with varying clearance.

the piston 3·75 cm. Over the working range of clearance from 0·25 mm. to 1·5 mm. these observations agree sufficiently well with the formula

$$D = \frac{12{\cdot}6}{t} + 1{\cdot}2 \text{ dyne cm. per radian per sec.}$$

If we assume, in accordance with the vane-damping results, that the difference of pressure per unit angular velocity is nearly independent of the dimensions of the piston, it is obvious that the damping constant can be written in the form

$$D = \left(\frac{a}{t} + k\right) Aa^2$$

where A is the area of the cylinder and a the radius of action. In this case the area of the cylinder is 2·68 sq. cm., and the formula then becomes

$$D = \left(\frac{0{\cdot}325}{t} + 0{\cdot}032\right) Aa^2$$

on the above assumptions. We may also suppose that the increase of 30 per cent. for a flanged piston holds as for a vane with upturned edges.

Air Damping Formula

The whole question of air damping needs more thorough investigation, but from the above we may provisionally adopt the following formulae :

TABLE VIII.—*Air Damping Formulae.*

Type of damper.	Damping constant in dyne cm. per radian per sec.
Plain rectangular vane in sector-shaped box	$D = \left(\frac{0 \cdot 173}{t} + 0 \cdot 118\right) ba^2$
Boxed ditto	$D = \left(\frac{0 \cdot 225}{y} + 0 \cdot 153\right) ba^2$
Paddle ditto	$D = \left(\frac{0 \cdot 19}{t} + 0 \cdot 14\right) ba^2$
Boxed paddle ditto	$D = \left(\frac{0 \cdot 248}{t} + 0 \cdot 169\right) ba^2$
Plain piston damper ditto	$D = \left(\frac{0 \cdot 335}{t} + 0 \cdot 032\right) ba^2$
Boxed piston damper	$D = \left(\frac{0 \cdot 435}{t} + 0 \cdot 0415\right) Aa^2$

Tests on Actual Instruments

In order to check the above formulae and to show the actual amounts of damping employed in practice, experiments were made on a number of air-damped instruments, and the results are recorded in Table IX. The observed values of the damping constant and those calculated from the above formulae are shown in adjoining

columns, and exhibit fair agreement in most cases, when it is remembered that the clearances are not only difficult to determine, but are usually more of less irregular. In the Weston instruments, however, the damping is greatly in excess of the calculated value, due apparently to exceedingly perfect workmanship and reduction of the air leakage. In the case of the soft iron ammeter the damping is nearly three times that calculated by the formula, and there are actually two holes in the cover of the damping chamber!

Inertia of Air Dampers

In comparing the performance of the various types of dampers their own inertia must not be forgotten, especially in the case of piston dampers, where the piston is supported by a long wire which is not easily made rigid enough without considerable weight. For vane dampers a thickness of 2 mils or 0·05 mm. of aluminium appears to be suitable.

For a plain vane $K = 1/3\ \delta\ bxa^3$, and for one with turned-up edges $K = \delta\ xa^2\left\{\frac{a}{3}(b + 2b_1) + bb_1\right\}$, where δ is the density of the material, x its thickness, and b_1 the width of the turn-up portion.

As an example, let us take a case where the vane has a radius of 3 cm., a breadth of 2 cm., and a thickness of 0·05 cm. Then for a plain vane $K = 1/3\ \delta bxa^3 = 0{\cdot}243$ gm.-cm.², and the damping constant D for a 0·5 mm. clearance.

$$= \left(\frac{0{\cdot}173}{t} + 1{\cdot}2\right) ba^2 = 8{\cdot}4, \text{ the ratio } \frac{D}{K} \text{ being } 34{\cdot}6.$$

With a boxed vane having the same dimensions and edges turned up 2 mm.

$$K = \delta\ xa^2\left\{\frac{a}{\cdot 3}(b + 2b_1) + bb_1\right\} = 0{\cdot}34 \text{ gm.-cm.}^2$$

and

$$D = \left(\frac{0{\cdot}0225}{t} + 0{\cdot}153\right)\ ba^2 = 10{\cdot}9, \text{ giving a ratio } \frac{D}{K} = 32.$$

In the case of piston dampers we have

$$\text{mass of piston} = \frac{\pi}{4}\delta\ xd(d + 4b_1)$$

$$\text{mass of arc} = \delta\ Aa\ \beta^\circ/57{\cdot}3$$

$$\text{mass of radius arm} = \delta Aa.$$

TABLE IX.—*Particulars of Air-Damped Instruments.*

Instrument.	Maker.	Type of damper.	No. of vanes.	Max. radius (cm.).	Breadth of box (cm.).	Clearance (min.).	S. dyne (cm. per radian).	(K. gm.-$cm.^2$).	Tλ (periodic time).	Damping factor. Observed.	Damping factor. Calculated.	Damping ratio.
Portable standard wattmeter	Weston Elec. Inst. Co.	Vanes in annular chamber	2	2·5	1·86	0·3	103	3·55	1·16	30·6	23·3	0·8
Double switchboard wattmeter	Ditto	Vanes in sector box	2	2·93	1·36	0·3	248·5	6·3	1	45·5	21	0·575
Moving iron ammeter	„	Vane in sector box	1	3	1·35	0·5	75	2·3	1·1	18·4	6·5	0·7
Ditto	Reiniger, Gilbert & Schall	Ditto	1	3	1	0·75	76·95	1·89	0·986	3·98	4·08	0·165
„	Hartmann & Braun	„	1	3	0·9	0·5	42	1·8	1·3	3·38	3·33	0·195
M.I. voltmeter	Evershed & Vignoles	Vane in trough box	1	9	4·26	0·8	383·5	15·68	1·27	81·25	81	0·523
Portable wattmeter	Reiniger, Gilbert & Schall	Vane in sector box	1	3·2	0·95	0·75	615	7·21	0·68	8	4·86	0·235
M.I. ammeter	Keiser & Schmidt	Ditto	2	3·15	1·65	0·75	79·1	3·23	1·27	4·425	10·4	0·117
„	British Thomson-Houston	Piston	1	3·55	1·55	0·35	255·5	4·52	0·835	19·62	30·2	0·289
„	Nalder Bros. & Thompson	„	1	2·25	1·15	0·5	81·5	1·75	0·92	8·64	6·1	0·361
Switchboard wattmeter	Everett, Edgcumbe & Co.	„	1	3·2	1·75	0·55	368	20·25	1·47	34·45	20·5	0·1995
Ditto	Siemens & Halske	„	1	3·5	2	0·5	659	11·49	0·8	29·5	35	0·17
M.I. ammeter	G.E.C. (Stanley)	„	1	2·32	1·05	0·75	127·1	3·88	1·1	4·9	4·5	0·11
„	Record Elec. Co.	Vane in sector box	1	2·9	1·32	0·425	80	2·02	1·1	5·9	7·2	0·232
„	White Elec. Co.	Piston	1	3	1·1	0·7	77·8	3·48	1·4	10·5	6·64	0·319
„	Crompton	Vane in box	1	2·15	1·4	0·9	61·77	3·02	1·4	3·41	2·88	0·125
„	„	Piston	1	3	1·59	1·4	318·5	8·075	1	13·17	9·05	0·13

Hence

$$K = \left\{0{\cdot}785\,xd(d + 4b_1) + Aa\left(1/3 + \frac{\beta^\circ}{57{\cdot}3}\right)\right\} \delta a^2.$$

In this case suppose the piston is of 0·05 cm., aluminium 1·75 cm. diameter, and turned up 2 mm., supported on an aluminium wire 1 mm. diameter, the arc being 80° and the radius of action being 3·75 cm. Then K works out to be 8·58 gm.-cm.², and $D = \left(\frac{0{\cdot}435}{t} + 0{\cdot}0413\right)$. $Aa^2 = 34{\cdot}5$ for a 0·5 mm. clearance, and the ratio $\frac{D}{K} = 4{\cdot}025$.

Contrast between Vane and Piston Dampers

The general conclusions to be derived from the above are that vane dampers have a decided advantage as regards inertia, but are not easily made to give large damping constants, probably on account of the difficulty of reducing air leakage around the shaft. Piston dampers give a greater amount of damping for the size of damping chamber, but necessarily have a high inertia on account of the long rigid arm required. In consequence vane dampers are to be preferred in delicate instruments with low inertia, and piston dampers in instruments with heavy movements of high inertia which require considerable damping and in which the extra inertia of the piston damper has little influence.

Eddy-Current or Electro-magnetic Damping

Besides being a most valuable form of damping device owing to its strictly proportional law, and ease of construction and adjustment, the eddy-current brake has the advantage of lending itself to accurate or approximate calculation from first principles. In fact it would be most exclusively employed if it were not for the difficulty of the stray field from the damping magnets, which renders it unsuitable for dynamometer and similar instruments, apart from its somewhat high cost.

The most simple and definite case is when a closed coil or metallic former rotates in a uniform field, as in permanent magnet moving coil instruments. Suppose, for example, that we have a coil of N turns with axial length l and breadth b, moving in a uniform radial field of strength B. Then the flux Φ passing through the coil in

any position is $lbB\beta = AB\beta$, where A is the area of the coil. If the coil turns, an E.M.F. $V = N \dfrac{d\Phi}{dt} \times 10^{-8}$ volts is induced, which equals $-NAB\dfrac{d\beta}{dt} \times 10^{-8} = NAB\omega \times 10^{-8}$, where $\omega = \dfrac{d\beta}{dt}$, the angular velocity of the coil. Consequently, if R is the resistance of the coil in ohms, a current $I = \dfrac{NAB}{10^8 R}\omega$ amperes is induced in it by the motion, assuming it to be too slow for inductance or capacity to have any material effect. This current produces a force equal to $1/10\ (INlB)$ dynes in each side of the coil, and a torque $T_D = 1/10\ (IN\ lB \times b) = 1/10\ (NABI)$ dyne-cm. Putting $I = \dfrac{NAB}{10^8 R}\omega$, we have $T_D = \dfrac{N^2A^2B^2}{10^9 R}\omega$ and the damping constant

$$D = \frac{T_D}{\omega} = \frac{N^2A^2B^2}{10^9 R} \text{ dyne cm. per rad. per sec.}$$

The negative sign shows that the torque is one of retardation or damping.

In the case of a metallic former of width b_1 and thickness x, the resistance R is $\dfrac{2(l+b)}{10^6\, b_1 x}\rho$ ohms and $N = 1$, from which

$$D = \frac{A^2B^2b_1\, x}{2{,}000\,(l+b)\,\rho} \text{ dyne cm. per rad. per sec.}$$

By varying the thickness x of the former the desired amount of damping can be obtained.

For design purposes it is convenient to express the resistance of the frame as R_F. Then the damping coefficient for the frame alone is $\dfrac{A^2B^2}{10^9\, R_F}$ and the total damping coefficient due to coil and frame is

$$D = \frac{A^2B^2}{10^9}\left\{\frac{N^2}{R} + \frac{1}{R_F}\right\}$$

It should be noted that the resistance R includes not only the resistance of the coil itself, but also the resistance of the circuit in which it is connected, *looked at from the instrument.*

Again the damping coefficient for critical damping D_o is given by $D_o = 2\sqrt{SK}$, and so the damping ratio of a moving coil instrument is

$$n = \frac{D}{D_o} = \frac{A^2B^2}{2\sqrt{KS}}\left\{\frac{N^2}{R} + \frac{1}{R_F}\right\} \times 10^{-9}$$

An application of this simple calculation in the design of a moving coil instrument will be found in Chap. 6. It will, of course be obvious that if such a moving coil is used to shunt a low resistance, as is very commonly done, the damping may be materially increased and the instrument rendered sluggish in its indications.

In a number of instruments, however, such as supply meters and induction instruments generally, the brake is in the form of a disc or cylinder revolving between the poles of a magnet, and in this case the calculation is much less simple, as the eddy currents are diffused over the disc in somewhat the same manner as the lines of force of a bar magnet (Fig. 3.21). Accurate mathematical treat-

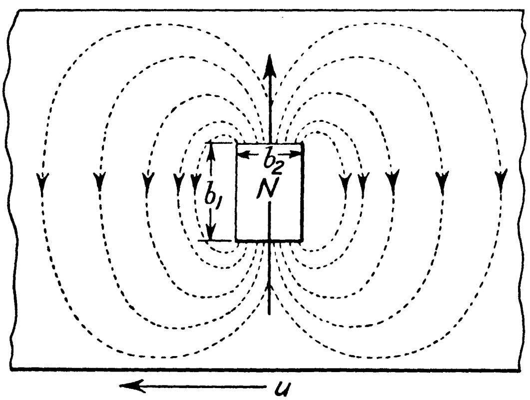


FIG. 3.21.—Stream lines of current in conducting plate moving under magnet pole.

ment of the subject has been given by W. Rogowski,* but is far too complex for ordinary application, and the following rough sketch of a theory will serve our purpose.†

Suppose a large sheet of metal is moving across a magnet pole with a velocity u, Fig. 3.21. If B is the flux density in the gap and

* *Archiv f. Elektrotechnik*, i, pp. 205–232, 1912.

† This treatment, although independently conceived, appears to be identical with that employed for cylindrical dampers by Mr. Evershed in his paper on "A Frictionless Motor Meter," *Journ. of Inst. Elec. Eng.*, vol. 22, pp. 772–773.

b_1 and b_2 the dimensions of the pole as shown, then an E.M.F. $V = \frac{b_1 B u}{10^8}$ volts is induced in the portion of the plate passing through the field, producing a belt of current flowing in the direction of the arrow and of breadth b_2. If x is the thickness of the disc and ρ its specific resistance, the resistance of the portion of the disc immediately in front of the pole is $\frac{b_1 \rho}{10^6 b_2 x}$ ohm. Obviously the whole resistance of the circuit in which the current flows is greater than this; let us suppose k times as great, where k is a coefficient difficult to calculate, but should not be very large compared with unity for a wide plate.

Then $R = \frac{k b_1 \rho}{10^6 b_2 x}$, and the current $I = \frac{V}{R} = \frac{b_1 B u}{10^8} \times \frac{10^6 b_2 x}{k b_1 \rho}$ or $I = \frac{b_2 x B u}{100 k}$ amperes. The force produced by reaction of this current on the field,

$$F = \frac{b_1 B I}{10} = \frac{b_1 b_2 x B^2 u}{1{,}000 k \rho} = \frac{x \Phi^2 u}{1{,}000 A k \rho}$$

dynes, where Φ is the total flux of the magnet and $A = b_1 b_2$, its polar area.

If the plate is a circular disc, and a the radius from the axis to the centre of the magnet pole $u = a\omega$, and the torque Fa. Hence $T_D = Fa = \frac{\Phi^2 a^2 x}{1{,}000 A k \rho}\ \omega$, and the damping constant

$$D = \frac{T_D}{\omega} = \frac{\Phi^2 a^2 x}{1{,}000\ A k \rho} \text{ dyne cm. per rad. per sec.}$$

It is clear that k will increase considerably if the magnet pole approaches the edge of the disc, as in that case the path of the current is restricted. When the middle of the pole reaches the edge of the disc there is no return path for the current, and k should become infinite, i.e. the torque should fall to zero.

It is obvious that if the pole is traversed outwards from the centre to the edge of the disc the damping constant should rise at first according to a square law, and go on increasing until k begins to increase owing to proximity to the edge of the disc. The damping constant should then rise more slowly, reaching a maximum not far from the edge, and then drop rapidly to zero as the centre of the pole reaches the edge of the disc.

The experimental investigation of the above conclusions was carried out by means of the apparatus illustrated in Fig. 3.22. Discs 10 cm. in diameter were successively mounted on a horizontal spindle which could be rotated at any desired speed by means of a worm gear driven by a small electric motor, and the speed of the disc at any time could be accurately ascertained.

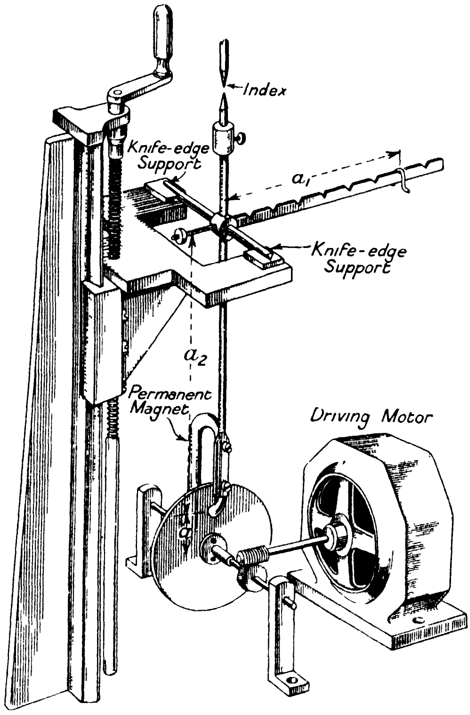


FIG. 3 22.—Apparatus for investigating the behaviour of disc dampers.

A permanent magnet of rectangular form was suspended from knife edges above the disc in such a way that the disc could rotate between the pole faces, which could be set to act at any desired radius from the axis of rotation of the disc by raising or lowering the knife edge platform by means of a vertical screw-driven slide.

The torque on the magnet due to the reaction of the currents induced in the disc was counterbalanced by means of a weight on a divided horizontal arm attached to the axis about which the magnet

was deflected, the position of equilibrium being determined by the coincidence of a fixed index with the pointed end of a vertical arm, which carried an adjustable weight employed for counterbalancing the magnet and adjusting the sensibility. In practice it was found most convenient to place a weight w at a definite radius a_1 on the horizontal arm and adjust the speed n of the disc until the pointer came to zero; this speed was then measured and the weight moved to another position and the observations repeated. A series of speed torque curves corresponding to the various relative positions of disc and magnet could thus be obtained. The flux Φ in the gap of the magnet was found by means of a search coil and flux meter.

Then we have the torque $T_D = \frac{wa_1a}{a_2}$ and the damping constant

$D = \frac{T_D}{\omega} = \frac{w}{2\pi n}\frac{a_1}{a_2}a$, and the coefficient

$$k = \frac{\Phi^2a^2x}{1{,}000\ AN\rho} = \frac{2\pi n}{1{,}000\ A\rho} \times \frac{\Phi^2ax}{w}\frac{a_2}{a_1}.$$

The curves in Fig. 3.23 were obtained by the apparatus, and confirm the general lines of the above theory as regards the variation of the damping constant, as the magnet is traversed outwards to the edge of the disc. The maximum torque in each case occurs when the centre of the magnet pole is about 4·1 cm. from the axis or at about 82% of the radius of the disc, which appears, therefore, to be the most effective radius of action.

For a given magnet pole in a fixed position the damping constant D should be proportional to $\frac{x}{\rho}$. Fig. 3.24 shows the relation between the maximum torque in each of the above cases to the value of $\frac{x}{\rho}$ for the disc.

It will be seen that the points do not lie on a single straight line, those for the aluminium discs being highest, those for the zinc discs considerably lower, and those for the copper discs lower still. The explanation is probably that the zinc and copper discs were not perfectly pure metal, and that their conductivity was probably considerably lower than that assumed; the higher line for aluminium discs is probably the correct one.

Since $D = \frac{\Phi^2a^2x}{1{,}000\ Ak\rho}$, $k = \frac{\Phi^2a^2x}{1{,}000\ NA\rho}$, which enables the values

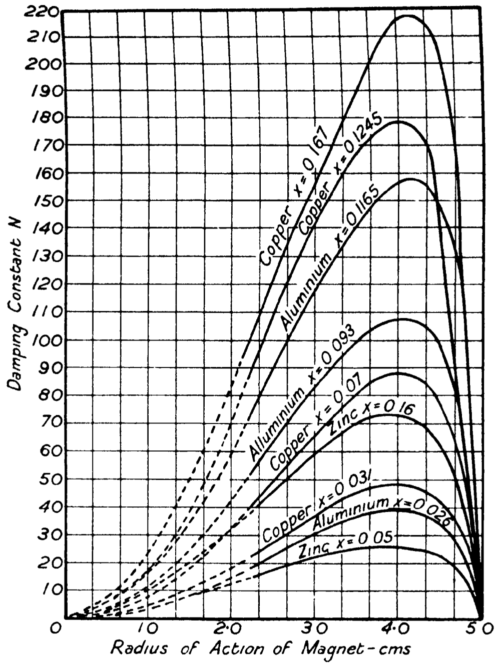


FIG. 3.23.—Curves for various materials used as disc dampers.

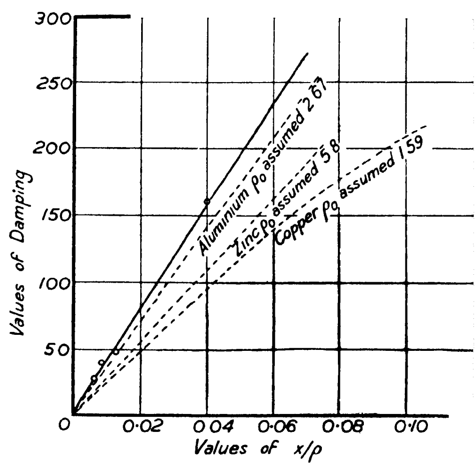


FIG. 3.24.—Relation between maximum torque and x/ρ for various discs.

of k to be found for the various observations. Fig. 3.25 shows the variation of k with the radius of action of the magnet for three of the discs, and shows, as would be expected, that it is fairly constant when far from the edge, but rises rapidly as this is approached. The value of k may be taken as about 9 for aluminium discs at the

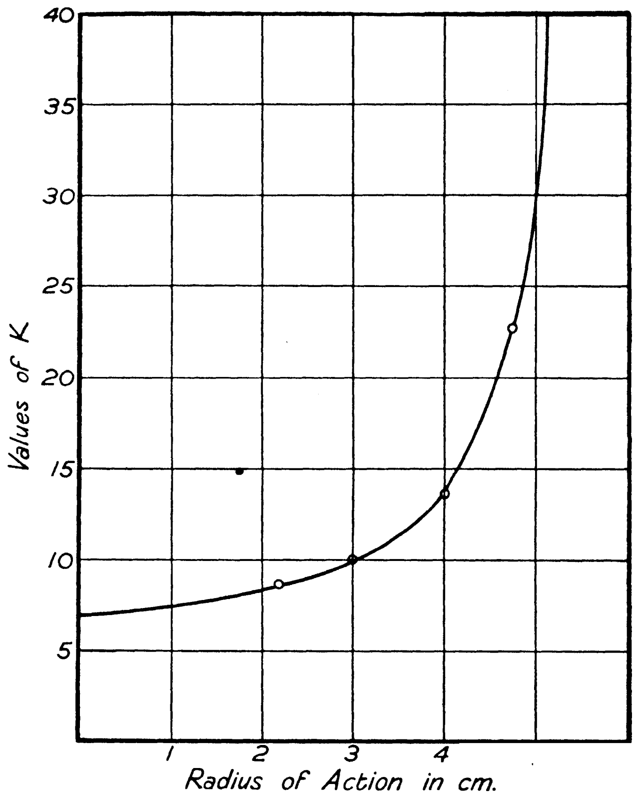


FIG. 3.25.—Curves showing variation of K with radius of action.

lower radii. The value for copper is higher, but this is probably due to the specific resistance being in reality higher than that assumed. Mr. Evershed appears to have obtained valves varying from about 3 for small square or circular poles to 6 or 7 for oblong poles and to 9 or 10 when the pole is near the edge, but our results are higher. One would not have expected high values for k, as the

area of the return path for the current is so large, but if there is much magnetic dispersion from the poles the current may have to take a long path.

Since the shape and size of the pole are of considerable importance, and the resistance of the return path increases so rapidly as we approach the edge of the disc, it is obvious that the more concentrated the field the greater will be the damping. For this reason the employment of broad rectangular poles to the damping magnets, such as one finds in supply meters, cannot be regarded as efficient, for although the increased pole area will lead to a greater flux through the disc, the additional flux so secured is far less effective for the reasons given above.

Similarly a circular pole face is more effective than one of rectangular form. It is interesting to note that in 1903 Lord Kelvin took out a patent in connection with Jas. White for a damping arrangement in which the conducting sector moved between two conically shaped pole pieces on the inner faces of a double magnet consisting of two horse-shoes placed end to end, thus realizing the importance of a highly concentrated field of small area, and our theory and experiments show conclusively that in many cases the large and expensive brake magnets employed in many meters are a needless expense.

It must not be forgotten, however, as an offset to the advantage of small poles that they tend to reduce the permanence of the magnet. This is of comparatively little importance in a deflectional instrument where the magnets are only employed for damping, but in supply meters, where the damping magnets furnish the retarding torque, permanence is of prime importance, and care must be taken to have as small a gap as possible and sufficiently large pole faces to avoid a high demagnetizing factor, while the value of cutting up the pole into a number of small projections has already been discussed.

Inertia and Efficiency of Electro-magnetic Dampers

The moment of inertia of a disc rotating about its axis is $1/2\, ma_3^2$ or $\pi/2\,(\delta\, x a_3^4)$, where m is its mass, a_3 its radius, x its thickness, and δ the density of the material. The damping constant on the other hand, $D = \dfrac{\Phi^2 a^2 x}{1{,}000\, Ak\rho}$, and we have seen that this is a maximum

when $a = 0{\cdot}82a_3$*, at which point k is about 13. This gives $D_{max} = 5{\cdot}18 \times 10^{-5} \frac{\Phi^2}{A} \frac{x}{\rho} a_3^2$, remembering, however, that this only applies if the size of the disc is large in comparison with that of the pole.

If K_1 is the moment of inertia of the remainder of the moving system the total moment of inertia $K = K_1 + \pi/2\,(\delta\, x a_3^4)$, so that the ratio

$$\frac{D_{max}}{K} = \frac{5{\cdot}18 \times 10^{-5} \frac{\Phi}{A} \frac{x}{\rho} a_3^2}{\pi/2\,(\delta\, x a_3^4) + K_1}$$

which is a maximum when $\pi/2\,(\delta\, x a_3^4) = K_1$, or the inertia of the damping disc is equal to that of the rest of the system.

In this case $\frac{D_{max}}{K} = \frac{5{\cdot}18 \times 10^{-5}}{2K_1} \frac{\Phi}{A} \cdot \frac{x}{\rho} a_3^2$.

* This proportion does not necessarily apply to different ratios between the size of the poles and of the disc.

CHAPTER 4

ELEMENTS OF ELECTRICAL THEORY AND DESIGN

In Chapter 1 the principles underlying the action of the chief types of electrical instruments were explained. In order to appreciate the details of their construction, however, or to be able to proportion the parts in a new design, these principles must be put into quantitative form. This trenches upon an extremely wide and complex subject, but in the present chapter it is proposed only to give the formulae which are fundamental in character and easily understood, reserving any more detailed or complex investigations to be treated in connection with the specific instruments concerned. For the present purpose it will be satisfactory to deal with the fundamental formulae, first of electrostatics, secondly of magnetisation, and thirdly of the electric current.

ELECTROSTATIC FORMULAE

Coulomb's Law of Inverse Squares

In 1785 Coulomb, by means of his torsion balance, which may be regarded as the first electrical measuring instrument, established the first quantitative electrical law: that the mechanical force of repulsion between two electrically charged bodies was proportional to the charge in each of them and inversely proportional to the square of the distance between them. This relation is expressed in the formula

$$F = \frac{ee_1}{\varepsilon D^2} \qquad . \quad . \quad . \quad . \quad . \qquad (1)$$

where e and e_1 are the amounts of the charges, D the distance between them, and ε is a constant depending upon the units chosen and the medium between the bodies.

Electrostatic Unit of Quantity

The above relation enables the first electrical unit, that of charge or quantity, to be defined, and it is obviously most simple to take it such that the unit electrical charge is that which at unit distance (1 cm.) from a similar charge, repels it with unit force (1 dyne)

in a standard medium which we may take as air (more accurately a vacuum).

Faraday's Law of Displacement

In 1843 Faraday showed by his " ice-pail " experiment that if a charged insulating body is introduced into a closed conducting vessel a charge immediately appears on the outside vessel which is exactly equal to the charge inside it, although no transfer by conduction has taken place.* This experiment indicates that it is impossible to increase the electrical charge within any volume of space, and that if any charge is introduced into a region, an equal charge must be " displaced " outwards so as to keep the total quantity within the region the same. The simplest way of visualising the phenomenon is to consider all space as being filled with an incompressible fluid, such as water, but entangled in such a way as to be unable to flow freely, as in the case of a jelly. If we take a mass of such a jelly, introduce a tube into it, as in Fig. 4.1, then force water through the tube so as to produce a water-filled space in the centre representing the charge, it is evident that, since the water is incompressible, an amount of water exactly equal to the quantity Q forced in must pass outwards through the boundary of any closed region surrounding the " charge." If we consider a sphere of radius a round the charge the area of its surface is $4\pi a^2$, and the quantity of water which passes out through unit area is $Q/4\pi a^2$. But the elasticity of the jelly produces a back pressure tending to force the water back again, so that if the tube is released the quantity of water Q is expelled and the jelly returns to its original form.

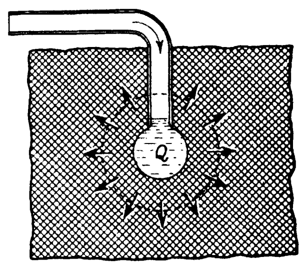


FIG. 4.1.—Water analogy of displacement of charge.

In like manner, when a charge e is brought into any space, the amount which passes out through unit area of a sphere of radius a surrounding it is $e/4\pi a^2$, and this is called the " displacement," or better, the "displacement density," δ, in the medium, which is resisted by a certain back pressure as in the case of the jelly. But

* In Faraday's experiment the medium inside the ice pail was air only, but one of the authors has repeated it, filling the whole of the cavity except a small space for the charged body with paraffin wax, or other dielectric of high dielectric coefficient, and has thus shown that the displacement is independent of the medium.

by Coulomb's Law we know that the force on unit charge at the same distance is $e/\varepsilon D^2$, and this is called the "intensity of the electric force" S, which we shall prefer to speak of as the "electric stress."

We therefore have the two relations

$$\delta = \frac{e}{4\pi a^2} \text{ and } S = \frac{e}{\varepsilon D^2}$$

from which $4\pi\delta = \varepsilon S$ when $D = a$. . . . . . . (2)

This is the fundamental law of electrostatics, and expresses the fact that the electrical strain or displacement is rigidly proportional to the electrical stress or intensity of electric force S. It is, in fact, the electrical analogue of Hooke's Law, viz. "stress is proportional to strain."

Difference of Electric Potential

When a body is moved a distance D cm. against a force F dynes, an amount of work is done which is equal to FD dyne-cm. or ergs. If a unit electrical charge is brought near a charged body it is repelled with a force S, and if it is moved a small distance dD against this force an amount of work is done $dV = SdD$. The total amount of work done on the unit charge in going from one point to another is $V = \Sigma dV = \Sigma SdD$, and this is called the difference of (electric) potential between the two points. This may be thought of as a difference of electrical level between them, as in the case of gravitation ; the work in ft. lb. done in moving a pound weight from one place to another is simply the difference of the height or level between them in feet. The actual potential at any point is the difference between its potential, and that of the earth, just as the actual height of any point is taken as its distance above sea-level, so that we have difference of potential $V = V_2 - V_1 = \int S\,d\,D$ . . (3)

Electrostatic Calculations

The two relations given above, viz. $4\pi\delta = \varepsilon S$ and $V_2 - V_1 = \int S\,d\,D$ enable most electrostatic calculations to be made with ease.

For example, let us take the case of two parallel plates of area A at a small distance D apart. Then if a charge e is imparted to one of these plates it is evident that nearly the whole of the displacement takes place uniformly across the thin intervening space between them, so that the displacement density $\delta = e/A$.

Since $$4\pi\delta = \varepsilon S,\ S = \frac{4\pi\delta}{\varepsilon} \quad . \quad . \quad . \qquad (4)$$

or in this case $S = \dfrac{4\pi e}{A\varepsilon}$, which gives us the electric stress, or the force on a unit charge anywhere between the plates. In this case the force is uniform, so that we have only to multiply it by the distance between the plates D to get the difference of potential and

$$V = V_2 - V_1 = SD = \frac{4\pi e D}{\varepsilon A} \quad . \quad . \quad . \qquad (5)$$

Electrostatic Capacity

When a body is charged its potential is raised, and the amount of charge for a unit rise in potential is called its capacity* C. It should be noted, however, that there are two kinds of capacity, which may be respectively termed the " self-capacity " of the body itself, or the "mutual capacity" of the body with respect to another body. The self-capacity of a body is the charge required to raise its potential (with respect to earth) by a unit amount.

The mutual capacity between two bodies is the total quantity of electricity displaced from one to the other when the difference of potential between them is altered by unit amount. It is important to bear these two definitions in mind, as the distinction between these two capacities is frequently overlooked, and it leads to much confusion in the case of open-circuit, such as the antennae of radio installations.

When two bodies are very close together, however, as in the case of our parallel plates, their mutual capacity is of chief importance. Since the capacity C is the charge for unit difference of potential, we have

$$e = C(V_2 - V_1) \text{ from which } C = \frac{e}{V_2 - V_1} \quad . \qquad (7)$$

Capacity of Parallel Plate Condenser

In the case of our two parallel plates we found that $V_2 - V_1 = \dfrac{4\pi e D}{\varepsilon A}$ and hence their mutual capacity

$$C = \frac{e}{V_2 - V_1} = \frac{A\varepsilon}{4\pi D} \quad . \quad . \quad . \qquad (8)$$

This is the formula for the capacity of an ordinary parallel plate

* The modern term is "capacitance."

condenser, and is of considerable importance, as it also enables us to calculate the forces in electrostatic instruments.

Mechanical Force between Parallel Plates

When two parallel plates are kept at a difference of potential there is an electrostatic attraction between them. We have already seen that the electric stress $S = 4\pi e/\varepsilon A$, which is the force on a unit charge anywhere between them. Now to find the force of attraction between the plates we have to remember in the first place that the charge on the second plate is also e, so that the total force should be $F = Se = 4\ \pi e^2/\varepsilon A$.

But it should be noted, in the second place, that a unit charge between the plates is repelled by the positively charged one and attracted to the other which is negatively charged, whereas the charge in the negative plate is only attracted by the positive plate and cannot repel itself. The force F is therefore only half the above, or $F = 2\pi e^2/\varepsilon A$, and since $V = 4\pi eD/\varepsilon A$, we have $e = \varepsilon AV/4\pi D$, from which

$$F = \frac{2\pi e^2}{\varepsilon A} = \frac{\varepsilon A V^2}{8\pi D^2} \quad . \quad . \quad . \quad (8)$$

This is the attraction between two parallel plates maintained at a given difference of potential, or the formula for the attracted disc electrometer.

Concentric Spheres

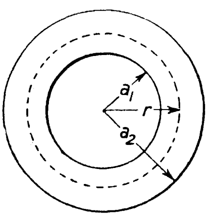


FIG. 4.2.—Diagram to illustrate the capacity between concentric spheres.

If we have two concentric spheres (Fig. 4.2) of radii a_1 and a_2 respectively, it is evident that if a charge e is imparted to the inner one it is displaced uniformly in all directions, so that the displacement density at radius r is $e/4\pi r^2$ and the electric stress $S = 4\pi\delta/\varepsilon = e/\varepsilon r^2$, exactly as for a charged point. Hence we have the difference of potential

$$V_2 - V_1 = \int_{a_1}^{a_2} S\ .\ dr = \frac{e}{\varepsilon}\int_{a_1}^{a_2} \frac{dr}{a^2} = \frac{e}{\varepsilon}\left(\frac{1}{a_1} - \frac{1}{a_2}\right)$$

Hence the mutual capacity between the spheres

$$C = \frac{e}{V_2 - V_1} = \frac{\varepsilon}{\dfrac{1}{a_1} - \dfrac{1}{a_2}} = \varepsilon\,\frac{a_1 a_2}{a_2 - a_1} \quad . \quad . \quad (9)$$

Now suppose that the radius of the outer sphere is increased so that $1/a_2$ is negligible compared with $1/a_1$, then

$$C = \varepsilon a_1 \text{ simply, and if } \varepsilon = 1 \text{ for air,}$$
$$C = a_1, \text{ the radius of the sphere} \quad . \quad . \quad . \quad (10)$$

This result establishes two important points. Firstly, the self-capacity of a sphere in electrostatic measure is simply its radius in centimetres if it is hung up in space away from other metallic objects. For this reason, Cavendish used a suspended sphere as his unity of capacity, and the capacities of radio-antennae are expressed in centimetres.

Secondly, this formula, like No. 7, gives us the meaning of the constant ε. The capacity of a condenser of any form with any medium between the plates is ε times the capacity of the same condenser with air (strictly a vacuum) between them. For this reason ε was called by Faraday the "specific inductive capacity" of the dielectric medium, which has since given place to the term "dielectric constant."

General Expression for the Mechanical Force between Charged Bodies

It was pointed out at the end of Chapter 1 that in any electrostatic instrument the moving element when charged tends to move so as to increase the mutual capacity between it and the fixed element. This gives rise to a general method of calculating the force or torque in any such instrument.

If two bodies are charged and are moved to, or farther away from one another, the difference of potential between them alters (see formulae 4 and 5). But potential is work done on unit charge, so that a change of potential means a change of the energy stored in the medium. By the principle of the conservation of energy this change of energy must equal the work done in moving the system, which equals either force multiplied by distance moved, or torque multiplied by angular displacement.

Now the energy required to charge a condenser is easily calculated. Let V be the difference of potential between its plates equal, by definition, to the work required to carry a unit charge from one plate to the other. Then to increase the charge by de, the work done must be Vde and the whole work

$$\int_0^v Vde \quad . \quad . \quad . \quad . \quad . \quad (11)$$

But we have seen that $V = e/C$, where C is the capacity, hence the energy is

$$\frac{1}{C}\int_0^e e.de = \frac{e^2}{2C} = \frac{1}{2}eV = \frac{1}{2}CV^2 \quad . \quad . \quad (12)$$

This is analogous to the case of filling a cistern with water. If w is the total weight of water and D its height in the tank, then if all the water had been lifted through the height D the work required would have been wD. But the water first entering the tank has not to be lifted at all, and on the average it has to be lifted half the total height so that the work done is $1/2\,wD$, corresponding to $\frac{1}{2}eV$ in the electrical case.

Next, suppose that the charge remains constant, but the distance between the plates is altered, then we have the change of energy equal to FdD or $T.d\theta = d.\frac{1}{2}eV_1$, from which

$$F = \frac{1}{2}\frac{d(eV)}{dD} \text{ and } T = \tfrac{1}{2}\frac{d(eV)}{d\theta}.$$

If the charge e remains constant $V = e/C$ and

$$F = \frac{1}{2}\frac{d}{dD}(eV) = \frac{e^2}{2}\cdot\frac{d}{dD}\cdot\frac{1}{C} = -\frac{e^2}{2C^2}\cdot\frac{dC}{dD} = -\frac{V^2}{2}\cdot\frac{dC}{dD} \quad (13)$$

or similarly

$$T = -\frac{V^2}{2}\cdot\frac{dC}{d\theta} \quad . \quad . \quad . \quad . \quad . \quad . \quad (14)$$

In order to get the force, therefore, all we have to do is to find dC/dD, i.e. the rate at which the capacity changes with the distance, and multiply it by half the square of the potential difference. Or if we find the rate at which the capacity changes as the system rotates through a given angle, and multiply this by half the square of the potential difference, we get the deflecting torque.

As a simple illustration of this let us take the case of the parallel plate arrangement already dealt with. Here the capacity $C = \frac{A\varepsilon}{4\pi D}$ (from 7).

If we move the plates apart so as to alter D,

$$\frac{dC}{dD} = -\frac{A\varepsilon}{4\pi D^2}$$

Therefore the force between them is by (13)

$$F = -\frac{V^2}{2}\frac{\delta C}{\delta D} = -\frac{V^2}{2} \times -\frac{A\varepsilon}{4\pi D^2} = \frac{A\varepsilon V^2}{8\pi D^2}$$

which is the same result as we obtained in formula (8). Other examples of the use of this valuable formula will be given in Chapter 10 on Electrostatic Instruments.

Electrostatic Field in Neighbourhood of Charged Surface

If an electrostatic charge is distributed over a surface, the amount of the charge per unit area is called the " surface density " of the charge, and may be denoted by the letter σ. If a unit charge is at the front O near the surface (Fig. 4.3), the force on it due to the

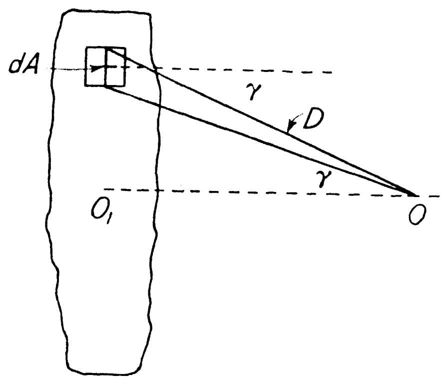


Fig. 4.3.—The force on a unit charge.

charge in the element dA will be $\sigma dA/\varepsilon D^2$. The total force cannot be obtained by simply summing this up over the surface, as the direction of the force due to each element is different, but if the surface is plane and the point O is symmetrically placed with respect to it, the components of the force perpendicular to the line OO_1 will cancel out, and we may add together the components in the direction OO_1, viz. $\frac{\delta dA}{\varepsilon D^2}\cos\gamma$. Hence the total electric stress

$$S = \frac{\sigma}{\varepsilon}\int\frac{dA\cos\gamma}{D^2} \qquad . \quad . \quad . \quad (15)$$

Solid Angle.—The expression $\int\frac{dA\cos\gamma}{D^2}$ taken over a surface has a simple geometrical meaning. If we have a small portion of a

curve dA (Fig. 4.4) and a point O in its vicinity, the angle $d\varphi$ which dA subtends at the point is arc/radius. The arc is obviously the component of dA perpendicular to the radius D or $dA \cos \gamma$, so that $d\varphi = dA \cos \gamma/D$, and the total angle subtended by the curve φ is $\int \frac{dA \cos \gamma}{D}$ taken along the curve. In like manner in the previous case $dA \cos \gamma$ is the component of the area dA perpendicular to the radius D, and this divided by the square of the radius is called the element of solid angle $d\Omega$. The reason for dividing by the square of the radius is obvious, as the area has two dimensions, length and breadth, and the length and breadth of the base of a cone formed by lines converging at a point are obviously both proportional to the radius, so that the area is proportional to the square of the radius. Consequently

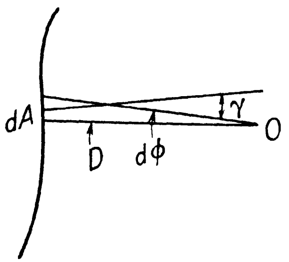


FIG. 4.4.

$$\int \frac{dA \cos \gamma}{D^2} = \Omega, \text{ the solid angle subtended by the surface} \qquad (16)$$

And therefore the electrostatic force at a point on the axis of a uniformly charged plane surface is

$$S = \frac{\sigma}{\varepsilon} \Omega \quad . \quad . \quad . \quad . \quad (17)$$

Calculation of Solid Angle.—The simplest case for the calculation of a solid angle is that of a zone of a sphere (Fig. 4.5). In that case every portion of the surface is normal to the radius and at the same distance, so that $\cos \gamma$ is everywhere unity and

$$\Omega = \frac{1}{a^2} \int dA = \frac{A}{a^2}$$

For a small zone $dA = 2\pi a \sin \gamma \times a . d\gamma = 2\pi a^2 \sin \gamma . d\gamma$.

$$\text{Hence } \Omega = \frac{2\pi a^2}{a^2} \int_{\gamma_1}^{\gamma_2} \sin \gamma \, . \, d\gamma = 2\pi (\cos \gamma_1 - \cos \gamma_2) \quad . \quad (18)$$

If we start from O_1, $\gamma_1 = 0$, $\cos \gamma_1 = 1$ and $\Omega = 2\pi(1 - \cos \gamma_2)$. Again, if we take a complete hemisphere, $\gamma_2 = \pi/2$, $\cos \gamma_2 = 0$ and

$\Omega = 2\pi$; while if we take the whole sphere $\gamma_2 = \pi$, $\cos\gamma_2 = -1$, and $\Omega = 4\pi$, as is obvious from the fact that the whole surface of the sphere is $4\pi a^2$.

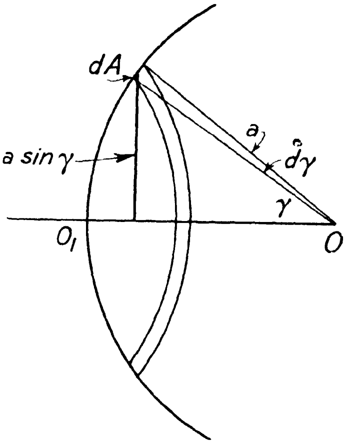


FIG. 4.5.—Calculation of solid angle.

Plane Circular Disc

The formula just obtained, viz. $\Omega = 2\pi(1 - \cos\gamma)$, also gives us of course the solid angle subtended by a plane circular disc of which

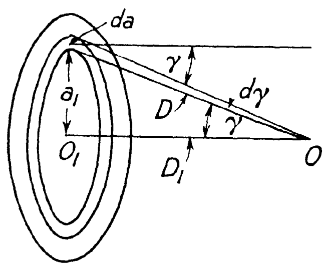


FIG. 4.6.—Solid angle of a plain circular disc.

the radius subtends the angle γ at the point O. But it will be well to calculate it from first principles in order to make such calculations familiar. Taking a circular strip or annulus of radius (a_1 (Fig. 4.6))

and width da, the element of surface dA is evidently $2\pi a_1 \, . \, da$, and the angle it makes with the surface is γ, hence

$$\Omega = \int \frac{dA \cos \gamma}{D^2} = 2\pi \int_o^\gamma \frac{a_1 da \cos \gamma}{D^2}$$

But $a_1 = D_1 \tan \gamma$, $da = D_1 \sec^2\gamma \, . \, d\gamma$ and $D = D_1 \sec \gamma$.

Hence $\Omega = 2\pi \int_o^\gamma \frac{D_1^2 \tan \gamma \sec^2\gamma \, . \, \cos \gamma \, . \, d\gamma}{D_1^2 \sec^2\gamma} = 2\pi \int_o^\gamma \sin \gamma \, . \, d\gamma =$

$2\pi \, (1 - \cos \gamma)$, as before.

Solid Angle close to Surface

If the disc is very large compared with the distance OO_1, or if the distance OO_1 is very small in comparison with the radius of the disc, γ becomes 90°, $\cos \gamma = 0$ and $\Omega = 2\pi$, as in the case of the hemisphere.

Electric Stress near Charged Surface

Since the electric stress S in the axis of a plane disc was found to be $\sigma\Omega/\varepsilon$ (see 17), where σ is the surface density of the charge, it follows that the electric stress $S = 2\pi\sigma/\varepsilon$ close to a charged surface. If there are two surfaces close together, one having a surface density σ and the other of $-\sigma$, as in the case of the parallel plate condenser, it is evident that the electric stress between them $S = \frac{2\pi\sigma}{\varepsilon} - \left(-\frac{2\pi\sigma}{\varepsilon}\right) = \frac{4\pi\sigma}{\varepsilon}$, which is the same result as we got from the displacement principle (see 4).

This again gives us the force of attraction between two parallel plates close together. Since each unit of charge on one plate is attracted with a force $2\pi\sigma/\varepsilon$ by the other plate and the total charge on it is $A\sigma$, the total attraction $F = 2\pi A\sigma^2/\varepsilon$, and since $V = 4\pi\sigma D/\varepsilon$ or $\sigma = CV/4\pi D^2$, $F = A\varepsilon V^2/8\pi D^2$ as in (8). Since in this case $S = V/D$, $F = A\varepsilon S^2/8\pi$ or $\varepsilon S^2/8\pi$ per unit area. This means that there is a tension of $\varepsilon S^2/8\pi$ dynes per sq. cm. along the lines of electric force.

MAGNETIC FORMULAE.

Although, physically speaking, magnetism is a complex electrical phenomenon caused by the rotation of electric charges or electrons, its elementary manifestations are simple and are akin to those of electrostatics. Both magnets and electrical charges have similar effects in the space surrounding them, but while a single electrostatic charge may be isolated (the equal and opposite charge which must be induced by it being too far away and too diffused to need consideration in many cases), a magnet pole is nearly always coupled to an equal and opposite pole within a short distance of it.

Coulomb's Law of Inverse Squares

By means of his torsion balance Coulomb also experimentally established a law of inverse squares for attraction or repulsion similar to the electrostatic inverse square law. He showed that the force of repulsion between two similar magnetic poles was proportional to the product of their strengths and inversely proportional to the square of the distance between them, so that we may write

$$F = \frac{m_1 m_2}{\mu D^2}, \text{ where } m_1 \text{ and } m_2 \text{ are the strengths of the two poles} \quad (19)$$

If we choose the unit magnetic pole as that which repels a similar pole at 1 cm. distance with a force of 1 dyne in air (strictly in vacuum) the formula becomes $F = m_1 m_2 / D^2$ in air, and μ is a number expressing by how much the force is diminished in any other medium.

Intensity of Magnetic Force or Field

As in the case of electrostatics, the intensity of a magnetic field which we might call the magnetic stress H is defined as the force in dynes upon a unit magnetic pole in the field : hence at a distance D from a pole of strength m

$$H = \frac{m}{\mu D^2} \quad . \quad . \quad . \quad . \quad . \quad (20)$$

Magnetic Flux Density or Induction

In harmony with electrostatics, the introduction of a magnet pole into any region causes a strain, or what might be called a magnetic displacement to be set up in the medium. The term magnetic induction or flux density, corresponding to what we called displace-

ment density in electrostatics, has been employed for this phenomenon. In electrostatics Faraday's ice-pail experiment, repeated with different media, proves with considerable accuracy that the total displacement in the medium around the charge is equal to the charge, from which $\delta = e/4\pi D^2$ at a distance D from the change e.

Unfortunately we have no magnetic experiment corresponding to the ice-pail experiment, and it is usual to assume, as Faraday did in the case of electrostatics, that the induction is independent of the medium. In the case of magnetism, however, instead of taking it as $m/4\pi D^2$ in analogy with the electric displacement density, it has been defined as having unit amount at unit distance from unit magnetic pole, or $B = m/D^2$, so that as $H = m/\mu D^2$

$$B = \mu H \text{ Gauss} \quad . \quad . \quad . \quad . \quad (21)$$

while in electrostatics $4\pi\delta = \varepsilon S$. The justification for the assumption will be considered later.

Magnetic Lines of Induction

The conception of lines of induction is of considerable use in electro-magnetic calculations, the number of lines of induction per unit area representing the value B. The total flux passing through any area A normal to the lines of induction is $\Phi = AB$ lines or Maxwells, and since B has the value m/D^2 at a distance D from a pole of strength m, and the area of the sphere of radius D around m is $A = 4\pi D^2$, we have

$$\Phi = AB = 4\pi D^2 \times \frac{m}{D^2} = 4\pi m \text{ Maxwells} \quad . \quad (22)$$

so that 4π lines of induction radiate outwards from a unit pole independently of the medium surrounding it.

Magnetic Needle

A single magnet may be regarded as two poles of equal and opposite strengths m and $-m$ separated by a distance l (Fig. 4.7). If such a magnet is in a magnetic field of strength H one pole is urged in the direction of the field with a force $F = Hm$ and the other with an equal force in the opposite sense. The result is that the needle does not tend to move as a whole in either way, but tends to turn with a couple or torque T of $Fl \sin \gamma$ or

$$T = Hml \sin \gamma \quad . \quad . \quad . \quad (23)$$

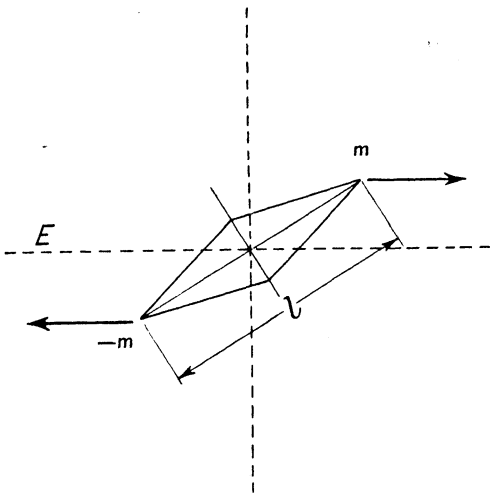


FIG. 4.7.—Force on a magnetic needle.

Magnetic Moment

In the last formula, if $\gamma = 90°$, or the magnetic needle is perpendicular to the field, the torque $T = Hml$. If, also, the field is of unit strength, $H = 1$ and $T = ml$. The quantity ml is therefore the moment or torque when the needle is at right-angles to a unit magnetic field and is called the "magnetic moment" M_m of the needle. Hence, whenever a galvanometer needle is in a field, we may write

$$T = Hml \sin \gamma = HM_m \sin \gamma \qquad . \quad . \quad (24)$$

Intensity of Magnetization

In order to express the degree to which a magnet is magnetised the term "intensity of magnetization" I_m is used, and gives the magnetic moment per unit of volume or $I_m = M_m/v$. Since, however, $M_m = ml$ and $v = Al$, when A is the area of the section of the magnet, $I_m = ml/Al = m/A$, or it may be expressed as the pole strength per unit area, which is analogous to the surface density of electrification σ in electrostatics.

Field in Neighbourhood of Magnet Pole

On page 154 it was shown that the force in the neighbourhood of an electrostatically charged surface was $\sigma\Omega/\varepsilon$, where σ is the surface

density of the charge and Ω the solid angle which the surface subtended at the point considered.

Since the law of attraction for a magnet pole is similar, the same formula applies for the strength of the field in the vicinity of a magnet pole, except that the intensity of magnetization I_m replaces the electrostatic surface density σ. Hence

$$H = \frac{I_m \Omega}{\mu} \quad . \quad . \quad . \quad . \quad (25)$$

If the point is close to the pole $\Omega = 2\pi$ and $H = 2\pi I_m/\mu$. Again, if the magnet is bent round so that the two poles are close together with only a small gap between, a unit pole in this gap is repelled from the N pole with a force $2\pi I_m$ and attracted to the S pole with a similar force m so that the total force or field in the gap is

$$H = \frac{4\pi I_m}{\mu} \quad . \quad . \quad . \quad . \quad (26)$$

which corresponds to the electrostatic field $S = 4\pi\sigma/\varepsilon$ in the gap between two parallel plates (see page 149).

Self-demagnetizing Effect

In consequence of the effect just investigated, every magnet having external or salient poles has an internal magnetic force tending to demagnetize it. This force is $2\pi I_m/\mu$ just inside the pole face, and gets smaller as the centre of the magnet is approached, becoming larger again and attaining the value $2\pi I_m/\mu$ once more just before reaching the other pole face.

It is evident from this that a straight permanent magnet will retain its magnetism more strongly the longer it is in comparison with its sectional area, so that a large proportion of it is practically without any appreciable demagnetizing force. By bending the magnet into a horseshoe or circular form the demagnetizing effect of each pole is practically annulled by the proximity of the other, and if the two ends are brought quite close together with only a small gap between, the solid angles subtended by both poles at any point are practically equal, and there is no resultant demagnetizing force. For this reason the gap in permanent magnet-moving coil instruments, where constancy of magnetic field is most important, is made as small as possible; while whenever permanent magnets

are to be preserved from loss of magnetization they are short-circuited by an armature or keeper.

Induced Magnetization

When a piece of ferro-magnetic material such as iron or steel, nickel, cobalt or the copper-manganese-aluminium alloys discovered by Heusler is placed in the field produced by a magnet, it becomes magnetized and increases the induction in its neighbourhood. The amount of the induction in the material is taken as that which would exist in a narrow transverse gap (Fig. 4.8). If H is the magnetic

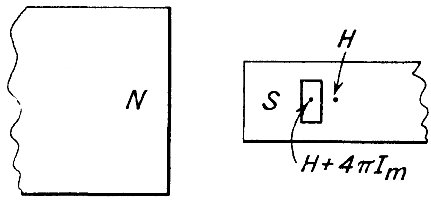


FIG. 4.8.—Induced magnetization.

force or stress at this gap due to the inducing magnet, it produces a certain intensity of magnetization I_m in the material, and this produces an additional magnetic stress $= 4\pi I_m$ in the small gap. Hence the total force or induction in the gap

$$B = H + 4\pi I_m \quad . \quad . \quad . \quad . \quad (27)$$

while in the material itself the poles in the two surfaces of the gap cancel each other and the force is therefore H only. Whenever, therefore, a magnetic force or stress H acts upon a piece of material, an induction B is produced in a small air gap in that substance $= H + 4\pi I_m$, and as the intensity of magnetization I_m is very large in many materials, B is much greater than H, or the induction close to the iron is much greater than it would have been without it.

Susceptibility and Permeability

The ratio of the intensity of magnetization I_m produced by a magnetic stress H to the stress is called the "susceptibility" of the material K, so that $I_m = HK$; also the ratio of the induction B to the stress H is called the permeability μ, so that $B = \mu H$.

Hence, since $\quad B = H + 4\pi I_m$

$$\mu H = H + 4\pi KH$$

from which $\quad \mu = 1 + 4\pi K \quad . \quad . \quad . \quad . \quad (28)$

This relation is of importance in instrument work, as in dealing with permanent magnets it is quite common to give their intensities of magnetization, while with electro-magnets the total induction B is more generally employed.

Magnetic Potential

In conformity with electrostatics the magnetic potential at a point is defined as the work required to bring up a unit magnetic pole from an infinite distance to that point. Since H is the force on unit pole, we have $V = \int H dl$, where dl is an element of the path. As $B = \mu H$, we have $H = B/\mu$ and

$$V = \int H dl = \int \frac{B}{\mu} \cdot dl \qquad . \quad . \quad . \quad (29)$$

Since in the neighbourhood of a pole of strength m, $H = m/\mu D^2$, the potential

$$V = \int_{\infty}^{D} H dD = \frac{m}{\mu} \int_{\infty}^{D} \frac{dD}{D^2} = \frac{m}{\mu D} \qquad . \quad . \quad (30)$$

corresponding to $V = e/\varepsilon D$ for electrostatics.

Field Due to Magnet

In order to find the magnetic force or field in the vicinity of a magnet we have to find the resultant of the two forces due to its poles. There are three positions in which it is useful to do this: (A) at a point on the axis of the magnet, (B) at a point on a transverse axis through its centre, and (C) at a point in a transverse axis through one pole,

The A position of Gauss: In the position A (Fig. 4.9) we have

$$H = \frac{m}{\mu\left(D - \frac{l}{2}\right)^2} - \frac{m}{\mu\left(D + \frac{l}{2}\right)^2} = \frac{2mlD}{\mu\left(D^2 - \frac{l^2}{4}\right)^2}$$

If D is large in comparison with $l/2$

$$H = \frac{2mlD}{\mu D^4} = \frac{2M_m}{\mu D^3} \qquad . \quad . \quad . \quad (31)$$

In the position B—the B position of Gauss—the force due to each pole is in the direction of the line joining it to B, and it is evident

that the components of the force along the transverse axis OB are equal and opposite, and therefore cancel, so that the resultant force is parallel to the magnet and twice that due to each pole.

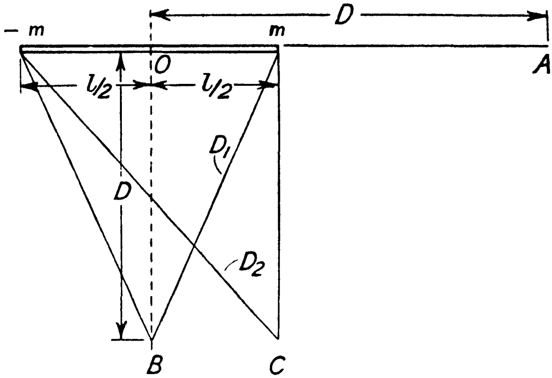


FIG. 4.9.—Magnetic force in the vicinity of a magnet.

The axial component of the force due to each pole is $\frac{m}{\mu D_1^2} \cdot \frac{l/2}{D_1} = \frac{ml}{2\mu D_1^3}$, and therefore the total force for the two poles

$$H = \frac{2ml}{2\mu D_1^3} = \frac{M_m}{\mu D_1^3} \quad . \quad . \quad . \quad (32)$$

In position C, employed by Ewing for magnetic measurement, the resultant force is not axial, but if we take the axial component only, it is evident that the pole m has no influence and the force is the axial component of

$$\frac{m}{\mu D_2^2} \text{ or } \frac{m}{\mu D_2^2} \cdot \frac{l}{D_2} = \frac{M_m}{\mu D_2^3} \quad . \quad . \quad . \quad (33)$$

Deflection of Magnetic Needle by Magnet

When a magnet is brought near to a pivoted magnetic needle in such a way that the field due to it, H_1, is inclined to the earth's magnetic field H, the needle is deflected so as to set itself along the resultant field. The most important case is when H_1 is perpendicular to H (Fig. 4.10) and in that case it is evident that

$$\tan \gamma = \frac{H_1}{H} \quad . \quad . \quad . \quad (34)$$

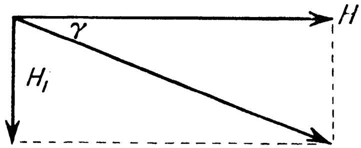


FIG. 4.10.—Forces acting on a magnetic needle.

If a magnet is placed at right angles to the earth's field and pointing to the centre of the needle it is in the A position of Gauss, and $H_1 = 2\,M_m/D^3$, from which

$$\tan \gamma = \frac{2M_m}{HD^3} \quad . \quad . \quad . \quad . \quad (35)$$

The deflection in other cases can be calculated as above.

Time of Oscillation of a Magnetic Needle

When a pivoted magnet is disturbed from its position of rest it oscillates before coming to rest again if the friction is sufficiently low. The torque due to its inertia is $\frac{K \cdot d^2\gamma}{dt^2} = K\ddot{\gamma}$, where K is its moment of inertia, and the controlling torque has been proved in (24) to be $HM_m \sin \gamma$.

Hence the total torque is $I\ddot{\gamma} + HM_m \sin \gamma = O$, when the disturbing force is removed and if friction is neglected. For small oscillations γ may be substituted for $\sin \gamma$, so that the equation of motion is $K\ddot{\gamma} + HM_m\gamma = O$; from which

$$\ddot{\gamma} = -\frac{HM_m}{K} \cdot \gamma$$

The equation to a simple harmonic oscillation is

$$\gamma = \beta \sin (\omega t - \varphi) \text{ from which } \dot{\gamma} = \beta\omega \cos (\omega t - \varphi)$$

and $\ddot{\gamma} = -\beta\omega^2 \sin (\omega t - \varphi) = -\omega^2\gamma$.

It is evident that γ returns to the same value every time ωt increases by 2π, or t increases by $2\pi/\omega$. Hence $2\pi/\omega$ is the periodic time T_λ and $1/T_\lambda = \omega/2\pi = f$ the frequency of oscillation or the number of oscillations per second.

Comparing the expressions $\ddot{\gamma} = -\frac{HM_m}{K}\gamma$ for the magnet and $\ddot{\gamma} = -\omega^2\gamma$ for the harmonic oscillation, it is evident they are the same

if $\omega^2 = \frac{HM_m}{K}$. It therefore follows that the motion of the needle is represented by $\gamma = \beta \sin \left\{ \sqrt{\frac{HM_m}{K}} \cdot t - \varphi \right\}$, which is a simple harmonic oscillation of amplitude β and periodic time

$$T_\lambda = \frac{2\pi}{\omega} = 2\pi \sqrt{\frac{K}{HM_m}},$$

$$\text{or of frequency } f = \frac{1}{2\pi}\sqrt{\frac{HM_m}{K}} \quad . \quad . \quad (36)$$

For a given magnetic needle M_m and K are constant, and therefore the frequency $f \propto \sqrt{H}$ or $H \propto f^2$, which affords a means of comparing the strengths of magnetic fields.

In moving needle galvanometers, etc., where the control is due to the external magnetic field H_1, the sensitiveness is inversely proportional to H, and this is proportional to T_λ^2, or to the square of the periodic time. A galvanometer having a 10 sec. swing has, therefore, four times the sensitiveness of the same instrument with a 5 sec. swing.

Determination of H and M_m

The absolute value of both the magnetic moment of a magnet M_m and of the earth's field H may be obtained by two simple experiments. Firstly, the magnet is suspended in a loop and set oscillating, T_λ being noted, from which the product HM_m can be obtained, since

$$T_\lambda = 2\pi \sqrt{\frac{K}{HM_m}} \text{ or } HM_m = \frac{4\pi^2 K}{T_\lambda^2} \quad . \quad . \quad (37)$$

Next, the same magnet is placed near a suspended needle, say in the A position of Gauss, producing a deflection γ, and since $\tan \gamma = 2M_m/HD^3$ by (35) we have

$$\frac{M_m}{H} = \frac{D^3 \tan \gamma}{2} \quad . \quad . \quad . \quad (38)$$

Multiplying (37) and (38) together we have

$$M_m^2 = \frac{4\pi^2 K}{T_\lambda^2} \cdot \frac{D^3 \tan \gamma}{2} \quad . \quad . \quad . \quad (39)$$

and dividing (37) by (38)

$$H^2 = \frac{4\pi^2 K}{T_\lambda^2} \cdot \frac{2}{D^3 \tan \gamma} \quad . \quad . \quad . \quad (40)$$

from which M_m and H are both determined if K is known. The moment of inertia of a rectangular bar magnet about an axis through its centre is 1/12 mass $\times$ length^2, and is therefore known from its weight and dimensions.

The determination of H and M_m can therefore be made with considerable accuracy, and this is of importance as enabling both magnetic and electro-magnetic units to be fixed independently of standards.

Attraction between Adjacent Poles

If we have two poles in contact with one another or closely adjacent, then if I_m is the intensity of magnetization, the attraction for a unit pole in one face by the whole of the other face is $2\pi I_m/\mu$ (page 160), and since the total strength of the other pole $m = I_m A$ and μ is the permeability of the medium between the poles, total force of attraction between poles,

$$F = \frac{2\pi I_m m}{\mu} = \frac{2\pi}{\mu} A I_m^2 \quad . \quad . \quad . \quad (41)$$

Also since $B = 4\pi I_m + H$, $I_m = B/4\pi$ if H is zero, as in the case of a permanent magnet.

Hence

$$F = \frac{2\pi}{\mu} A I_m^2 = \frac{2\pi}{\mu} A \cdot \frac{B^2}{16\pi^2} = \frac{AB^2}{8\pi\mu} \quad . \quad . \quad (42)$$

which corresponds to the electrostatic formula (page 150), viz. $F = \varepsilon S^2/8\pi$ for the attraction between parallel plates.

The above formula (42) is correct for the attraction between the adjacent poles of a permanent magnet when the magnetizing force H is zero, but in the case of induced magnetism it is clear that

$$F = AI_m \left(\frac{2\pi}{\mu} I_m + H \right);$$

and since

$$I_m = \frac{B - H}{4\pi}$$

$$F = A \frac{B - H)}{4\pi} \left\{ \frac{B - H}{2\mu} + H \right\} \quad . \quad . \quad . \quad (43)$$

or

$$F = \frac{A(B^2 - H^2)}{8\pi} \text{ in air} \quad . \quad . \quad . \quad . \quad (44)$$

These formulae are of importance in connection with traction permeameters for testing the magnetic qualities of iron, etc. By making A unity in formula (42) we have the force per unit area of tension along the lines of force $= B^2/8\pi\mu$ or $H^2/8\pi$ in air, which corresponds to the tension along the electrostatic lines of force of $S^2/8\pi$.

ELECTRO-MAGNETIC FORMULAE.

The experiments of Ampère in 1821 laid the foundation of quantitative electro-magnetic calculation. He found that each element of a current-carrying circuit produced a magnetic force at any point, proportional to the strength of the current and to the length of the element, inversely proportional to the square of the distance and proportional to the sine of the angle which the element made with the line joining it to the point. If I is the current strength, dl the length of the element of conductor (Fig. 4.11), D the distance to the

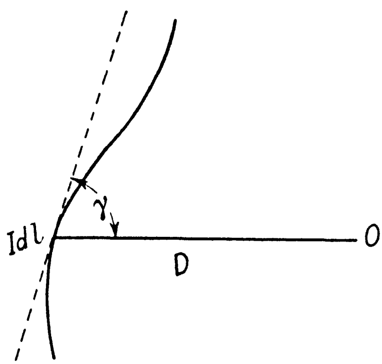


FIG. 4.11.—Force due to a conductor carrying a current.

point O, and γ the angle between dl and D, then we have for the element of force dH at O, $dH \propto \frac{Idl}{D^2} \sin \gamma$, and it is perpendicular both to dl and to D, i.e. to the plane of the paper in Fig. 4.11.

Definition of Unit Current

The C.G.S. unit of current is so chosen that it turns the above relation into an equality for air or

$$dH = \frac{Idl}{D^2}\sin\gamma \quad . \quad . \quad . \quad . \quad (45)$$

If we take a circular coil of radius a, the field at its centre is therefore $H = \frac{2\pi aI}{a^2} = \frac{2\pi I}{a}$, since γ is everywhere 90°. If the coil is of unit radius $H = 2\pi I$, so that the C.G.S. unit of current may be best defined as that current which produces a force of 2π dynes on a unit magnetic pole at the centre of a circular loop of 1 cm. radius. The practical unit of current, the ampere, is one-tenth or 10^{-1} C.G.S. unit.

Field in Coils of Various Forms

As the important class of electromagnetic measuring instruments depend for their action on the magnetic forces produced by the currents in their coils, it is eminently desirable to be able to form a fairly close estimate of the magnetic fields produced by coils of various forms.

Circular Coil

For a simple circular coil in which the section is small in comparison with its radius, the field at its centre is evidently

$$\frac{2\pi NI}{a} \text{ or } 0{\cdot}628\,\frac{\text{ampere turns}}{\text{radius in cm.}} \quad . \quad . \quad . \quad (46)$$

For a point on the axis of the coil at a distance D from the centre (Fig. 4.12) we have $dH = \frac{Idl}{D_1^2}$. This force can be divided into two

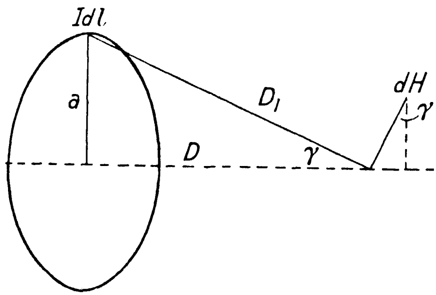


FIG. 4.12.—Force at a distance from a circular coil.

components, one, $dH \sin \gamma$, along the axis, and the other, $dH \cos \gamma$ perpendicular to it. It is evident from the symmetry of the system that the perpendicular components from the various opposite elements cancel each other, and since γ is the same for each position of D_1,

$$H = \frac{Il}{D_1^2} \sin \gamma = \frac{2\pi aNI}{D_1^2} \cdot \frac{a}{D_1} \cdot = \frac{2\pi a^2 NI}{D_1^3} = \frac{2\pi a^2 IN}{(a^2 + D^2)^{\frac{3}{2}}}$$

or

$$H = 0{\cdot}628\ a^2 \frac{\text{ampere turns}}{\text{cube of the slant distance}} \quad . \quad . \quad . \quad (47)$$

This rapid diminution of the force with the distance from the centre was made use of by Lord Kelvin in his heavy-current galvanometer, in which a circular coil was provided with a compass-box, which could be slid along the axis and fixed at points for which the constant had a definite value. At a distance large in comparison with a, $D_1 = D$ and $H = \frac{2A}{D^3} IN$, where A is the area of the circle.

Cylindrical Coil

In order to get the magnetic force at any point on the axis of a cylindrical coil we use the formula just obtained and sum the force due to each portion of the coil. If N_o is the number of turns per

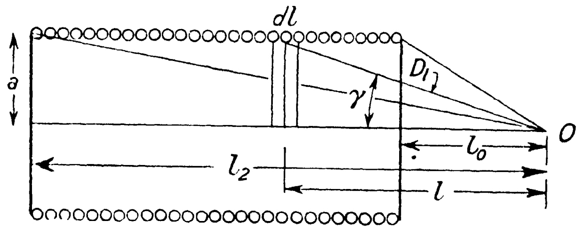


FIG. 4.13.—Force at any point on a solenoid.

cm. length of the coil, then $N_o dl$ is the number of turns in an element dl of its length, and the force due to this element

$$dH = \frac{2\pi a^2 I N_o dl}{D_1^3}$$

The total force due to the coil is therefore $H = 2\pi a^2 I N_o \int_{l_1}^{l_2} \frac{dl}{D_1^3}$; by reference to the diagram (Fig. 4.13), $l = a \cot \gamma$, from which $dl = a \operatorname{cosec}^2 \gamma d\gamma$ and $D_1 = a \operatorname{cosec} \gamma$.

Hence

$$H = 2\pi IN_o \int_{\gamma_1}^{\gamma_2} \frac{\operatorname{cosec}^2 \gamma d\gamma}{\operatorname{cosec}^3 \gamma}, \text{ or } H = 2\pi IN_o \int_{\gamma_1}^{\gamma_2} \sin \gamma d\gamma,$$

from which

$$H = 2\pi IN_o (\cos y_2 - \cos y_1) \quad . \quad . \quad (48)$$

But $2\pi (\cos \gamma_1 - \cos \gamma_2) = \Omega$ by formula (18), hence

$$H = IN_o\Omega \quad . \quad . \quad . \quad . \quad (49)$$

If the point O is inside a very long narrow coil, then $\gamma_2 = O$ and $\gamma_1 = \pi$, and

$$H = 4\pi IN_o \quad . \quad . \quad . \quad . \quad (50)$$

Hence at any internal point on the axis of a long narrow cylindrical coil the magnetic force is $\frac{4\pi}{10}$ or 1·2566 times the ampere turns per centimetre length. This is a very useful formula in many cases.

Circular Coil of Rectangular Section

This is the most common form of coil (Fig. 4.14), and the field produced by it at any point on the axis can be obtained by using

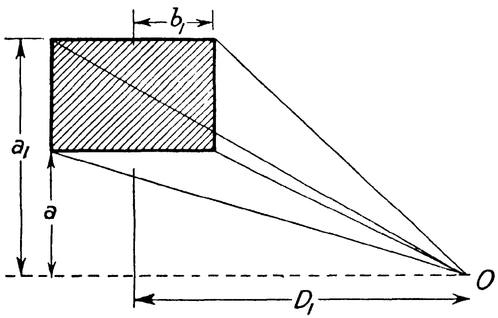


FIG. 4.14.—Force due to a circular coil of rectangular section.

the last formula for each layer, and summing for the different layers. The integration will not be given here, but the result is as follows

$$H = \frac{2\pi}{10} IN_o \left\{ (D_1 + b_1) \log \frac{a_1 + \sqrt{a_1^2 + (D_1 + b_1)^2}}{a + \sqrt{a^2 + (D_1 + b_1)^2}} \right.$$

$$\left. - (D_1 - b_1) \log \frac{a_1 + \sqrt{a_1^2 + (D_1 - b_1)^2}}{a + \sqrt{a^2 + (D_1 - b_1)^2}} \right\} \quad . \quad (51)$$

Straight Conductor

By Ampère's formula the force due to a current Idl is $dH = \dfrac{Idl \cos \gamma}{D_1^2}$, from which the total force due to any length of conductor is $H = I\displaystyle\int_{l_1}^{l_2} \frac{dl \cos \gamma}{D_1^2}$. If D is the perpendicular distance of the point O (Fig. 95) from the conductor, we have $l = D \tan \gamma$, $dl = D \sec {}^2\gamma d\gamma$, and $D_1 = D \sec \gamma$, and the formula transforms to

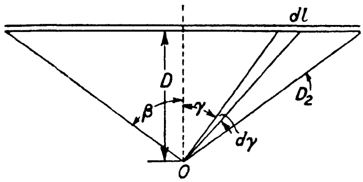


FIG. 4.15.—Force due to a straight conductor.

$$H = \frac{I}{D}\int_{\gamma_1}^{\gamma_2} \cos \gamma d\gamma = \frac{I}{D}(\sin \gamma_2 - \sin \gamma_1) \quad . \quad (52)$$

If the conductor is of length l (Fig. 4.15), and subtends an angle of 2β with the point O, $H = \dfrac{2I}{D} \sin \beta = \dfrac{2I}{D}\dfrac{l}{2D_2} = \dfrac{Il}{D_2 D}$. If the wire is very long in comparison with the distance D, $\sin \beta = 1$ and

$$H = \frac{2I}{D} \quad . \quad . \quad . \quad . \quad . \quad (53)$$

Rectangular Coil

If we consider a rectangular coil (Fig. 4.16) of sides b and b_1, then as above the force at a point O on the axis is $H = \dfrac{Ib}{D_1 D_2}$, due to the side b. This force, however, is perpendicular to the plane of b and D_1, and is inclined at an angle φ to the axis. The axial component is

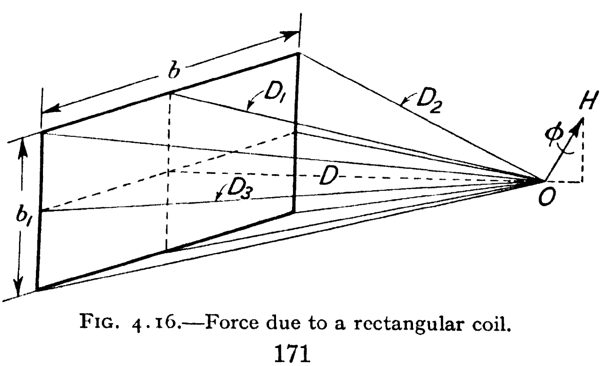


FIG. 4.16.—Force due to a rectangular coil.

$H \cos \varphi = H \dfrac{b_1}{2D_1} = \dfrac{Ibb_1}{2D_1^2 D_2}$ for one long horizontal side, or $\dfrac{bb_2 I}{D_2 D_1^2}$ for the two, their vertical components cancelling.

Similarly, for the two vertical sides the axial component is $\dfrac{bb_1 I}{D_2 D_3^2}$ so that the total force for the rectangle is

$$H = \frac{bb_1 I}{D_2}\left(\frac{1}{D_1^2} + \frac{1}{D_3^2}\right) \quad . \quad . \quad . \quad (54)$$

For a coil of IN ampere turns

$$H = \frac{bb_1}{10D_2}\left(\frac{1}{D_1^2} + \frac{1}{D_3^2}\right) IN \quad . \quad . \quad (55)$$

At a distance large in comparison with b or b_1, $D_1 = D_3 = D_2 = D$, and

$$H = \frac{2bb_1}{10D^3} IN = \frac{2A}{10D^3} IN \quad . \quad . \quad (56)$$

the same result as for a circular coil.

The above cases comprise the principal ones met with in practice, and enable the fields in most electro-magnetic instruments to be estimated with fair accuracy.

For points off the axis the calculation, except in the case of rectangular coils, is very complex, necessitating the employment of elliptic integrals or a series of spherical harmonics.

Magnetic Potentials due to Current

The difference of magnetic potential between any two points $V_2 - V_1 = \int_1^2 H dl$, if the motion is in the direction of the force. In the case of a long straight conductor carrying a current we have seen that the force at any point $H = \dfrac{2I}{D}$, where D is the perpendicular distance between the point and the conductor. If a unit magnetic pole is moved in a circle round the conductor the force is everywhere the same, and the distance moved to bring it back to the starting-point is $2\pi D$, hence

$$V = \frac{2I}{D} \times 2\pi D = 4\pi I \quad . \quad . \quad . \quad (57)$$

which is independent of the radius of the circle. Hence every time a unit magnetic pole encircles a long current-carrying conductor,

an amount of work $4\pi I$ is done. If we now take a coil (Fig. 4.17) of N turns of radius a, the force H at a point distant D is $\frac{2\pi a^2 IN}{D_1^3}$ (see page 169). Hence the work done in moving a unit pole from one point to another is $V_2 - V_1 = 2\pi a^2 IN \int_{D_1}^{D_1} \frac{dD}{D_1^3}$, which is the same integral as we had in (48) when adding up the effect of a long cylindrical coil.

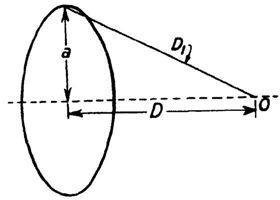


FIG. 4.17.—Work done in moving a unit pole round a current carrying conductor.

Hence $V_2 - V_1 = 2\pi IN(\cos\gamma_1 - \cos\gamma_2)$.

If we start from an infinite distance $\gamma_1 = O$ and $\cos\gamma_1 = 1$, from which $V = 2\pi IN(1 - \cos y) = IN\Omega$.

On bringing up the unit pole to the centre of the coil $\Omega = 2\pi$ and $V = 2\pi IN$, while on removing it to infinity on the other side an equal amount of work is done. Hence the total increase of potential on passing right through the coil is $4\pi IN$ or $4\pi I$ for a coil of one turn.

This is the same result obtained on encircling a long conductor, and it will be proved later that this result holds generally, and that the work done by a unit pole on passing through any coil and returning to the same position is $4\pi IN$ or $\frac{4\pi}{10}$ times the ampere turns in the coil . . . . (58)

Hopkinson's Formula

If we have a magnetic circuit made up of different portions of different materials with a magnetizing coil, we have by the foregoing:

Total work in taking unit pole round the circuit is $\frac{4\pi}{10}$ ampere turns, $= \int H dl = \frac{B_1}{\mu_1} l_1 + \frac{B_2}{\mu_2} l_2 +$ etc., since $\frac{B}{\mu} = H$. Since also $\Phi = AB$, $B = \frac{\Phi}{A}$, and $\frac{4\pi}{10}$ ampere turns $= \Phi \left\{ \frac{l_1}{A_1\mu_1} + \frac{l_2}{A_2\mu_2} + \frac{l_3}{A_3\mu_3} + \text{etc.,} \right\} = \Phi \sum \frac{l}{A\mu}$, or the flux

$$\Phi = \frac{\frac{4\pi}{10} \text{ ampere turns}}{\sum \frac{l}{A\mu}} \quad . \quad . \quad . \quad . \quad (59)$$

Force on Current-carrying Conductor in Magnetic Field

Reverting to Ampère's formula (45) for the magnetic field at a point due to a current element, viz. $dH = \frac{Idl \sin y}{D^2}$, we therefore have as the force on a pole of strength m,

$$dF = \frac{mIdl \sin \gamma}{D^2}$$

But $\frac{m}{D^2}$ is the magnetic field H_1 at the element of current due to the pole m, and the force dF, which represents the reaction of the magnet on the current, must equally represent the reaction of the magnet on the current. Hence

$$dF = H_1 Idl \sin \gamma \quad . \quad . \quad . \quad (60)$$

and if the field H is uniform and we have a straight conductor of length l perpendicular to the field, $\sin \gamma = 1$ and $F = H_1 Il$. If I is in amperes

$$F \text{ dynes} = \frac{1}{10} H_1 Il \quad . \quad . \quad . \quad (61)$$

which is the fundamental formula for moving coil instruments.

Rectangular Coil in Uniform Field

If we have a rectangular coil of axial length l and breadth b, and carrying N turns (Fig. 4.18), the force on each conductor will be $\frac{1}{10} HIl$ as above, and the total force on each vertical side of the coil will be $\frac{1}{10} HINl$ dynes each at a radius $\frac{b}{2}$ from the axis, the total torque $T = \frac{2}{10} HINl \times \frac{b}{2} \cos \gamma$, or

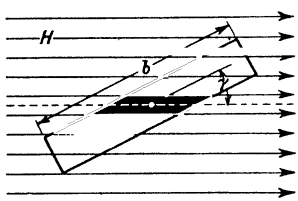


FIG. 4.18.—Torque on a rectangular coil in a uniform field.

$$T = \frac{1}{10} HINlb \cos \gamma = \frac{1}{10} HIN A \cos \gamma \quad . \quad (62)$$

where A is the area of the coil.

Magneto-electric Induction

In 1831 Faraday discovered that when a magnet was introduced into or withdrawn from a coil of wire connected to a galvanometer

a current was produced during the motion of the magnet only, being opposite in direction on withdrawal to that on entrance. He came to the conclusion that the E.M.F. induced in each turn of the coil was proportional to the rate of change of magnetic flux or number of lines of induction passing through it, so that we may write

$$E \propto N \frac{d\Phi}{dt}.$$

In 1834 Lenz showed that the direction of the E.M.F. was such that the current induced tended to stop the motion which produced it. Also the B.A. Committee in 1867 decided to define the electromagnetic unit of E.M.F. in accordance with Faraday's law.

The C.G.S. unit of E.M.F. is that E.M.F. produced in a circuit when the flux linked with it alters at the rate of one line or Maxwell per second. One volt equals 10^8 C.G.S. units of E.M.F. Hence we may write

$$E = -N\frac{d\Phi}{dt} \text{ C.G.S. units, or } \frac{-N}{10^8}\frac{d\Phi}{dt} \text{ volts} \quad . \quad (63)$$

E.M.F. Induced in Moving Conductor

If a conductor of length l slides with a uniform velocity u perpendicular to a field of strength H (Fig. 4.19), the area of the

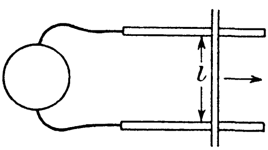


FIG. 4.19.—EMF induced in a sliding conductor.

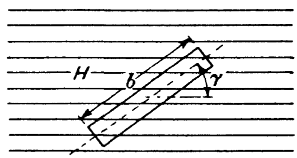


FIG. 4.20.—EMF induced in rotating coil.

circuit increases by lu per sec., and the flux Φ therefore increases by Hlu lines per sec. Hence the induced E.M.F. in volts

$$E = \frac{Hlu}{10^8} \quad . \quad . \quad . \quad . \quad (64)$$

E.M.F. Induced in Rotating Coil

If we take a rectangular coil (Fig. 4.20) of axial length l and breadth b, revolving at a speed of n revolutions per sec., in a field

of strength H, the speed of movement of the vertical conductors will be $2\pi \times \frac{b}{2} \times n = \pi bn$. The component of the velocity perpendicular to the magnetic field, which is alone concerned in increasing the area of the coil open to the field, is $\pi bn \cos \gamma$, and the increase of area per sec. is therefore $2l \times \pi bn \cos \gamma$, or

$$2\pi lbn \cos \gamma \quad . \quad . \quad . \quad . \quad (65)$$

If there are N turns in the coil the same E.M.F. will be generated in each turn, and the total E.M.F.

$$E = \frac{2\pi lbHn \cos \gamma}{10^8} = \frac{2\pi n \Phi N \cos \gamma}{10^8} \quad . \quad . \quad (66)$$

where $\Phi = lbH$, the total flux through the coil.

Potential of Current-carrying Coil in Magnetic Field

We have seen that if the magnetic flux passing through the circuit varies, the induced E.M.F. is $E = \frac{-d\Phi}{dt}$. If the coil is carrying a current I, the power at any distant is the product EI, and the work done $= \int EIdt = \int \mathrm{I} \frac{d\Phi}{dt} dt = \int Id\Phi$. If I is constant the work done by it will be $\mathrm{I} \int d\Phi = I\Phi$, and this must be equal to the work done on the coil by the field or the potential V of the coil. If the coil has N turns, the E.M.F. will be N times as great, and

$$V = IN\Phi \quad . \quad . \quad . \quad . \quad (67)$$

Potential of Magnetic Pole in Neighbourhood of Current-carrying Coil

We have seen that the total flux from a pole of strength m is $4\pi m$ lines, so that the flux passing through any area subtending a solid angle Ω is $\Phi = m\Omega$ lines. Hence by what we have just proved, the potential of such a pole in the neighbourhood of a coil N turns carrying a current I is

$$V = mIN\Omega \quad . \quad . \quad . \quad . \quad (68)$$

Mutual Inductance

When two coils are near one another, the starting of a current in one coil causes a magnetic flux to be produced which passes through the second coil and induces an E.M.F. in it. The number of " link-

ages" (lines of force $\times$ turns in the coil) produced in the second coil by unit current in the first is called the Coefficient of Mutual Induction, or more briefly, the Mutual Inductance between the coils, and is denoted by M.

Hence, if a current I_1 passes through the first coil, an E.M.F. E_2 is set up in the second, such that

$$E_2 = -M\frac{dI_1}{dt} \qquad (69)$$

and similarly $E_1 = -M\frac{dI_2}{dt}$, if, as will be seen in a moment, M is the same in both cases.

Potential of Two Current-carrying Coils

If the second coil is carrying a current, the starting of a current in the first produces the E.M.F., $E_2 = -M\frac{dI_1}{dt}$ and the work done is $\int E_2 I_2 dt = MI_2 \int \frac{dI_1}{dt}\,dt = MI_1I_2$.

Hence the potential

$$V = MI_1I_2 \qquad (70)$$

If we had considered the effect on coil 1 of starting the current in coil 2 we should have found $V = E_1I_1dt = MI_1\frac{dI_2}{dt}\,dt = MI_1I_2$, the same result as above, which shows that M must be the same in both cases.

Self-induction or Inductance

When a coil of wire carries a current the flux produced not only passes through the circuits in its vicinity, but also through itself, so that if the current through it is varied, an E.M.F. is induced in its own windings.

The Coefficient of Self-induction

The Coefficient of Self-Induction, or, more briefly, the Inductance of the coil is the number of linkages of flux with itself due to unit current in it, so that we have the linkages N in it equal to LI. Consequently, if I or L vary

$$E = -\frac{d\Phi N}{dt} = -\frac{d(LI)}{dt} = -L\frac{dI}{dt} \qquad (71)$$

if L is constant.

It will be seen below that if the permeability of the medium is taken as unity, either self or mutual inductance is represented by a length, and the C.G.S. unit of inductance is therefore 1 cm., as the C.G.S. unit of capacity was in the electrostatic system in which the dielectric constant was taken as unity (see page 157).

Practical Units of Inductance

The C.G.S. units of self or mutual inductance are defined as above, but for practical purposes a much larger unit is employed. If we take the relation $E = L\dfrac{dI}{dt}$ or $M = \dfrac{dI_2}{dt}$, L or M are such that if I is in C.G.S. units, E is given in C.G.S. units. Since the volt is 10^8 and the ampere 10^{-1} C.G.S. units, the inductance must be 10^9 times as great for these equations to hold when E is in volts and I in amperes.

The practical unit of self or mutual inductance is therefore 10^9 C.G.S. units, or 10^9 cm., which was called by Profs. Ayrton and Perry a quadrant (a quadrant of the earth being 10^9 cm.), or Secohm, but which is now called a Henry.

An inductance of 1 henry is therefore such that when the current is increasing or diminishing at the rate of 1 ampere per second 1 volt of E.M.F. is induced.

Forces in Electro-magnetic Instruments

We have seen that when a current-carrying coil is in a field produced either by a magnet or a second coil, its potential is $V = NI\Phi$, where N is the number of turns in the coil, I the current, and Φ the flux passing through it. If the coil moves so as to alter the flux passing through it we have $dV = Fds$ or $F = \dfrac{dV}{ds}$, where F is the mechanical force and ds is the element of distance moved.

If I is constant therefore

$$F = \frac{dV}{ds} = NI\frac{d\Phi}{ds} \quad . \quad . \quad . \quad . \qquad (72)$$

and by similar reasoning the torque $T = \dfrac{dV}{d\gamma}$, or

$$NI\frac{d\Phi}{d\gamma} \quad . \quad . \quad . \quad . \qquad (73)$$

These relations and the method of obtaining them should be compared with (13) and (14) in the case of electrostatic instruments, and they are of fundamental importance. As action and reaction are equal and opposite, they give equally the force or torque in moving magnet instruments.

In the case of dynamometer instruments we have seen that the number of linkages $N\Phi$ in the first circuit are equal to MI_2, so that

$$\frac{d\Phi}{ds} = I_2 \frac{dM}{ds} \text{ and}$$

$$F = I_1 I_2 \frac{dM}{ds} \text{ or } T = I_1 I_2 \frac{dM}{d\gamma} \quad . \quad . \quad (74)$$

In the case of a single circuit we have by (71) $E = L\frac{dI}{dt}$, so that the potential $V = \int EIdt = L\int I\frac{dI}{dt}dt = L\int IdI = \frac{1}{2}LI^2$. This is only half the value which would have been expected from the formulae $V = N\Phi I$ and $N\Phi = LI$, which would give $V = LI^2$. The discrepancy is accounted for by the fact that in the original case constant current was assumed in the coil while the field was growing, while in the case of the inductive coil the field and current grow together.

Applying the relation $F = \frac{dV}{ds}$ and $T = \frac{dV}{d\gamma}$, we have

$$F = \frac{1}{2}I^2\frac{dL}{ds}, \text{ and } T = \frac{1}{2}I^2\frac{dL}{d\gamma} \quad . \quad . \quad (75)$$

giving the force or torque in electro-magnetic instruments.

Another illustration of this latter case is of importance, as it brings out the meaning of the self and mutual induction and their relationship. If we suppose two coils in series near one another having self-inductances L_1 and L_2 and mutual inductance M, we may either consider them as two separate circuits carrying equal currents or as a single circuit. In the latter case it is evident that the total inductance, i.e. the number of linkages per unit current,

$$L = L_1 + L_2 + 2M \quad . \quad . \quad . \quad (76)$$

since the first coil has the number of linkages L_1 due to itself and of M due to the second, and reciprocally. If the coils are moved relatively to one another, L_1 and L_2 remain constant and M varies,

so that the change of the whole inductance L is twice the change in M. This is the basis of the variable inductances or "variometers" so much used as standards and for radio telegraphy. Hence

$$\frac{dL}{ds} = 2\frac{dM}{ds} \text{ and } \frac{dL}{d\gamma} = 2\frac{dM}{d\gamma}.$$

Now when considering the force or torque in a system we had $F = I_1 I_2 \frac{dM}{ds}$ for two circuits and $F = \frac{1}{2} I^2 \frac{dL}{ds}$ for a single circuit. Taking the latter, putting $\frac{dL}{ds} = 2\frac{dM}{ds}$ as above, remembering that $I_1 I_2 = I^2$, since the currents are equal when in series, we get $F = \frac{1}{2} I^2 \times 2\frac{dM}{ds} = I^2 \frac{dM}{ds}$, which agrees with the expression for the two separate circuits with equal currents.

All these relations harmonize with the statement made on page 6 of the introduction that a current-carrying circuit always attempts to increase the magnetic flux linked with it, either by moving itself or by causing magnets or masses of magnetic material in its vicinity to move.

For example, if we take the case of the permanent magnet moving-coil instrument having a coil of length l and breadth b and number of turns N in a field of strength H, the area of the coil is lb and the projected area perpendicular to the field is $lb \sin y$, from which the flux $\Phi = Hlb \sin y$.

By formula (73) $T = NI \frac{d\Phi}{d\gamma}$, so that the torque $T = NIHlb \frac{d \sin \gamma}{d\gamma} = NIHlb \cos \gamma$, which agrees with the formula (62) on page 174.

Relation between Self and Mutual Inductance

If we consider two coils having N_1 and N_2 turns respectively and close together, it is clear that if a current flows in one producing a flux Φ through it, the bulk of that flux will pass through the other coil also. Let us suppose for the moment that the whole of it passes through the second coil. Then if I_1 is the current in the first coil its inductance L_1 is $\frac{N_1 \Phi}{I_1}$, while the mutual inductance M is $\frac{N_2 \Phi}{I_1} = \frac{N_2}{N_1} L_1$. By similar reasoning it is evident that $M = \frac{N_1}{N_2} L_2$,

so that by multiplying we have

$$M^2 = \frac{N_2}{N_1} \times \frac{N_1}{N_2} L_1 L_2 \text{ and } M = \sqrt{(L_1 L_2)} \quad . \quad . \quad (77)$$

This is very nearly true where the coils are close together on an iron core, but in other cases there is always considerable " Magnetic dispersion " between the coils, so that M is less than $\sqrt{L_1 L_2}$. The ratio of $M/\sqrt{(L_1 L_2)}$ is called the " coupling " coefficient, and when it is greater than 0·5 the circuits are said to be " close coupled," or when less than 0·5 they are said to be " loose coupled."

Calculation of Inductance

The calculation of the inductance of circuits is from the fundamental point of view the most important of electrical calculations, as it is from known calculated inductances that the units of current, P.D. and resistance are derived. Unfortunately such calculations are, as a rule, very complicated, but there are a few cases in which inductances can be worked out from elementary principles with moderate accuracy.

Long Helix

We have seen that in the long helix the magnetic force in its interior is uniform and equal to $4\pi I N_o$, where N_o is the number of turns per cm. length (see (50), page 170). If we have a helix l cm. long of N turns and radius a cm., $H = \frac{4\pi I N}{l}$ and the area $A = \pi a^2$.

Hence

$$\Phi = \mu H A = \frac{4\pi^2 \mu a^2 I N}{l} \text{ and the inductance}$$

$$L = \frac{N\Phi}{I} = \frac{4\pi^2 \mu a^2 N^2}{l} \text{ approximately} \quad . \quad . \quad . \quad (78)$$

It will be seen that $4\pi^2 a^2 N^2$ is the square of $2\pi a c$, the total length of wire in the helix which we may denote by L_1, and hence

$$L = \mu \frac{L_1^2}{l} \quad . \quad . \quad . \quad . \quad (79)$$

In air where μ is unity $L = \frac{L_1^2}{l}$, which is dimensionally a length, just as the capacity C was a length in the electrostatic system when ε was unity (see page 151).

Hence the C.G.S. unit of inductance is 1 cm. as mentioned above; and since 1 henry is 10^9 cm.,

$$L_{\text{henries}} = \frac{4\pi^2\mu a^2 N^2}{10^9 l} = \frac{\mu}{10^9}\frac{L_1^2}{l} \quad . \quad . \quad (80)$$

If instead of considering the inductance of the helix itself we take the mutual inductance between it and a coil in its interior of radius a_2 and number of turns N_2,

$$\Phi = \mu H A_1 = \mu \frac{4\pi I N}{l}\pi a_2^2$$

and

$$M = \frac{N_1 \Phi}{I} = \frac{4\pi^2 \mu a_2^2 N_1 N_2}{l} \quad . \quad . \quad . \quad (81)$$

Two Parallel Wires

On page 171 it was shown that the magnetic stress H at a distance D from a long straight conductor was $2I/D$ or $B = \frac{2\mu I}{D}$.

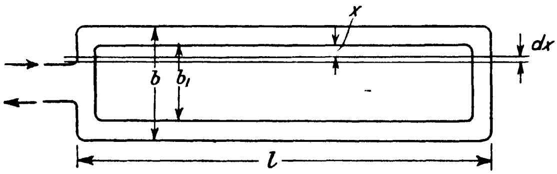


FIG. 4.21.—The inductance of a single loop.

If in Fig. 4.21 we take the element of area dA between two lines of length l and width dx, $d\Phi = Bldx = 2\mu Il\frac{dx}{x}$, and the total flux between any two values of x is

$$\Phi = 2\mu Il\int_{x_1}^{x_2}\frac{dx}{x} = 2\mu Il\log_\varepsilon\frac{x_2}{x_1} \quad . \quad . \quad (82)$$

If we have two circuits of the same length l and of widths b and b_1 respectively, $x_1 = \frac{b - b_1}{2}$ and $x_2 = \frac{b + b_1}{2}$, from which

$$M = \Phi/I = 2\mu l\log_\varepsilon\frac{b + b_1}{b - b_1},$$

or if there are N_1 turns on one loop and N_2 turns on the other,

$$M = 2\mu l N_1 N_2\log_\varepsilon\frac{b + b_1}{b - b_1} \quad . \quad . \quad . \quad (83)$$

In order to get the inductance of the single loop we have $x_2 = b$, but for x_1 there is some doubt. If we take the current as being distributed only on the surface of the wire, as is the case for alternating currents of very high frequency, there will be no magnetic field inside the wire, and x_1 is the radius of the wire $\frac{d}{2}$, from which

$$L = 2\mu l \log_\epsilon \frac{2b}{d} \quad . \quad . \quad . \quad . \quad (84)$$

This is called the high-frequency inductance of the loop, but at lower frequencies the inductance is somewhat greater as there is an internal field, though of rapidly decreasing intensity as the centre of the conductor is approached. If we assume the current to be uniformly distributed throughout the section of the conductor, as is the case at very low frequencies, the current density will be $\frac{I}{\pi a^2}$, and the current I_o within a circle of radius a_o is $\frac{\pi a_o^2 I}{\pi a_2} = \frac{a_o^2}{a^2} I$. Hence the field B at any radius a_o is

$$\frac{2\mu I_1}{a_o} = \frac{2\mu I}{a^2} a_o \text{ and } d\Phi = \frac{2\mu I}{a^2} a_o \times l da_o = \frac{2\mu l I}{a^2} a_o da_o,$$

from which

$$\Phi = \frac{2\pi l I}{a^2} \int_0^a a_o da_o = \frac{2\mu l I}{a^2} \frac{a^2}{2} = \mu l I \quad . \quad . \quad (85)$$

The inductance due to this field is $\frac{\Phi}{I} = \mu l$, or $2\mu l$ for the two conductors.

Hence the total low-frequency inductance

$$L = 2\mu l \log \frac{2b}{b} + 2\mu l = 2\mu l \left\{ \log_\epsilon \frac{2b}{d} + 1 \right\} \quad . \quad (86)$$

This formula is of value in giving the residual inductance in the case of the ordinary double wound "non-inductive" resistances. In this case $b = d_c$, the diameter of the covered wire, and

$$L = 2l \left\{ \log_\epsilon \frac{2d_c}{d} + 1 \right\} \quad . \quad . \quad . \quad (87)$$

Concentric Circular Coils

Suppose we have two very nearly concentric circular coils of radii a_1 and a_2, as in Fig. 4.22. Then Maxwell has shown that the mutual inductance between them can be calculated by direct integration.

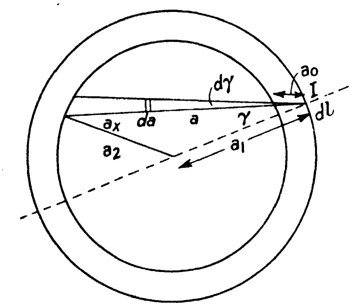


FIG. 4.22.—The mutual inductance between concentric coils.

If we consider an element dl of the outer circle carrying a current I, then the magnetic field at any point distant a from this element in a direction making an angle γ with the diameter will be by Ampère's formula (see page 168).

$$dH = \frac{Idl}{a^2}\cos\gamma \text{ and } dB = \frac{\mu Idl}{a^2}\cos\gamma.$$

If we consider a small element of area bounded between the two radii and two concentric circles of radii a and $a + da$, the element of area $dA = ad\gamma \times da$ and the magnetic flux passing through it

$$d\Phi = dBda = \frac{\mu Idl}{a^2}\cos\gamma adad\gamma$$

$$= \mu Idl\frac{da}{a}\cos\gamma d\gamma.$$

The total flux passing through the portion of the inner circle bounded by the two radii will therefore be

$$\mu Idl\cos\gamma d\gamma\int_{a_o}^{a_x}\frac{da}{a} = Idl\cos\gamma d\gamma\log\frac{a_x}{a_o},$$

where a_o and a_x are the two distances from dl to the first and second intersections with the smaller circle respectively. By trigonometry it is easily seen that

$$a_o = a_1\cos\gamma - \sqrt{a_1^2\cos^2\gamma - (a_1^2 - a_2^2)} = \frac{a_1 - a_2}{\cos\gamma}$$

when $a_1 - a_2$ is small and

$$a_x = a_1\cos\gamma + \sqrt{a_1^2\cos^2\gamma - (a_1^2 - a_2^2)} = 2a_1\cos\gamma,$$

when $a_1 - a_2$ is small.

Hence the flux passing through this area is

$$\mu I dl \cos \gamma \,.\, d\gamma \,.\, \log \frac{a_x}{a} = \mu I dl \cos \gamma \,.\, d\gamma \log \frac{2a_1 \cos^2 \gamma}{a_1 - a_2}$$

and the total flux due to the element dl passing through the inner circle

$$= 2\pi I dl \int_o^\beta \log \frac{2a_1 \cos^2 \gamma}{a_1 - a_2} \cos \gamma d\gamma,$$

where β is the extreme angle which the tangent to a_2 makes with the diameter, and is such that $\sin \beta = \frac{a_2}{a_1}$.

If we write q for $\sin \gamma$ the integration becomes

$$\text{Flux} = 2\mu I dl \int_o^{\frac{a_2}{a_1}} \log \frac{2a_1}{a_1 - a_2} (1 - q^2) dq,$$

which works out to

$$2\mu I dl \left[q \log \frac{2a_1(1 - q^2)}{a_1 - a_2} + \log \frac{1+q}{1-q} - 2q \right]_o^{\frac{a_2}{a_1}}$$

$$= 2\mu I dl \left\{ \log \frac{8a_1}{a_1 - a_2} - 2 \right\}$$

It is clear from symmetry that every element d on the outer circle has a similar effect, and if the outer circle has N_1 turns, $l = 2\pi N_1 a_1$, and the flux through the small coil

$$\Phi = 4\pi a_1 I N_1 \left\{ \log \frac{8a_1}{a_1 - a_2} - 2 \right\}.$$

If the inner coil has N_2 turns, then we have the coefficient of mutual inductance between the coils,

$$M = \frac{\Phi N_2}{I} = 4\pi \mu a_1 N_1 N_2 \left\{ \log \frac{8a_1}{a_1 - a_2} - 2 \right\} \text{cm.} \quad . \quad (88)$$

Since the lines of force close to a conductor are in the form of concentric circles round it, it is evident that the flux passing through the second coil will be sensibly the same whether it be inside the first coil, assumed in the calculation, or in any other position in which its circumference is at the same distance from the first (Fig.

4.23). Hence for $a_1 - a_2$ we may simply write a_m, the distance between the mean circumference of the coils, and

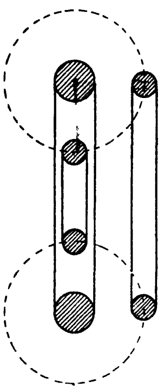


FIG. 4.23.—Mutual inductance between parallel coils.

$$M = 4\pi\mu a_1 N_1 N_2 \left\{ \log \frac{8a_1}{a_m} - 2 \right\} \text{cm.} \qquad (89)$$

When the values of a_m are considerably different for the various turns, it is evident that we must use a mean value, and that this value R should be such that $\log R = \text{mean} \log a_m$.

R was then called by Maxwell the "geometric mean distance" (G.M.D.) of the turns of the coils. The values of R for various cases have been calculated, and these are given in Table X (page 187).

Hence for any pair of coaxial coils whose diameters are large in comparison with the distance between their turns

$$M = 4\pi\mu a_1 N_1 N_2 \left\{ \log_\varepsilon \frac{8a_1}{R} - 2 \right\} \text{cm.} \qquad (90)$$

If instead of the mutual inductance between two coils we require the self-inductance of one, it will be seen by a little consideration that it will be the same as the above, except that we write N^2 for N_1N_2 and R is the G.M.D. of the various turns of the coil from each other.

Hence

$$L = 4\pi\mu a_1 N^2 \left\{ \log_\varepsilon \frac{8a}{R} - 2 \right\} \text{cm.} \qquad (91)$$

If the coil is wound so as to have a square section of side b, then $R = 0{\cdot}447b$, and

$$L = 4\pi\mu a_1 N^2 \left\{ 2{\cdot}3 \log_{10} \frac{a_1}{b} + 0{\cdot}885 \right\} \qquad (92)$$

and for a circular section of diameter d, $R = 0{\cdot}39d$ and

$$L = 4\pi\mu a_1 N^2 \left\{ 2{\cdot}3 \log_{10} \frac{a}{d} + 1{\cdot}02 \right\} \qquad (93)$$

This calculation, although somewhat complex, has been gone into here because it illustrates the general principle underlying all such calculations, and the nature of the result.

TABLE X.—*Geometrical Mean Distance.*

A line from itself .	length a .	$\text{Log } R = \log a \frac{3}{2}$ or $R = 0{\cdot}22313\, a$.
Square from itself .	side $= a$.	$\text{Log } R = \log a + \frac{1}{3} \log 2 + \frac{\pi}{3} - \frac{25}{12}$ or $R =$ $044705\, a$.
Circular area from itself	radius a .	$\text{Log } R = \log a - \frac{1}{4}$ or $R = 0{\cdot}7788\, a$.
Ellipse from itself .	semi-axes . a and b	$\text{Log } R = \log \frac{a+b}{2} - \frac{1}{4}$.
Rectangular area from itself	length a, . breadth b	$R = 0{\cdot}2235\ (a + b)$ very approximately for all values of a and b.
Annular ring from itself	radii a_1 . and a_2	$\text{Log } R = \log a_1 - \frac{a_2}{(a_1^2 - a_2^2)^2} \log \frac{a_1}{a_2} + \frac{1{\cdot}3}{4} \frac{a_2^2 - a_1^2}{a_1^2 - a_2^2}$.

We have also $2\pi a_1 N =$ the length of wire L_1, and

$$L = 2\mu L_1 c \left\{ \log \frac{8a_1}{R} - 2 \right\},$$

which is similar in form to that obtained for the parallel wires.

A very simple method of approximately estimating the inductance of a coil was given by P. R. Coursey,* following on Nagaoka.

In this case the inductance is put in the form

$$L = k\pi^2 d^2 N^2 / l = k \frac{L_1}{l} \quad . \quad . \quad . \quad (94)$$

where d is the mean diameter of the coil (Fig. 4.24) and l its axial length, or its radial depth in the case of a flat spiral. The value of the factor k is plotted against the values of $\frac{l}{d}$, and the curves are given in Fig. 4.25. If the depth of the winding is appreciable a further correction is necessary. The ratio of length of the coil to the depth of winding and also the ratio of the depth of winding to the mean diameter are found and the value of k_1 obtained from the curves in Fig. 4.26. The value so obtained must be subtracted from the original value of k, so that for deep coils formula (94) must be written

$$L = (k - k_1)\ \pi^2 d^2 N^2 / l = (k - k_1) \frac{L_1}{l}$$

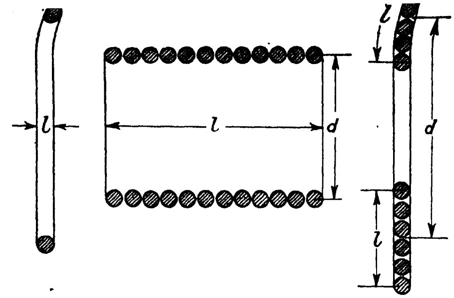


FIG. 4.24.—Figure used in Coursey's inductance formula.

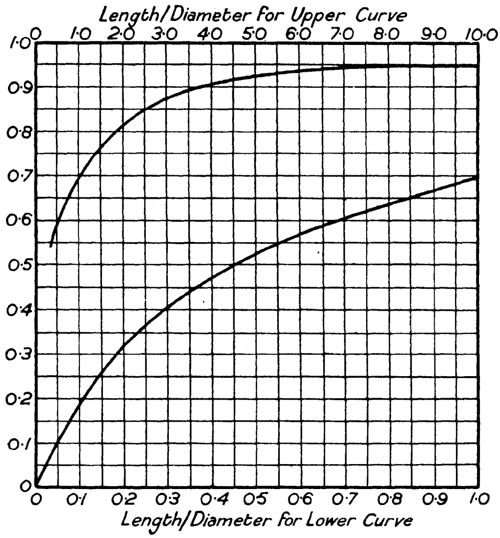


FIG. 4.25.—Coursey's curves for calculating the self inductance of coils.

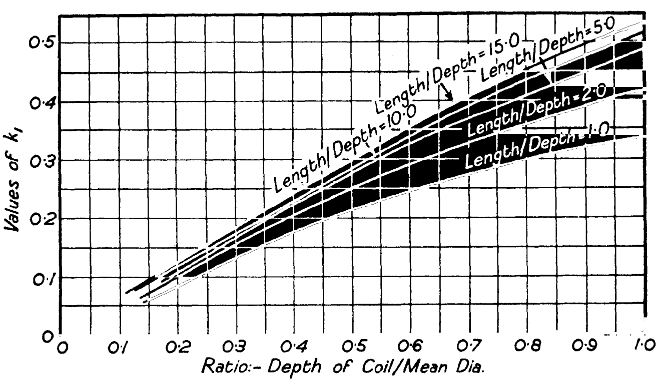


FIG. 4.26.—Coursey's curves for calculating the self-induction of coils.

Inductance of Circuits with Iron Cores

When coils of wire are wound on iron cores, with closed or nearly closed magnetic circuits, the approximate calculation of their inductances is easy. By Hopkinson's formula (59) we have

$$\Phi = \frac{4\pi IN}{\sum \frac{l}{A\mu}}$$

and consequently the self-induction of a coil of N turns,

$$L = \frac{\Phi N}{I} = \frac{4\pi N^2}{\sum \frac{l}{A\mu}} \text{ or } \frac{4\pi}{10^9} \frac{N^2}{\sum \frac{l}{A\mu}} \text{ henries} \quad . \quad (95)$$

and

$$M = \frac{\Phi N_2}{I} = \frac{4\pi}{10^9} \frac{N_1 N_2}{\sum \frac{l}{A\mu}} \text{ henries} \quad . \quad . \quad (96)$$

These formulae are of considerable importance in the design of alternating current electromagnets and choking coils.

ELECTRIC CURRENT OR ELECTRO-KINETIC FORMULAE

The treatment of electric current phenomena should logically have followed that of electrostatics, as an electric current is simply a stream of moving charges; but as the unit of current has been defined in terms of its magnetic effects, these effects required to be dealt with first. A few of the fundamental laws of the current follow, however, at once from electrostatic principles.

The Electric Current

When a charged body is brought into contact with a metallic wire or other conductor, the whole or part of the charge passes along the conductor to the earth or some other body. The conducting wire acts in fact like a rift in the jelly of Fig. 4.1, through which the pressure due to the distension of the charge forces the water to flow away. The rate at which this flow takes place or the amount of charge which passes away per second is the electric current I, so that

$$I = \frac{de}{dt} \quad . \quad . \quad . \quad . \quad (97)$$

Conversely, $de = Idt$ or $= e = \int Idt$.

When I is given in amperes e is the quantity in ampere-seconds or coulombs, and is generally denoted by the letter Q, so that

$$Q_{\text{coulombs}} = \int I_{\text{amperes}}\, dt \qquad . \quad . \quad . \quad (98)$$

Since one ampere is 10^{-1} C.G.S. unit of current, one coulomb $= 10^{-1}$ C.G.S. unit of quantity.

Faraday's Laws of Electrolysis

When the current passes through a compound conducting liquid or electrolyte chemical action is produced, resulting in the deposit of metal or the liberation of gas at the electrodes. Faraday showed that the amount of an " ion " (product of electric decomposition) was proportional to the current, to the time, and to the chemical equivalent of the ion, i.e. its atomic or molecular weight divided by its valency, or

$$w = Izt \qquad . \quad . \quad . \quad . \quad (99)$$

where w is the weight of the ion liberated and z the " electrochemical equivalent " (grammes liberated per coulomb) of the ion.

International Unit of Current

The C.G.S. unit of current defined by its magnetic effect, as on page 168, cannot be accurately determined without elaborate apparatus and calculations. Hence it has been decided to give its equivalent in terms of the deposition of some metal in a voltameter. The International Conference on Electrical Units and Standards in 1908 gave the following definition :

The International Ampere is the unvarying electric current, which, when passed through a solution of silver nitrate in water, in accordance with Specification II attached to these Resolutions, deposits silver at the rate of 0·001118800 of a gramme per second.

The specification referred to stated that the solution shall be of 15 to 20 parts by weight of silver nitrate in 100 parts of distilled water, with a silver anode and platinum cathode.

The value of the ampere given by the above is probably between 2 and 3 parts in 10,000 lower than the true C.G.S. ampere, which appears to deposit 1·11828 milligrams of silver per coulomb.

Energy and Power in the Electric Current

If a body is at a potential V and a small charge de is brought up to it, an amount of work is done $dW = Vde$ by formula (11).

Hence $\frac{dW}{dt} = V\frac{de}{dt}$ if V is kept constant.

But $\frac{dW}{dt}$ is the power P in ergs per sec. and $\frac{de}{dt}$ is the current I, so that we have

$$P = VI \qquad . \quad . \quad . \quad . \quad (100)$$

In electro-magnetic measure one volt $= 10^8$ C.G.S. units, and $I = 10^{-1}$ C.G.S. units, so that $P = 10^7\, V_{\text{volts}}\, I_{\text{amperes}}$ ergs per sec.

But 10^7 ergs per sec. $= 1$ watt.

Also
$$P_{\text{watts}} = V_{\text{volts}}\, I_{\text{amperes}} \qquad . \quad . \quad . \quad . \quad (101)$$

Hence
$$W_{\text{joules}} = \int P_{\text{watts}}\, dt = \int VIdt \qquad . \quad . \quad (102)$$

These relations were determined by Joule in 1840 by calorimeter experiments, but they follow logically from electrostatic principles and the definition of the electro-magnetic units once the identity of the "Voltaic current" with a stream of moving charges is established.

Ohm's Law

In 1827 Dr. G. S. Ohm put forward his well-known law, which had been established by Cavendish in 1781, though not published. This law may best be expressed by stating that in any metallic conductor, at a constant temperature, the difference of potential between two fixed points on it is rigidly proportional to the steady current flowing through it. The ratio of this P.D. in volts to the current in amperes is called the resistance of the conductor between the two points in ohms.

Hence
$$V = RI \text{ and } R = \frac{V}{I} \text{ ohms} \qquad . \quad . \quad (103)$$

International Unit of Resistance

The value of the ohm has been determined in electro-magnetic measure by the Lorenz apparatus and in other ways, but, as with the ampere, a more easily realizable unit has been defined by the International Conference in terms of a column of mercury.

The International Ohm is the resistance offered to an electric current by a column of mercury at the temperature of melting ice,

14·4521 gm. mass of a constant cross-sectional area and of a length of 106·300 cm.

The particular mass is intended to represent a cross-section of the bore of the tube of one square millimetre exactly.

The International Ohm probably agrees with the true C.G.S. to within a few parts in 100,000.

Heating Effect of Current

The power developed in any circuit is $P = VI$ watts or joules per sec. (101). In the case of a resistance we have seen that $V = RI$, and as the power does not appear either mechanically or in the form of chemical action, it is converted into heat. By Joule's experiments on the mechanical equivalent of heat we have one calorie (gm. °C.) equals 4·2 joules, or one joule = 0·24 calorie.

Hence $$P = VI = I^2R \text{ watts} \qquad (104)$$

and $$\text{Heat} = 0{\cdot}24\, I^2R \text{ calories per second} \qquad (105)$$

International Unit of P.D. or E.M.F.

From the above definitions of the International ampere and ohm combined with Ohm's law, $V = RI$, we have the International volt: " The International Volt is the electrical pressure which, when steadily applied to a conductor whose resistance is one International ohm, will produce a current of one International ampere."

Electromotive Force

Whenever a current flows through a circuit a certain total driving force or electromotive force is required. This E.M.F. may be derived from the cutting of lines of force (see page 175) or from chemical action in a voltaic cell or battery, or from heat in the case of a thermopile, and, like P.D., it is expressed in volts. The E.M.F. represents, in fact, the total force or head in the case of a stream of water, and the P.D. between any two points of the stream is that part of the force which is employed in urging the current past the obstacles (resistance or back E.M.F.) in that portion of the stream.

We have seen that in the case of magnetic induction

$$E = \frac{N}{10^8} \cdot \frac{d\Phi}{dt} \text{ volts (see (63))}$$

For chemical action we have seen (98)

$$W = Qz = z \int Idt$$

but whenever chemical action takes place heat is either absorbed or generated, and if h is the heat of formation of the action in calories per gramme,

$$\text{Total heat} = Wh = h_z \int Idt$$

which corresponds to an amount of energy $\frac{1}{4 \cdot 2} hz \int Idt$ joules. By the principle of the conservation of energy

$$E \int Idt = \frac{1}{4 \cdot 2} hz \int Idt$$

where E is the back E.M.F. produced by the chemical action.

Hence $$E = \frac{1}{4 \cdot 2} hz, \text{ which is constant} \quad . \quad . \quad (106)$$

The most convenient practical unit of E.M.F. is therefore a standard cell, and of these the Clark and Weston or cadmium cells are the most constant. The latter, having electrodes of cadmium, amalgam and mercury, with a paste of mercurous sulphate and solution of cadmium sulphate, is now taken as having an E.M.F. of 1·0183 volts at 20° C.

Kirchhoff's Laws

We have already seen by Faraday's " ice-pail " experiment that electricity behaves like an incomparable fluid, and that if any additional charge is brought into an enclosure an equal quantity must escape. In 1847 Kirchhoff put forward two laws relating to networks of conductors, the first of which is equivalent to a reassertion of the incompressibility of electricity, and the second is a derivation of Ohm's Law. These two laws can be stated as follows :

1. In any network of conductors the algebraic sum of the currents at any point is zero, i.e. the sum of the currents flowing away from the point equals the sum of those flowing towards it.

2. In any mesh of a network the algebraic sum of the P.D.'s (current $\times$ resistance) in each side of the mesh equals the algebraic sum of all the E.M.F.'s in the mesh. Or in symbols

$$\Sigma I = O \quad . \quad . \quad . \quad . \quad (107)$$

and

$$\Sigma V = \Sigma RI = \Sigma E \quad . \quad . \quad . \quad (108)$$

These formulae are of considerable value in connection with bridge measurements, polyphase circuits, etc.

Varying Currents

The laws of Ohm and Kirchhoff above described apply in their simple form to steady currents only, if by the E.M.F. is meant merely the electromotive force applied from a battery or steady current generator. But we have seen that when a current passing through a coil of wire varies in any way an E.M.F. is set up in the coil equal to $-L\dfrac{dI}{dt}$ volts, if L is the inductance in henrys and dI/dt is the rate of change of current or electrical acceleration in amperes per second. Similarly if the coil is in the neighbourhood of a second circuit in which a current I_2 is flowing, an E.M.F. equal to $-MdI_2/dt$ is induced. Again, if the circuit contains a condenser of capacity C farads, the P.D. required to charge it is $\dfrac{1}{C} \cdot Q = \dfrac{1}{C}\int Idt$. So that if we have a circuit containing resistance, self and mutual inductance, and capacity

$$RI = E - L\frac{dI}{dt} - M\frac{dI_2}{dt} - \frac{1}{C}\int Idt$$

or

$$RI + L\frac{dI}{dt} + MdI_2 + \frac{1}{C}\int Idt = E \quad . \quad . \quad (109)$$

which is the general equation to the E.M.F. in the circuit.

Alternating Currents

The subject of alternating current phenomena is too complex to be adequately dealt with here, but the following brief sketch of the theory may be useful. The importance of a thorough understanding of the theory of instruments for alternating current, P.D., and power measurement is very great, and those unfamiliar with the subject may be referred to the standard works on alternating current theory.

When a coil rotates in a uniform magnetic field, an E.M.F. is induced in it which reverses in direction twice in every revolution. In equation (66) page 176 we saw that this E.M.F. was given by the formula $E = \frac{2\pi n \Phi N \cos\gamma}{10^8} = \frac{\Phi N \omega \cos\gamma}{10^8}$, where $\omega = 2\pi n$ the angular velocity of the coil, Φ is the total flux through it and N the number of turns. If the coil is rotating uniformly in a constant field $\Phi N\omega/10^8$ remains constant and is the maximum value of the E.M.F., so that

$$E = E_{\max} \cos \omega t$$

In this case $t = O$, $E = E_{\max}$ or the time is reckoned from the constant of maximum E.M.F., but if we reckon it from the moment of zero E.M.F.

$$E = E_{\max} \sin \omega t \quad . \quad . \quad . \quad (110)$$

The E.M.F. is thus said to vary according to a sine law or in a simple harmonic manner (Fig. 4.27).

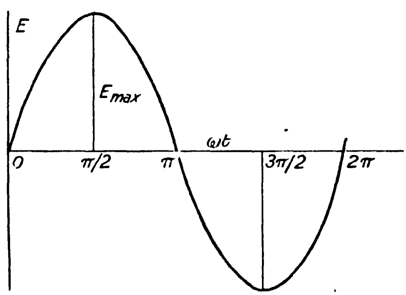


FIG. 4.27.—Sine wave of EMF.

When such an E.M.F. operates on a circuit the result is to produce a current alternating in a similar manner. Let us suppose that the current varies according to a simple harmonic law and that

$$I = I_{\max} \sin \omega t \quad . \quad . \quad . \quad (111)$$

If this current passes through an ordinary non-inductive resistance R, we have by Ohm's law

$$V = RI = RI_{\max} \sin \omega t \quad . \quad . \quad . \quad (112)$$

So that the P.D. across the coil required to produce the above current rises and falls " in phase " with the current (Fig. 4.28).

Next, suppose that the same current passes through a coil having an inductance L henrys and negligible resistance. Then we have seen on page 177 that a P.D. $= LdI/dt$ is needed to overcome the back E.M.F. of inductance.

Hence

$$V = L\frac{dI}{dt} = \omega L I_{\text{max}} \cos \omega t \quad . \quad . \quad . \quad (113)$$

This means that the P.D. varies in a simple harmonic manner, having a maximum value $V_{\text{max}} = \omega L I_{\text{max}}$ but "leading 90°" in front of the current, *i.e.* it is at a maxmium value a quarter of a period or revolution earlier (Fig. 4.29).

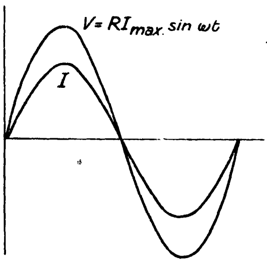


FIG. 4.28.—Current and voltage waves in phase.

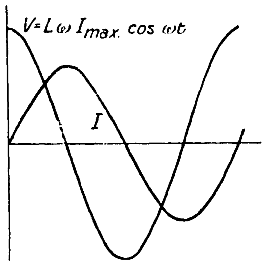


FIG. 4.29.—Current maximum lagging on the voltage.

Again, if the current passes through a condenser of C farads, then we have

$$V = \frac{1}{C} Q = \frac{1}{C}\int Idt = -\frac{1}{\omega C} I_{\text{max}} \cos \omega t \quad . \quad (114)$$

but it occurs a quarter of a period later or " lags 90° " behind the current (Fig. 4.30).

Reactance

It will be noted that in both of the two latter cases the P.D. is proportional to the current at a fixed given frequency, so that Ohm's law holds for alternating currents, except that ωL or $1/\omega C$ takes the place of R and the phase is altered to quadrature. The

quantity ωL is called the reactance X, and $1/\omega C$ the "capacity reactance" which is negative.

Resistance, Inductance and Capacitance in Series

Hence if the current $I = I_{max} \sin \omega t$ passes through a circuit having resistance R and inductance L in series with a capacitance C, the total P.D. will be the sum of the potential differences required to force it through each of them separately, and

$$V = RI_{max} \sin \omega t + \omega L I_{max} \cos \omega t - \frac{1}{\omega C} \cdot I_{max} \cos \omega t$$

$$= \left\{ R \sin \omega t + \left(\omega L - \frac{1}{\omega C} \right) \cos \omega t \right\} I_{max} \quad . \quad . \quad (115)$$

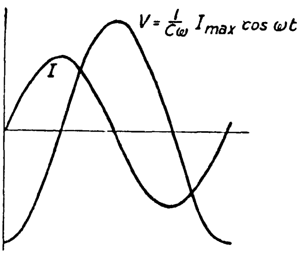


FIG. 4.30.—Current maximum leading on the voltage.

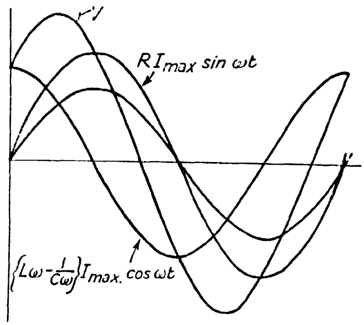


FIG. 4.31.—Summation of voltage waves.

Hence the total P.D. required is got by adding the two curves $RI_{max} \sin \omega t$ and $\left(\omega L - \frac{1}{\omega C} \right) I_{max} \cos \omega t$ as in Fig. 4.31. The result is obviously a rising and falling P.D., and it is easily seen by trigonometry that

$$V = \left\{ R \sin \omega t + \left(\omega L - \frac{1}{\omega C} \right) \cos \omega t \right\} I_{max} = Z I_{max} \sin (\omega t + \beta),$$

where

$$Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C} \right)^2} \text{ and } \tan \beta = \frac{\omega L - \frac{1}{\omega C}}{R} \quad . \quad . \quad (116)$$

The P.D., therefore, has a maximum value $V_{max} = ZI_{max}$ and varies according to a sine law, leading in front of the current by the angle β such that

$$\tan \beta = \frac{\omega L - \frac{1}{\omega C}}{R} = \frac{X}{R} = \frac{\text{Reactance}}{\text{Resistance}}$$

Consequently, if we wish to find the current produced in such a combination of resistance, inductance, and capacitance by a P.D., $V = V_{max} \sin \omega t$, we have

$$I = \frac{V_{max}}{Z} \sin (\omega t - \beta) = \frac{V_{max}}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}} \sin (\omega t - \beta).$$

The quantity Z is termed the "impedance" of the circuit, and takes the place for alternating current circuits which resistance occupies in direct current circuits.

Effective or R.M.S. Value

In the introductory chapter it was shown that the effective value of an alternating P.D. or current was its square root of mean square or R.M.S. value. For a sine curve this has the value $\sqrt{\text{mean} \sin^2 \omega t}$.

Now since $\sin^2 \omega t + \cos^2 \omega t = 1$ for all values of t, mean $\sin^2 \omega t$ + mean $\cos^2 \omega t = 1$.

But $\sin \omega t$ and $\cos \omega t$ vary in exactly the same manner, except for a difference in phase, so that their mean values over a whole period are equal. Hence mean $\sin^2 \omega t$ = mean $\cos^2 \omega t$ and this leads to $\sin^2 \omega t = 1/2$, from which

$$\sqrt{\text{mean} \sin^2 \omega t} = \frac{1}{\sqrt{2}} = 0{\cdot}707.$$

Since the maximum value of $\sin \omega t = 1$, it follows that the effective values of any P.D. or current varying according to a sine law is got by dividing their maximum values by $\sqrt{2}$. We have seen that for a circuit having resistance, inductance and capacity in series

$$I = \frac{V_{max}}{Z} \sin (\omega t - \beta), \text{ when } Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

$$\text{and } \tan \beta = \frac{\omega L - \frac{1}{\omega C}}{R}$$

Hence

$$I_{\max} = \frac{V_{\max}}{Z} \text{ and } \frac{1}{\sqrt{2}} I_{\max} = \frac{\frac{1}{\sqrt{2}} V_{\max}}{Z}$$

or

$$I_{\text{effective}} = \frac{V_{\text{effective}}}{Z} = \frac{V_{\text{effective}}}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$

In this brief sketch of the theory we cannot go completely into the various cases, but it should be noted that with resistance and inductance only

$$I_E = \frac{V_E}{\sqrt{R^2 + \omega^2 L^2}} \text{ and } \tan \beta = \frac{\omega L}{R} \quad . \quad . \quad (117)$$

As the frequency increases, therefore, the current gets smaller and smaller for a given P.D., and the angle of lag of the current gets larger up to nearly 90°.

With a combination of resistance, inductance and capacity it is interesting and important to note that there is a certain frequency at which the circuit behaves as if it were a non-inductive resistance.

Since for this combination

$$I_E = \frac{V_E}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}} \text{ and } \tan \beta = \frac{\omega L - \frac{1}{\omega C}}{R}$$

it is evident that if $\omega L = 1/\omega C$ or $\omega = 1/\sqrt{CL}$, $I_E = V_E/R$ and $\tan \beta = O$. This is called the condition of resonance. At low frequencies ωL is small and $1/\omega C$ large, so that the current is low and $\tan \beta$ is large and negative, so that β is leading and nearly 90°. As the frequency increases the current gets larger and more nearly into phase, until it reaches its maximum at the frequency $\omega = 1/\sqrt{CL}$; at still higher frequencies the current again falls and becomes lagging. In this way the changes which occur in a case where frequency increases may be calculated.

Vector Representation

By far the most convenient method of dealing with alternating current problems is to represent the simple harmonic currents or P.D.s as vectors. If a uniformly revolving crank moves a pencil on a steadily moving sheet of paper, as in Fig. 4.32, it will evidently trace out a sine curve, since the distance of the pencil above its

mid-point is (a sin β) where β is the angle which the crank has turned from the horizontal. By altering the radius of the crank and its angle with the horizontal at the moment from which the time is reckoned, it is evident that simple harmonic curves of any amplitude and phase can be drawn.

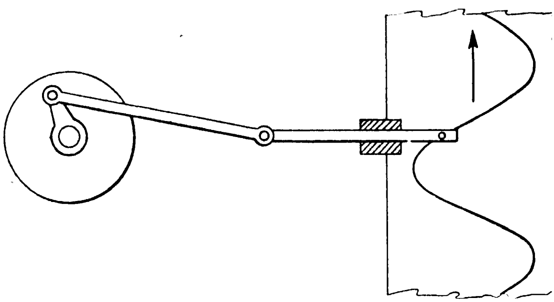


FIG. 4.32.—Sine curve traced by revolving crank.

In the case of a circuit having resistance, inductance, and capacity in series, we can set off the resistance R horizontally, and the reactance $X = \omega L - \frac{1}{\omega C}$ vertically upwards as in Fig. 4.33; the resultant is obviously $\sqrt{R^2 + X^2} = Z$; the impedance and its angle with R is such that $\tan \beta = X/R$, so that β is the angle by which the P.D. leads in front of the current. Hence if the current is represented by a horizontal crank or vector, the P.D. will be represented

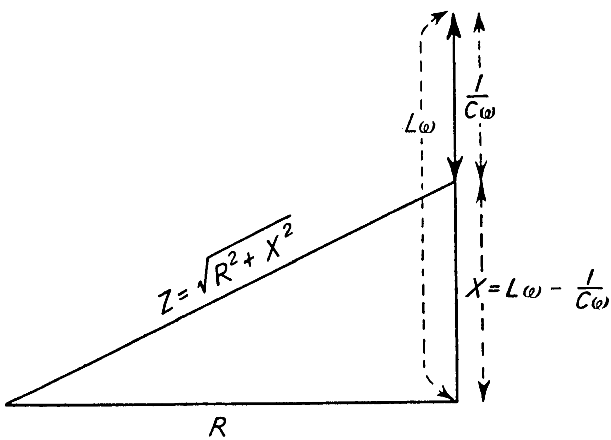


FIG. 4.33.—Vector diagram of impedance.

by a second vector in the direction of the resultant Z and of Z times the magnitude.

It is evident that instead of laying down the vectors to represent the maximum values of the currents or P.D.'s we may equally well employ them to represent their *effective* values, so long as it is done consistently, as it is simply equivalent to reducing the scale of all the diagrams in the ratio of $\sqrt{2}$ to 1.

Vector Calculation

As vector diagrams are the most convenient method of illustrating alternating current phenomena, it is also convenient to have a means of vector calculation, in order to work out the values of the currents and P.D.'s, etc., in various cases. Supposing that we have a P.D. of V effective volts leading by an angle φ in front of the current, and represent it by the expression V/φ. Now by means of what has just been said, this can be treated just like a force or other vector and be split into two components, one horizontal or in phase with the current, $V_1 = V \cos \varphi$, the other vertical or leading 90° in front of the current, $V_2 = V \sin \varphi$, and we may write V = resultant of V_1 and V_2.

What is evidently wanted is a simple means of expressing mathematically that V_2 is in quadrature, or of turning a vector through a right angle. Let us suppose that there is a factor j which, when a vector is multiplied by it, turns that vector through a right angle in the counter clockwise or leading direction. Then if in Fig. 4.34

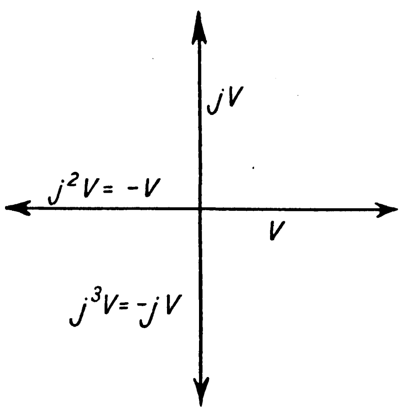


FIG. 4.34.—To illustrate the meaning of " j.

V is a horizontal vector, jV is an equal vector vertically upwards. But by repeating the process $j \times jV$ must be the same vector turned through a second right angle or to a direction diametrically opposite to the first, which gives as a vector $-V$.

Hence $j^2V = -V = -1 \times V$, from which $j^2 = -1$ and $j = \sqrt{-1}$. We therefore represent all vertical vectors or those leading by 90° by the simple device of putting j in front of them ; and those leading 270°, which is equivalent to lagging 90°, by putting $-j$ in front of them.

For example, the impedance $Z = \sqrt{R^2 + X^2}\ /\tan^{-1} X/R$ may be written as a vector $Z = R + jX$.

If we have a certain P.D. V we may proceed to find the current $I = \dfrac{V}{Z} = \dfrac{V}{R + jX} = \dfrac{R - jX}{(R + jX)(R - jX)} \cdot V = \dfrac{R - jX}{R^2 + X^2} \cdot V$ since $j^2 = -1$ or $I = \dfrac{R}{R^2 + X^2} \cdot V - j \cdot \dfrac{X}{R^2 + X^2} \cdot V$.

This shows that the current is represented as a vector having a horizontal or " power " component $\dfrac{R}{R^2 + X^2} \cdot V$, and a downwards lagging or " wattless " component $j\dfrac{X}{R^2 + X^2} \cdot V$, the tangent of the angle of lag being given by the ratio of the vertical to the horizontal component.

Hence $\tan \varphi = \left(\dfrac{X}{R^2 + X^2}\right) \Big/ \left(\dfrac{R}{R^2 + X^2}\right) = \dfrac{X}{R}$, or the same as for the angle of lead of the P.D. in front of the current, as must evidently be the case.

Admittance, Conductance and Susceptance

In dealing with combinations of resistance, etc., in series we introduced the impedance of the combination and its two components, the resistance and the reactance. When dealing with circuits in parallel we have to add the currents (by Kirchhoff's Law) instead of the P.D.'s, so that with direct currents we find it convenient to introduce the conductance G or current per volt, so that $I = GV$, and the conductance of any number of circuits in parallel is got by adding their separate conductances together.

With alternating currents in parallel circuits we add the currents together to get the total current, but to do this we must add the

components of the current in phase with the P.D. together to get the total current in phase, and similarly with the components in quadrature.

Now we have just seen that when a P.D. V is applied to a circuit of impedance $Z = R + jX$, the component of the current in phase is $\frac{R}{R^2 + X^2} \cdot V$ and the component in quadrature $\frac{X}{R^2 + X^2} \cdot V$.

Hence the current in phase per volt or " conductance " $G = \frac{R}{R^2 + X^2}$ and the current in quadrature per volt or " susceptance " $B = \frac{X}{R^2 + X^2}$.

Hence the total current per volt or " admittance "

$$Y = \frac{1}{Z} = \frac{1}{R + jX} = \frac{R - jX}{R^2 + X^2} = G - jB$$

Hence for any number of circuits in parallel :

Total conductance = total current in phase per volt $= G = G_1 + G_2 +$ etc.

Total susceptance = total current in quadrature per volt $= B = B_1 + B_2 +$ etc.

Total admittance = total current per volt $Y = G - jB$.

Total impedance

$$Z = \frac{1}{Y} = \frac{1}{G - jB} = \frac{G + jB}{G^2 + B^2} = \frac{G}{G^2 + B^2} + j \cdot \frac{B}{G^2 + B^2}$$

and so

$$\text{Total equivalent resistance} = R = \frac{G}{G^2 + B^2}$$

$$\text{Total equivalent reactance} = X = \frac{B}{G^2 + B^2}.$$

Power in A.C. Circuits

We have seen that the power in a circuit carrying a steady current is $W = VI$ volts, and this is the power at any instant when V or I or both are varying. The average power supplied in the case of an alternating current is the mean value of VI during one complete period.

If the P.D. and current are in phase so that $V = V_{\max} \sin \omega t$ and $I = I_{\max} \sin \omega t$, the power

$$W = V_{\max} I_{\max} \sin^2 \omega t$$

$$\text{mean } W = V_{\max} I_{\max} \text{ mean } \sin^2 \omega t.$$

But we have seen (page 198) that the mean value of $\sin^2 \omega t = \frac{1}{2}$.

$$\text{Hence mean } W = \frac{1}{2} V_{\max} I_{\max} = \frac{V_{\max}}{\sqrt{2}} \times \frac{I_{\max}}{\sqrt{2}} = V_E I_E \quad . \quad (118)$$

So that with the P.D. and current in phase the average power in watts is the product of the voltmeter and ammeter readings as with steady currents.

Next, suppose that the P.D. and current are in quadrature, so that

$$V = V_{\max} \sin \omega t, \; I = \pm I_{\max} \cos \omega t$$

$$\text{Then mean } W = \pm V_{\max} I_{\max} \text{ mean } \sin \omega t \cdot \cos \omega t$$

$$= \pm \frac{1}{2} V_{\max} I_{\max} \text{ mean } \sin 2\omega t.$$

Now the mean $\sin 2\omega t$ over the whole period is evidently zero, since there are two equal positive and negative portions of the curve in each complete period (Fig. 4.35). Hence when the P.D. and

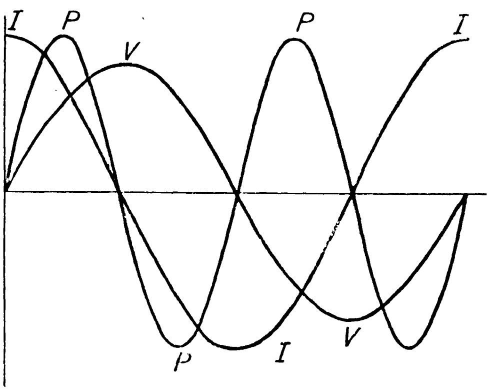


FIG. 4.35.—Current voltage and power waves.

current are in quadrature they may both of them have very high values, but the average power supplied to the circuit is nothing. Now if we take the case of a current lagging by an angle φ, $V = V_{\max}$

sin ωt and $I = I_{max} \sin(\omega t - \varphi)$. Splitting the current I into its components (Fig. 4.36), $I \cos \varphi$ in phase with the P.D. and $I \sin \varphi$ in quadrature with it, the former gives with the P.D. an average power $= VI \cos \varphi$, while the latter produces no power at all.

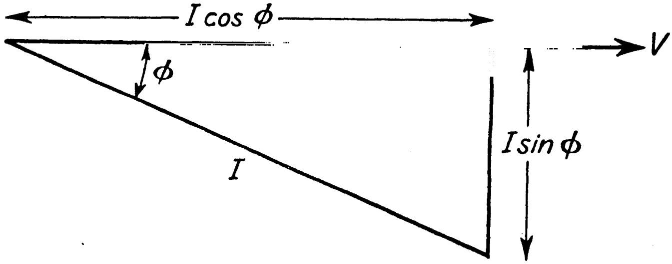


FIG. 4.36.—The components of a lagging current.

Hence with sinusoidal P.D. and current $W = VI \cos \varphi$. (119)

With irregular wave forms there is no simple angle of lag, but we may write $W = fVI$, where f is a factor equal to unity or less, which is called the " power factor " of the circuit. Hence for sine waves $f = \cos \varphi$ and is unity when the current is in phase with the P.D. and zero when they are in quadrature.

Complex Wave Forms

The above theory is based entirely upon currents of simple harmonic or sinusoidal wave form, and is sufficient for many purposes, as modern alternating current supply is generally fairly closely sinusoidal. In certain cases, however, such as supply from small alternators or through choking coils with iron cores, the wave may be very considerably distorted. It is possible, nevertheless, to express any such distorted wave as a series of sine waves of different frequencies, or into the " fundamental wave " and its higher harmonics, and each of these component waves can be treated separately by the above theory. To take up this subject generally would require considerable space, but it will be sufficient to deal with such cases when they arise in connection with particular instruments.

CHAPTER 5

PROPERTIES OF ELECTRICAL MATERIALS

THE electrical materials used in instrument manufacture may be broadly classified into three groups, viz.:

(a) Conductors.
(b) Insulators or dielectrics.
(c) Magnetic materials.

CONDUCTORS

Materials under this heading naturally fall into two sub-groups, viz.:

1. Those of high conductivity.
2. Resistance materials.

In the first group the best electrical conductor is silver, which has a specific resistance of 1·46 microhm-cm. at 0° C., and a temperature coefficient of 0·377% per degree Centigrade, but on the score of expense the employment of this material is very limited, and as there is but little gain in conductivity over that of the next best substance, copper, its employment in instrument construction is limited to a few special cases, notably those in which it is necessary to keep resistance to its lowest value in certain parts of the circuit, as, for instance, in suspensions and leading-in filaments for moving coils.

It is sometimes advantageous to employ fine insulated silver wire for moving coils in dynamometers and other instruments, as it is usually more easily freed from magnetic impurity than copper.

Copper, therefore, takes the first place among practical conductors, and since the introduction of electrolytic methods of refining, it can be produced economically in large quantities in a very high state of purity. Pure annealed copper has a specific resistance of 1·584 microhm-cm. at 0° C., and when hard drawn the value increases to 1·619 microhm-cm.

Practically all pure elementary metals increase in resistance with temperature; as this increase is in most cases quite appreciable, it is necessary not only to give the value of the resistance, but also the temperature at which it was measured. Moreover, in order to correct for temperature variation it is necessary to know the tem-

perature coefficient or fractional rate of increase in resistance for a small difference in temperature.

For most metals the relation between temperature and resistance is not strictly linear, but has a slight curvature, but over moderate ranges of temperature this curvature is very slight in pure metals. Hence, for extreme ranges we have

$$R_\theta = R_0 (1 + a\theta + b\theta^2 + c\theta^4 + \ldots)$$

while over moderate temperature range b and c are zero, and we may then write

$$R_\theta = R_{\theta_1} \{1 + \alpha_{\theta_1} (\theta - \theta_1)\}.$$

From measurements made at the Bureau of Standards in 1911, it was found that between 10° C. and 100° C. the variation of resistance with temperature for copper was linear, and that the temperature coefficient at 20° C. was proportional to the percentage conductivity. For 100% conductivity α_{20} has the value 0·00393, so that $\alpha_{20} = \dfrac{R_\theta - R_{20}}{R_{20}(\theta - 20)} = 0{\cdot}00393 \times k$, where k is the conductivity expressed decimally, and hence α for any other temperature calculated from 20° C. is

$$\alpha_\theta = \frac{1}{\dfrac{1}{0{\cdot}00393 \times k} + (\theta - 20)}$$

Since, however, the resistance of copper varies linearly we may correct to any temperature θ if we know the resistance at two other temperatures θ_1 and θ_2, for then we have

$$R_\theta = R_{\theta_2} - \frac{R_{\theta_2} - R_{\theta_1}}{(\theta_2 - \theta_1)} (\theta_2 - \theta)$$

and

$$\alpha_\theta = \frac{R_{\theta_2} - R_{\theta_1}}{R_{\theta_1}(\theta_2 - \theta_1) - R_{\theta_2}(\theta_1 - \theta)}$$

It is also convenient to determine the mean temperature of a winding by determining the increase in resistance. Thus, if R_θ is the temperature at θ° C., and R_{θ_1} at θ_1° C., then

$$R_\theta = (R_o(1 + \alpha\theta) \text{ and } R_{o_1} = R_\theta(1 + \alpha\theta_1)$$

So that

$$\frac{R_\theta}{R_{\theta_1}} = \frac{1 + \alpha\theta}{1 + \alpha\theta_1} \text{ which gives } \theta_1 = \frac{R_{\theta_1} - R_\theta}{\alpha R_\theta} + \frac{R_{\theta_1}}{R_\theta} \cdot \theta$$

and

$$(\theta_1 - \theta) = \left(\frac{R_{\theta_1}}{R_\theta} - 1\right)\left(\frac{1}{\alpha} + \theta\right)$$

The value of α_0 adopted by the Engineering Standards Committee is 0·00428. The American Institute of Electrical Engineers recommend a lower value, 0·0042, and the German rules employ the value 0·00426.

Copper casts well, but when used in this form its specific resistance is about three times the value of pure copper, and the material works well under the usual workshop process. Weinstrub has, however shown that the conductivity of cast copper may be materially increased by employing a small percentage of "boron sub-oxide" in the melt. Only 0·1% of this substance gave a material of 94% conductivity, and subsequently it was found that wide variations in the amount of sub-oxide had but little influence either in the electrical or mechanical properties of the copper, and conductivity as high as 97·5 can be reached, with only a trivial increase in cost per pound.

Magnesium has been found to act in similar fashion, and a small quantity of stock magnesium, thrust under the surface of the molten metal in the casting ladle, will effect considerable improvement in the copper castings.

Normally, in the perfectly pure condition, it is a slightly diamagnetic body, but it is comparatively rare to find a sample, particularly in the form of drawn wire, which exhibits this property; more usually the reverse is the case owing to the fact that the wire is drawn through hard steel dies, and then appears to be decidedly magnetic. This effect becomes troublesome in the case of moving-coil instruments, particularly those of the narrow coil type, and can only be eliminated by employing special wire or by suitable treatment. The prolonged action of a bath of hot diluted sulphuric acid, followed by copious washing and final drying at a gentle heat in a dust-free atmosphere, will be found fairly effective in removing the magnetic impurity which is usually confined to the surface of the wire. It is, however, possible to procure fine insulated wires which have been drawn through dies of non-magnetic material, and such wires may be used without further treatment.

Six kinds of insulated coverings are used for instrument wires, viz. :

Pure silk.
Rayon.
Fused rayon.
Cotton.
Enamel.
Synthetic resin.

Silk coverings are generally satisfactory, either in natural colour or dyed by means of non-conducting synthetic dyes, such as aniline and the like. Rayon of the regenerated cellulose type is a satisfactory wire covering, but is not generally so resistant to abrasion as pure silk ; it can generally be used successfully. Fused rayon is made by a different process and is usually an acetate type. It gives a low space factor and has good electric strength. All three of these types are free from fluff and are reasonably nonhygroscopic, particularly the last.

Cotton is not so good an insulator, and in an untreated condition is somewhat hygroscopic ; it does not give so high a space factor ; but on the score of cheapness it is often used in the stationary and more robust coils of the instrument, where it is usually impregnated with some kind of insulating varnish or compound after winding.

Enamel insulation is considered extremely suitable in many instances and gives an excellent space factor, together with high electric strength. Most enamels contain either linseed, china wood or castor oil suitably blended with natural gums or pitches, and possess very high electric strength and good adhesion to the wire. They are somewhat adversely affected by the action of varnish solvents, and are not therefore to be recommended for use in those cases where vacuum impregnation is used.

As regards synthetic resins, nylon and Formvar resins have been in use for several years as wire coverings, and possess the advantages of enamel as regards dielectric strength, and are comparable with cotton and silk in abrasion resistance. Wires covered with either resin can be handled more safely in manufacturing processes employed in winding coils than most other types of covering, and are practically unaffected by the varnish solvents used in impregnation. Nylon is somewhat hygroscopic, and tends to lose insulation resistance when exposed to very moist atmospheres

and is better impregnated, when this undesirable feature is entirely eliminated. It should be noted, however, that for normal conditions in which fibrous coverings such as silk and cotton would be used, nylon is more satisfactory than either in the impregnated state. Formvar coverings crack when the wire is bent round a small radius, but those cracks can be closed by heating the covered wire to 105° C. for a few minutes, and are not found to be detrimental. Both synthetic resin coverings are fully satisfactory for working at temperatures higher than those which can be used for silk, although the ultimate safe working temperature has yet to be determined.

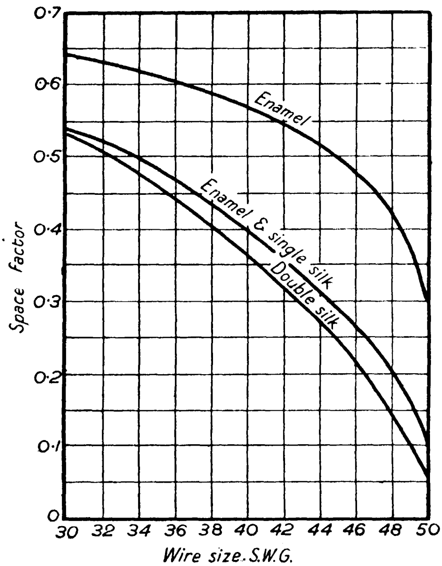


FIG. 5.1.—Space factors for various insulating coverings.

Fig. 5.1 gives a series of space-factor curves for various insulating coverings, and will be found in general to give a value which is slightly worse than that which can be obtained with careful winding.

The tension used in the wire during winding should not exceed the yield point of copper, 6 tons per square inch, otherwise inconsistent resistance values will be obtained. All forms of covering if properly applied will withstand such tension without damage.

It is remarkable how little rectangular wire is used in instrument work, despite the fact that its employment would be equivalent

to finding a conductor with a 21% better conductivity ; it obviously gives a better space-factor, and materially enhances the finished appearance of the instrument. Where massive conductors are necessary, edge strip winding is occasionally employed, using bare conductors ; this method of winding makes a rigid and handsome job, and the finished coil can be mounted so as to have very high insulation. Fig. 5.2 shows a coil of this description.

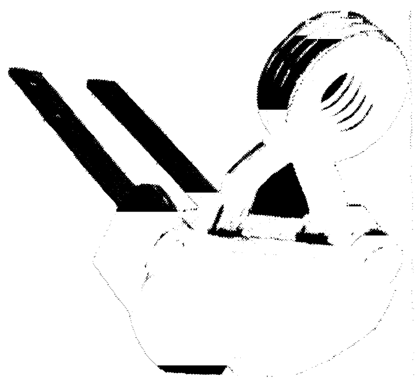


FIG. 5.2.—Coil wound with rectangular-section wire.

In some cases bare square section conductors are used in the same way, and in a few instances the distance between successive turns is regulated by running on a cord with the wire, which acts as a gauge or distance piece between the turns.

The current densities employed by various manufacturers in copper coils vary considerably, particularly in wires of small gauge. The following table gives a few actual examples taken at random from various instruments.

From this it will be seen that only in the case of wattmeters are the results at all uniform.

Now the voltage drop in a conductor is given by the relation

$$V = IR = I\rho\frac{l}{A} = \Delta\rho l$$

where Δ is the current density, ρ the specific resistance of the material, and A its cross-section, and l the length of the conductor, therefore, where it is important to keep the voltage drop and the

TABLE XI.—*Copper Current Densities.*

Type of instrument.	Maker.	Max. range of current.	Conductor dimensions in millimetres.	Current density amperes per sq. mm.
Ammeter	Hartmann & Braun	40	3·5 diameter	4·15
	Keiser & Schmidt	100	14 strands 2·3 diameter.	3·125
	Schuckert (Old)	200	24·0 × 11·0	0·758
	Everett Edgcumbe	100	2 (20 × 0·7)	3·76
	Lund und See Kabelwerke	10	2·8 diameter	1·623
	Nalder Bros. & Thompson	10	2·25 diameter	2·515
	Reiniger, Gilbert & Schall	100	28 × 7 × 0·5 dia. stranded	2·87
	" "	10	2·5 diameter	2·036
Wattmeter (Main coil)	Nalder Bros. & Thompson	100	4 × 4·25 diameter	1·763
	" "	30	3·78 diameter	2·68
	A.E.G.	30	4·3 diameter	2·06
	Kelvin & James White	75	20 × 1·0 strip	3·75
	Reiniger, Gilbert & Schall	40	1 × 25 strip	1·6
	Weston	10	3·38 × 2·5 rectangular strip	1·183
	"	100	4 × 15·3 rectangular strip	1·635
	Siemens & Halske	1	0·785 diameter	2·065
	" "	25	1·5 × 16 strip	1·04
Voltmeter	Everett Edgcumbe	..	0·122	8·075
	Evershed	..	0·19 diameter	1·48
	Abrahamson	..	0·19 diameter	3·7

watts lost in the instrument to a low value, a correspondingly low value of the current density has to be chosen.

In general it would be well, where space permits, to design to a current density of 1·5 to 2 amperes per sq. mm. Annealing the conductor has an important effect, and Addocks has shown that the highest conductivity is obtained in hard drawn wire by raising the temperature until the wire is just barely visible in complete darkness (500 – 600° C.) and then allowing it to cool. Annealing at higher temperatures lowered the conductivity on account of the crystallization which results on cooling.

Alloying copper with other materials invariably leads to a loss of conductivity. The copper bronzes in which copper is alloyed with phosphorus, manganese, silicon and chromium have excellent mechanical properties, but their higher specific resistance limits their application in instrument work strictly to those cases in which such qualities are essential.

Phosphor bronze is an alloy of copper 79%, tin 10%, lead 10%, phosphorus 1%; the composition may, however, be raised considerably in the proportion of the last two constituents, and phosphorus up to 9% is sometimes employed. It is a hard alloy of great tenacity and durability; the specific resistance is dependent on composition.

Closely allied are the silicon and chromium bronzes, in which the phosphorus is replaced by silicon and chromium respectively. Silicon bronze, however, is said to have better conductivity and lower resistance temperature co-efficient than phosphor bronze, and is much more resistant to corrosion and oxidization.

Copper manganese alloys increase in strength with the amount of manganese added up to 26·5%, but the electrical conductivity, specific gravity and melting-point decrease very considerably with each addition of manganese.

The low thermo-E.M.F. of phosphor bronze has been referred to by Murphy and Warren, who have advocated its use for potentiometer construction.

Brass and gun-metal are alloys of copper with zinc and tin, and are valuable materials on account of their good working qualities and their power of taking a high finish. The inclusion of much zinc in the alloy nearly always leads in time to a want of mechanical strength, and with these materials there is sometimes a tendency to become brittle and short with age.

On the score of economy and neatness die castings are often used in instrument construction. Alloys rich in zinc are sometimes used, but care must be employed in choosing such alloys, as there is a definite tendency to become fragile, to crack and warp with time. Much better alloys with an aluminium base are available. Die-castings give a neat appearance to an instrument, and can be produced in shapes which would be awkward by any other method.

Great care should be exercised in choosing these materials to ensure that they are free from iron, since the presence of a minute quantity of iron can render brass and similar materials feebly magnetic, and with the intense magnetic fields which are now available with modern permanent magnet materials, any slight trace of magnetic properties in materials which are assumed to be non-magnetic may cause serious trouble.

The possibility of using aluminium as a conductor where weight is of importance has to be considered. Aluminium has a specific resistance of 2·56 microhm-cm. at 0° C. This value is, however, liable to some variation on account of slight impurities always present in the commercial product. Roughly, we may regard it as having a conductivity of about 60·7% that of copper, and on this basis, for the same voltage drop, a current density of 0·94 amperes per square millimetre would be permissible. The specific gravity is, however, only 2·7 as against 8·9 for copper, and hence the weights of electrically equivalent conductors are in the ratio of 0·48 to 1·0. Several manufacturers use insulated aluminium wire for the construction of moving coils in indicating instruments. Messrs. Hartmann and Braun have wound coils with this material which, after winding, are put into a press and subjected to considerable pressure, which practically alters the conductor to one of rectangular section.

The material casts well and is soft and ductile, but as an offset against this is the difficulty of making a good electrical joint other than by clamping, welding or casting. Where clamped joints are employed, the surface in contact must be of ample area, because of the high contact resistance due to the presence of a transparent oxide film over the surface of the metal, which on account of its transparency hardly affects the surface brightness of the material, but is nevertheless always present and forms a protective coating. The presence of mercury will destroy this protective skin, and then the formation of alumina proceeds very rapidly on the surface,

the oxide being produced in the form of a pile or plush standing out in threadlike crystals from the surface.

Exposure to the action of water, particularly when the surface is liable at the same time to abrasion, will rapidly produce corrosion. Under these circumstances the surfaces must be protected by a coating of organic or bituminous varnish, but care must be exercised in excluding any material containing lead oxide because of the destructive chemical action which is likely to arise between the oxide and the metal. Protection is also afforded by contact with steel in the presence of water, but only at the expense of accelerated corrosion of the steel.

A number of special solders for aluminium have been produced, and while some of these appear to produce satisfactory mechanical joints, there is as yet insufficient evidence to show whether the joints so produced are satisfactory from an electrical point of view and remain so over a considerable period of time.

Owing to its soft and ductile qualities, aluminium lends itself well to edge strip winding, and by oxidizing the adjacent surfaces, coils can be wound in otherwise uninsulated material which are very compact and light, and which will permit of a very fair voltage per turn being employed.

A form of protection for aluminium and aluminium alloys which is in great use nowadays is the process known as anodizing. This consists in producing on the surface of the metal a layer of hard oxide which is an effective insulator. Several processes have been developed all bearing a strong resemblance to electroplating practice, and have been very fully described from time to time. The process can be applied to bare wire, and produces an insulated conductor of the same diameter as the bare wire, and owing to the thinness of the oxide layer, the reduction in the effective cross-section of the conductor is negligible.

Aluminium Alloys

The attractiveness of the low specific gravity of aluminium is very considerably discounted in constructional work by its comparatively poor mechanical properties, and on this account much attention has been given to the production of light alloys, which have now resulted in making available materials of the greatest engineering importance. In this direction, according to Dr.

Rosenhain, there are possibly only two rivals to aluminium, viz. magnesium and beryllium.

Strong and tough alloys of aluminium have now been produced, but it must be remembered that, with the exception of the special aluminium magnesium alloys (magnalum), all aluminium alloys are necessarily heavier than aluminium itself.

The addition of copper to aluminium materially increases the tensile strength. An alloy containing 12% of copper has been used, but Dr. Rosenhain has shown that if the copper content is reduced to 4%, an equally strong alloy can be produced which has the additional advantage over the 12% admixture in that, as well as being appreciably lighter, it can be drawn and rolled and is far less brittle. If part of the copper is replaced by manganese a still further improvement is effected, particularly as far as resistance to corrosion is concerned.

Alloys of aluminium and zinc also possess valuable features, but such alloys, if rich in zinc, are very soft and fragile while hot, and are liable to crack from shrinkage if not properly manipulated; it is therefore desirable to keep the zinc content as low as possible.

A small admixture of copper to such alloys improves their mechanical strength, but of course increases the specific weight. All aluminium zinc copper alloys are, however, very subject to corrosion, particularly in the presence of sea-water, and they have comparatively poor ductility.

The hardening effect of magnesium in aluminium alloys was discovered by Wilson, and led to the introduction of this element into aluminium copper manganese alloys, the resultant material being given the trade name of "Duralumin." The virgin hot-rolled material has a high tensile strength, but if reheated to a temperature a little below 500° C. and quenched with water it slowly hardens at ordinary temperatures, the change taking about four days to complete, and at the end of the process the tensile strength has increased very considerably.

Dr. Rosenhain, at the National Physical Laboratory, has succeeded in applying the same principle to the aluminium copper zinc series of alloys, and has produced materials with properties even more remarkable than those of duralumin. The hardest and strongest is 50% better than duralumin, and represents the best combination of strength and lightness in these alloys yet attained; it is, however, subject to corrosion as mentioned above.

By substituting nickel for the zinc an alloy results which is not quite as strong as the aluminium zinc copper magnesium alloy, but which retained its strength at a higher temperature than any other wrought aluminium alloy, and at the smae time showed a high resistance to corrosion even in sea-water, while the elimination of the zinc rendered the alloy comparatively easy to roll.

Resistance Materials

The systematic study of the electrical properties of resistance materials began with Dr. Mathiessen's researches, made on behalf of the British Association in 1861–65. Since that time our knowledge of the electrical properties of alloys has steadily grown, until at the present time there are upon the market a large number of materials possessing excellent characteristics. The properties of an ideal material for resistances may be specified as follows :

I. High specific resistance.
II. Low temperature coefficient.
III. Low thermo-electric force to copper.
IV. It should be highly resistant to oxidation even at high temperatures.
V. It should be permanent and stable in its general properties.
VI. It should be ductile and malleable.
VII. It should joint easily and be capable of working well with ordinary shop processes.
VIII. It should preferably not amalgamate with mercury.

No single alloy completely fulfils such a specification, but there are several which come near to it.

Mathiessen classified the pure metals into two groups, the first being a small one consisting only of lead, tin, cadmium and zinc. The other group contained practically all other metals. From these he made binary alloys, and investigated their properties, and found that for simple mixtures of two metals, both of which are in the first group, the electrical conductivity could be calculated from those of the components.

If, however, both metals were in the second group no such simple law existed, but it was found that with the first very small admixture of the alloying constituent there was a very marked fall in the conductivity of the resultant alloy, which continued at a gradually-decreasing rate until 50% was reached—that is, the

amount of each metal was equal. As the percentage of the alloying constituent continued to increase beyond this point the conductivity again began to rise, at first slowly, and eventually sharply, to the value of that of the alloying constituent.

Alloys consisting of one metal from each group show also the large drop in conductivity on the first addition of the second constituent, but afterwards there is a steady fall until the value of the second constituent is reached ; in this case the second constituent refers to a metal in the first group.

The lowering of conductivity by alloying is accompanied by a corresponding fall in the temperature coefficient, and this led Mathiessen to formulate the law : That the ratio of the temperature coefficient of an alloy to the mean temperature coefficient of its constituents is equal to the ratio of the conductivity of the alloy to the mean conductivity of its constituents.

Since this excellent investigation was undertaken the work has been extended to more complex alloys, and some compositions have been found with the quite remarkable property of a negative temperature coefficient.

This is true of some alloys of copper and nickel, and would appear as a contradiction to Mathiessen's rule, but it should be remembered that nearly all such alloys contain traces of other materials—as, for instance, iron, and it is not improbable that it is the presence of these which account for the peculiar deviations. As a result of his investigations, Mathiessen selected an alloy of 66 parts of silver to 33 parts of platinum as the most suitable for the construction of permanent standards of resistance. Its specific resistance was about 31·6 microhm-cm., and its temperature coefficient about 0·027% per degree Centigrade. The thermo-E.M.F. to copper is not excessive, and it does not amalgamate with mercury. He concluded that it would be difficult to obtain an alloy of lower temperature variations than this, but more recent investigations have shown that this is by no means the case. Many of the more complex resistance alloys, however, exhibit peculiar unsteadiness even when carefully annealed, which renders them unsuitable for precision resistances. This unsteadiness usually manifests itself in a tendency of the resistance value to increase continually as time goes on.

The alloy German silver was very largely used in resistance construction in the early days, but of recent years much more

suitable materials have been found, for it is an alloy of 60 parts of copper, 25 zinc, 15 nickel, and so large an admixture of zinc appears to be objectionable and leads to instability, the material becomes brittle and rotten, and in some cases resistance coils wound in this material have entirely fallen to pieces after a lapse of time. The addition of about 1% of tungsten to German silver considerably improves the alloy, both with respect to its permanence and temperature variation, and the resultant compound is known under the trade name of Platinoid. The constants of this material are a little uncertain, as the properties may vary from batch to batch of the material, since the amount of tungsten in the alloy depends to some extent on the skill and judgment of the metal mixer. The specific resistance varies from 43 to 34 microhm-cm., and the temperature coefficient from 0·031 to 0·0224% per degree C.

In 1889 Dr. Weston, of Newark, N.J., investigated the properties of alloys consisting of copper, manganese and nickel, with a view to their utilization for permanent resistances. It was found that they had very peculiar temperature resistance variation. Subsequently Feussner and Lindeck concluded from their researches at the Reichsanstalt that the most suitable alloy for standard resistances consisted of copper 84%, manganese 12% and nickel 4%, and the name "Manganin" was given to the material. There seems, however, to have been considerable variation in the electrical properties of various samples of this material, particularly in respect to its temperature coefficient. The temperature resistance curve shows that the material has, at low temperatures, a positive temperature coefficient, but as temperature increases the coefficient falls to zero and subsequently becomes negative. Indeed, the temperature resistance curve is sufficiently nearly parabolic to be represented by the formula

$$R_t = R_{\max} 1 + \beta (T - t)^2$$

where $R_{\max}$ is the maximum resistance at the temperature T.

Feussner and Lindeck, examining 42 manganin resistances, found that the majority had temperature coefficients of the order of 1×10^{-5}, but samples as high as $8{\cdot}1 \times 10^{-5}$ were found. Dr. Rosa examined samples having values ranging from 4 to 50 parts in a million per degree Centigrade, and some samples of German origin examined by the authors show values of about 25×10^{-5}

between 0° and 10° Centigrade. In general the temperature coefficient becomes negative at temperatures between 20° and 25° C., but this characteristic depends upon the amount of iron present in the alloy, and when the iron content is very low the maximum of the temperature resistance curve shifts towards higher temperatures. Thus in the samples we have examined the iron content was merely a trace due to normal impurity and the maximum occurred at 40° C., so that it is only above this temperature that the material exhibits a negative temperature coefficient.

The thermo-E.M.F. to copper is very low, being in the samples we have examined only 1·4 micro-volts per degree Centrigrade, and this is a characteristic of very great value. The specific resistance is high and ranges from 34 to 55 microhm-cm., the variation between samples being mainly due to small percentages of impurity.

In the year 1919 Hunter and Bacon made a systematic investigation of the alloy, which may be summarized in the following table :

TABLE XII.—*Properties of Various Manganin Alloys.*

Composition per cent.				Temperature coefficient 18°–24° C.	Resistivity microhm-cm. at 20° C.	Thermo-E.M.F. to copper micro-volts.
Cu.	Mn.	Ni.	Fe.			
80·02	9·93	1·74	0·24	$1{\cdot}2 \times 10^{-5}$	34·2	4
87·24	10·26	1·77	0·52	$1{\cdot}5 \times 10^{-5}$	37·4	5
88·2	8·84	1·78	0·93	$0{\cdot}33 \times 10^{-5}$	55·6	3
83·6	12·03	3·41	1·04	$0{\cdot}22 \times 10^{-5}$	47·8	8
84·72	12·83	2·08	0·73	$0{\cdot}38 \times 10^{-5}$	50·8	4
84·07	12·98	2·6	0·82	$0{\cdot}57 \times 10^{-5}$	51·1	..

From the above they conclude :

That manganese between the limits employed affects the specific resistance most, but has practically no effect on the temperature coefficient of the material.

That the presence of iron has the greatest influence on the temperature coefficient, and the presence of iron up to 1% improves the material in this respect by lowering the coefficient and bringing the point of maximum of the resistance temperature curve within the range of normal working temperatures.

In general the mechanical qualities of manganin are good, but unfortunately, as first pointed out by Feussner and Lindeck, when the material is annealed at low temperatures, in air, selective

oxidation of the manganese occurs, with the result that a layer of metallic copper is left on the surface of the wire, which leads to a high positive temperature coefficient. It is therefore essential to carry out all annealing operations with the material out of contact with the atmosphere, either protecting it with a heavy coat of varnish, or employing an inert gas to surround it during the annealing process

Soft soldered joints should be avoided, particularly when the coils are wound with fine wire for high resistance units, as instability is liable to occur at the joints and cause unsteadiness of value. In practice it is therefore better to hard solder the ends of the resistance to copper collets, washers or end-plates, which can be screwed into position in the copper lugs and finally sweated solid with soft solder. This resistance is then stabilized by annealing at about 700° C. for a few minutes in CO_2, and then given a final annealing for four or five hours at 150° C. in an oil bath. This process was adopted by Hunter and Bacon in their experiments mentioned above. Many manufacturers, however, find it sufficient to coat the whole resistance carefully with shellac varnish, and then give a prolonged annealing at about 140° C.

The use of a heavy varnish coat has, however, a serious disadvantage, for in 1907 Dr. Rosa and Babiock published an account of measurements made upon manganin coils protected in this way, and showed that the varnish coat was liable to absorb moisture from the atmosphere, which causes it to swell and put stresses on the wire, which result in variable resistance values, and this has therefore led to the practice of hermetically sealing standards in order to exclude atmospheric changes. When manganin is employed in the form of sheets or plates for low-resistance standards, or for the shunts of moving-coil instruments, it does not appear to be so permanent as wire of the same material, in spite of all precautions in annealing and varnishing. This may possibly be due to the joints, which play a much more important part in strip than in wire resistances, but with even the most carefully silver-soldered joints, a fairly rapid rise of resistance usually takes place for several months after annealing. Fig. 5.3 shows the variation of resistance with time in the case of a newly-constructed 0·1 ohm manganin-resistance standard. This change is at the rate of about 2 parts in a million per day, or 0·07% in a year. A similar change of resistance has been found in a Wolff 0·001 ohm resistance after

standardization by the Reichsanstalt and the National Physical Laboratory, which apparently showed an increase in resistance of 0·18% in between two and three years.

Recently British alloys having similar composition and characteristics to manganin, and known as " Minalpha," " Telcuman " and " Tamac," have been introduced.

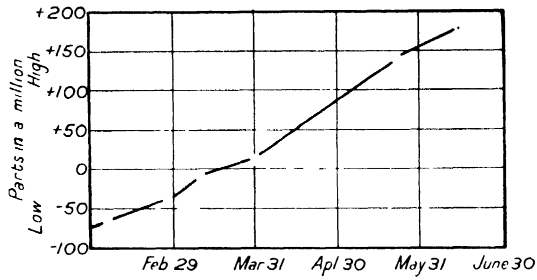


FIG. 5.3.—Variation of resistance with time in a new standard resistance.

A comprehensive investigation has been made of copper-manganese-aluminium alloys, and it has been found that for the construction of standard resistances the best composition is 85% copper, 9·5% manganese, 5·5% aluminium. This alloy is known as " Therlo " and has a resistivity of about 45 microhm-cm., while its change of resistance with temperature is less than that of manganin. The mean thermo-electric E.M.F. with respect to copper over the range 0–100° C. is about 0·3 micro-volt per degree Centigrade, and may be reduced to 0·1 micro-volt per degree Centigrade, i.e. to one-tenth of the value for manganin, by the addition of 0·15% iron. The stability of these alloys is reported to be as good as that of manganin, and they have the advantage of being suitable for use at higher temperatures.

A range of copper-nickel alloys having negligible resistance-temperature coefficients are available, and are known under the trade names Ferry, Advance, Eureka, Constantan, Hecnum, Copel and Ideal. It has been found that over the range 40% nickel to 50% nickel the temperature coefficient is practically zero, and all the materials mentioned have a nickel content within this range. It is also found that the actual temperature coefficient is very dependent on the presence of impurities in the material. Generally the relationship between resistance and temperature is not linear, and these alloys tend to have rather a high thermo-

electric E.M.F. against copper, and care is therefore necessary in their use. The specific resistance of these alloys is slightly higher than that of manganin.

Another group of alloys which are used for very high resistances are the nickel-iron and the nickel-chrome series. These usually have specific resistances of the order of 100 microhm-cm., and consequently lend themselves to the construction of very compact high resistances.

Another extensive group having a composition of 70 – 75% of copper, 25% nickel, together with small percentages of iron and manganese, are known under the trade name of Nickeline. The specific resistance is of the order of 40 microhm-cm., and the temperature coefficient is of the order of 0·02% per degree Centigrade, and is positive; but very wide variations occur among samples bearing the same trade name. The thermo-E.M.F. is in general lower than that of such alloys as constantan, being usually of the order of 20 micro-volts per degree Centigrade.

Two new alloys have been developed in Germany under the names "Isabellin" and "Norokonstant." The composition of "Isabellin" is 84% copper, 13% manganese and 3% aluminium, while that of "Norokonstant" is 82·5% copper, 12% manganese, 4% aluminium and 1·5% iron. The specific resistance of "Isabellin" is 50 microhm-cm. and the temperature coefficient – 0·00003. For "Norokonstant" the specific resistance is 45 microhm-cm., the temperature coefficient is about 0·000001, and the thermo-E.M.F. against copper is + 0·3 microvolts per degree Centigrade in the hard drawn condition, and – 0·3 microvolts per degree Centigrade in the final heat-treated condition.

Generally resistance materials exhibit a want of steadiness in value after winding and working in the shops, and should always be allowed a period of rest before being finally adjusted. Annealing has also an important effect, and it is therefore preferable to anneal after the rougher shop work has been completed and just before the final adjustment.

For coils and plates this process is preferably carried out by the passage of an alternating current of low frequency, whose effective value should be sufficient to produce a temperature of 100° C. to 140° C. This current may then be reduced in value as the resistance steadies down. The current densities employed in alloys by various manufacturers are shown in Table XIII.

TABLE XIII.—*Current Densities in Resistance Alloys.*

Instrument.	Maker.	Max. current.	Current density (amperes per sq. mm.).	Voltage drop (volts).	Resistance (ohms.).	Surface.	Total area (A).	Freely exposed area (a).	Watts (W).	$\frac{W}{A}$	$\frac{W}{a}$
Potentiometer shunt	Crompton & Co. Manganin	25	1·145	1	0·04	Black	1,388	1,388	25	0·01805	0·0180
Ammeter shunt	Compagnie des Compteurs	50	2·565	0·1	0·002	Bright	30	15	5	0·1665	0·333
,,	,, ,,	20	1·85	0·1	0·005	,,	25·1	12·55	2	0·08	0·16
,,	,, ,,	10	1·72	0·1	0·001	,,	12·48	6·24	1	0·08	0·16
,,	,, ,,	2	0·802	0·1	0·05	,,	14·17	14·17	0·2	0·0141	0·0141
,,	Crompton & Co. Manganin	100	1·45	0·075	0·00075	Black	275	68	7·5	0·0272	0·11
,,	Hartmann & Braun	150	2·89	..	..	Bright	250	62·5	..	..	..
,,	Siemens & Halske	400	2·2	0·0543	0·0001358	,,	63·5	31·75	21·7	0·342	0·684
,,	,, ,,	200	2·15	0·0541	0·0002725	,,	55·25	13·7	10·9	0·1975	0·797
,,	Nalder Bros. & Thompson	300	1·875	0·111	0·00037	,,	300	59·4	33·3	0·111	0·561
,,	,, ,,	150	1·83	0·111	0·00074	,,	142·3	63·25	16·7	0·01171	0·264
,,	,, ,,	15	1·84	0·111	0·0074	,,	37	16·12	1·665	0·045	0·1032
,,	,, ,,	1·5	0·4875	0·111	0·074	,,	32·2	32·2	0·1665	0·00517	0·0051
,,	Keiser & Schmidt	50	2·5	..	..	..	67	11·18	..	..	..
,,	Weston	50	3·88	0·005	0·001	Bright	13·75	6·88	2·5	0·1818	0·3636
,,	Elliott Bros.	100	1·37	0·165	0·00165	Dull	466	77·66	16·5	0·0354	0·2125
,,	,,	50	1·07	0·165	0·0033	Grey	310	77·5	8·25	0·0266	0·1063
Voltmeter bobbin	Nalder Bros. & Thompson	0·05	0·9125	..	..	..	..	..	..	..	.
,,	Hodges & Todd	0·084	0·82	..	..	..	..	..	..	..	.

The calculation and design of instrument shunts will be dealt with under the heading of Moving-coil Instruments.

On the whole, current densities in the neighbourhood of 2 amperes per square millimetre are general, but everything will, of course, depend upon the management and mounting of the shunt, so that full advantage may be taken of surface radiation and convection.

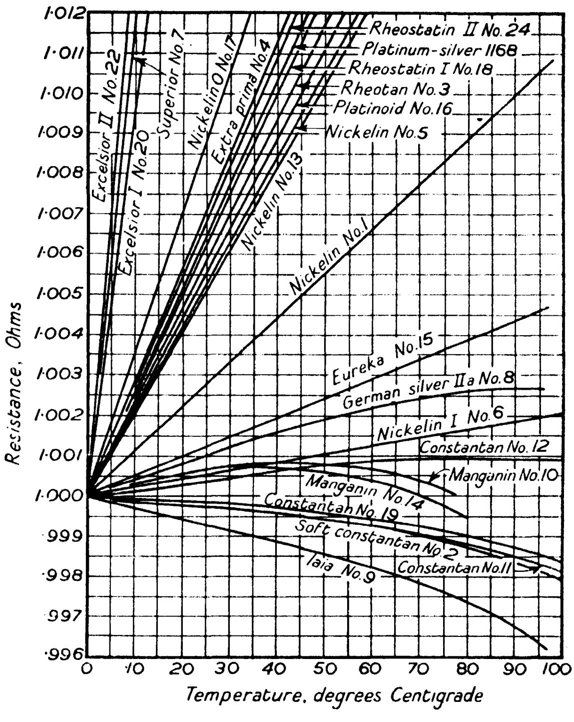


FIG. 5.4.—Temperature-resistance curves for various materials.

Fig. 5.4 is a set of resistance-temperature curves for various resistance materials. Although several of the pure metals have a high specific resistance, their use as resistances is limited to very special cases, where their high temperature coefficients are either of no account or can be turned to some advantage. Thus platinum is used in high-temperature thermometry, where its definite law of resistance-temperature change is utilized. Nickel, in the same

way, is used for resistance compensation. Iron, on account of its high specific resistance and cheapness, is often used for heavy current rheostats of the rougher kind, but its large coefficient is here a considerable drawback.

In this respect some of the alloy steels are better, and several of such steels are sold under trade names, such as Resista, Rheostene, Kruppin, etc., for the manufacture of heavy-current resistances. Many of them have very high specific resistance, but some care is required in their use, since they are liable to develop hard and brittle spots during the process of drawing.

DIELECTRICS

Much attention has been devoted to the experimental study of insulating substances, and a vast amount of data relating to them has accumulated. Unfortunately much of this work has been undertaken with some special object in view, and methods of test have been adopted which, although satisfactory for the special purpose of the investigation, render a comparison with the results obtained by other observers difficult, if not impossible. The use, therefore, of such data for purposes other than that for which it was obtained may be entirely misleading, if not disastrous, and figures, particularly for electric strength, should always be employed with the greatest of caution.

The three fundamental electrical qualities of dielectrics of great importance to the instrument maker are :

I. The dielectric strength.
II. The insulation resistance.
III. The specific inductive capacity or dielectric constant.

The relation of the first two of these quantities one to the other is not easy, if at all possible to define, for it does not necessarily follow that an insulator showing high dielectric strength should have a high insulation resistance, or vice versa, and consequently it is necessary to examine and deal with each quantity separately.

All insulating substances, whether gaseous, liquid or solid, enter into one or the other branches of instrument construction, and consequently there is a very wide range of substances to be examined.

Broadly considering the qualities we must have in the ideal

substance, the following properties are desirable, irrespective of whether it be liquid or solid :

I. High dielectric strength.
II. High insulation resistance.
III. It should be non-hygroscopic.
IV. It should be capable of withstanding without deterioration a repeated heat cycle.
V. Solids should have a high melting or softening point, and liquids should not evaporate or volatilize.
VI. Solids should have great mechanical durability.
VII. It should have small temperature coefficient.

In addition to these properties, solid or rigid insulators intended as supports to live parts should have—

(*a*) Rigidity, tenacity, compressive strength and freedom from brittleness.

(*b*) Freedom from surface leakage and metallic veins.

(*c*) Should be unaffected by oil, water and corrosive liquids.

(*d*) Should be easily workable and take a high finish.

(*e*) Should be fireproof.

(*f*) Should not warp or distort if subjected to repeated cycles of heat and humidity.

Solid insulators intended to be applied by hand should, in addition to the general qualities, have—

(*a*) Flexibility.

(*b*) Toughness.

(*c*) Be water and liquid-proof.

(*d*) Be capable of being moulded and reduced to thin sheets or strips.

Insulators to be applied in liquid form, such as varnishes, etc., should also have—

(*a*) Quick-drying properties.

(*b*) The solvents should not attack the base insulator or the conductor.

(*c*) They should be chemically stable, even under strong oxidizing influence, after they have solidified.

(*d*) They should set hard and with a good surface, and not be brittle.

(*e*) When dried off it should be liquid-proof and non-absorbent.

Simple liquid insulators should comply with the following:

(*a*) Should not absorb moisture.

(*b*) Should not carbonize or suffer chemical change under electric stress.

(*c*) Should not attack metallic conductors immersed in it.

(*d*) Should have a high flash-point and high evaporation temperature.

(*e*) Its viscosity should remain constant over a fair range of temperature.

(*f*) It should not creep.

Gaseous insulators, in addition to the general properties,

(*a*) Should not form explosive mixtures with the constituents of the atmosphere or with the gaseous products likely to be formed.

(*b*) Should not dissociate under the applied P.D.

The weakest link in the whole chain of electrical construction is undoubtedly the insulator. The universally bad mechanical properties and the uncertainty of its physical and chemical properties are alike a cause of anxiety to manufacturer and user of electrical apparatus. The failure of a comparatively insignificant piece of insulation in an instrument may lead to a wholesale shut down, with the result of, perhaps, loss of life and large sums of money, and hence too much care cannot be devoted to the study both of the physical properties and methods of application of insulators to constructional work, for not infrequently the cause of breakdown is not so much the fault of the insulating substance itself as the bad or careless design under which it is applied.

Turning now to the general characteristics of each of the main classes of insulating substance, we find the following special properties:

Gases

The behaviour of gases as insulators is of great importance in instrument work, particularly in connection with the design of high potential instruments. In general they are but very feeble conductors under normal circumstances, but their conductivity increases considerably under the action of an external ionizing agent, such as an electric discharge, radio-active bodies, ultra-violet light or an incandescent body. But even under these circumstances the conductory power is limited, and is independent

of the time the ionizing agent acts, and does not persist after its removal. If, however, an ionized gas is subjected to the action of a gradually increasing electrostatic field, the resultant current will at first increase fairly rapidly as the intensity of the field increases, but eventually a steady or saturation value is reached, when the ions, by whose agency the current is transmitted, are supplied by the P.D. as fast as they are utilized. The current, therefore, remains constant for a considerable range of P.D. Finally, however, a point is reached when a further slight increase of the electrical pressure causes a rapid rise in the current, due to a strong self-ionizing action in the immediate neighbourhood of the electrodes, and this effect gradually spreads, and apparently reduces the effective length of the dielectric between the electrodes, causing the field to become more concentrated, until at last breakdown occurs.

There are several other factors besides the state of ionization which influence the dielectric strength of a gas. Thus, the size and shape of the electrodes, and consequently the form of the electro-static field, the physical state of the gas as regards the amount of water-vapour, dust, etc., present in it, the pressure of the gas, all have their separate bearing on the final value.

The form of electrode naturally determines the distribution of the electrostatic field. Large flat plates will give a uniform field within their areas, but this will be modified as we approach the edges, where a sharp change in direction occurs, and at breakdown the spark nearly always occurs at this position, where the field is non-uniform. Spheres of large radius will therefore give a better distribution, since the change of direction is gradual and uniform everywhere, while the other extreme would be the case of very small spheres or needle-points, where the field is most intense at the electrodes, and a minimum half-way between them. Anything, then, in the nature of a sharp edge or point will lead to local concentration of the field and a consequent apparent weakening of the dielectric.

Usually freeing the gas from moisture and dust particles results in an increase in its electric strength, but this condition in most cases can only be artificially maintained. After the passage of a spark the gas between the electrodes becomes temporarily ionized, and the gap will break down at much lower potentials, unless the ionized gas so formed is immediately removed.

The relation between spark length and gas pressure has been investigated by Paschen, and he concluded from his observations that over a wide range the spark potential varied as the product of the spark length and pressure; or in other words, it was dependent upon the mass of gas under electric stress, except for gaps which were very small compared with the dimensions of the electrodes. By spark potential is meant the maximum P.D. which can be applied to the electrodes indefinitely without causing breakdown. Dr. Russell, investigating theoretically the behaviour of dielectrics, showed that if E is the dielectric strength of the medium, V the difference of potential established between spherical electrodes, such that $V_1 = V_2 = V/2$, where V_1 is the potential of one electrode and V_2 that of the other, and calling x the minimum distance between the spheres whose radius is a, then

$$E_{\max} = (V/x)f$$

where f is a function of a. This function is plotted in Fig. 5.5.

The maximum conductivity of a gas occurs at a pressure between 0·1 and 0·01 of an atmosphere, the actual value depending on the gap length, and if the pressure is reduced below this the conductivity again falls off rapidly. The influence of the form of electrode is well shown in Fig. 5.6, and the effects of gas pressure in Fig. 5.7, which are curves obtained by Watson from careful experiments in air, and from which he deduces the empirical law,

$$E = 20 + 25{\cdot}6\,p$$

where E is the dielectric strength of the gas and p is the pressure in atmospheres absolute.

Dr. Russell has put the dielectric strength of air at atmospheric pressure at about 38 kV maximum per centimetre, and Baille's results for parallel planes give the value 30·8 kV maximum for the same conditions.

In every case the effect of wave shape must not be forgotten in dielectric testing. It is the maximum value which is responsible for rupture, and although the effective value as given by a suitable voltmeter may be comparatively low, the maximum value may be very considerable, hence some knowledge of the amplitude factor or the ratio of the maximum to the effective ordinate of the wave form is necessary. In Table XIV are some values for geometric forms of curve.

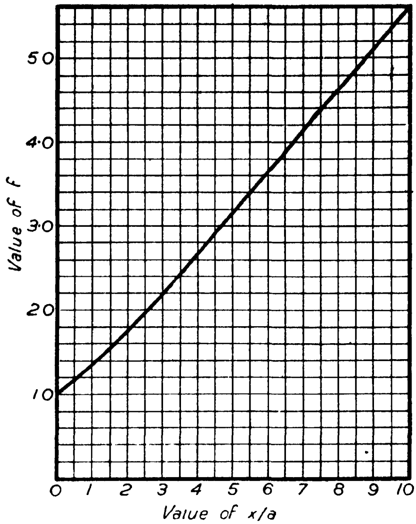


FIG. 5.5.—Russell's curve relating sparking distance and dimensions of sphere gaps.

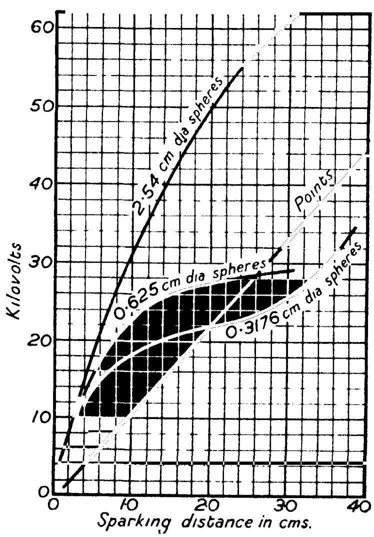


FIG. 5.6.—Effect of electrode form on sparking distance.

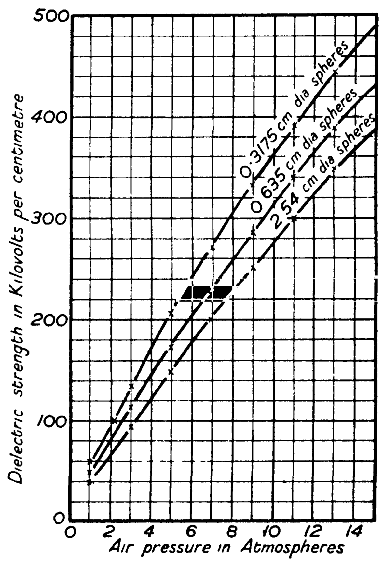


FIG 5.7.—Effect of gas pressure on dielectric strength.

TABLE XIV.

Form.	Amplitude factor.	From.	Amplitude factor.
Rectangular	1	Triangular	2
Semi-ellipse	1·22		
Semi-circle	1·22	Inverse sine	2·1
Sine	1·41	Inverse circle or ellipse	3·23

The amplitude factor employed to convert the root mean square values should be that which applies to the source of supply *when carrying the testing load.*

The dielectric strength of gases other than air in some cases is subject to curious variation. Thus hydrogen and oxygen at all pressures are not as strong as air; but while oxygen at atmospheric pressure is 95% of air strength, and falls to 90% at 5 atmospheres, hydrogen falls from 87% at normal pressure to 68% at 5 atmospheres of the equivalent air values. Carbon dioxide and nitrogen are stronger than air at normal pressures; CO_2 is 20%, and nitrogen 16% greater than air. At 5 atmospheres the former is only 2% greater than air at equivalent pressure, but nitrogen is much less affected by pressure, as it is still 14% above air at 5 atmospheres.

The importance of compressed gas insulation in high voltage instrument construction cannot be over-estimated, and great advances will probably be made in the future by employing this method of suppressing brush discharge in such apparatus.

Liquids

The bulk of the observations on liquid dielectrics refer to oils of mineral and vegetable origin, these being practically the only fluid insulators at present in use commercially. The specific resistance of oils is in general very high: at ordinary temperature values as high as $0{\cdot}52 \times 10^{12}$ ohm-cm. are obtained; but the figures for various grades of oil vary enormously from one to the other. The temperature coefficient is negative, and for mineral oils is in the neighbourhood of 1% per degree Centigrade. The heating and cooling curves connecting the specific resistance and temperature are, however, very different, the cooling curve showing

much lower values, probably on account of a molecular regrouping, occurring at the higher temperature, as shown in Fig. 5.8.

In testing the dielectric strength some diversity of opinion exists as to the best form of electrode to use in the testing cup. On the one hand it is claimed the more consistent and reliable results can be obtained by using needle points or a point and disc, and the bulk of the work in this country has been done with this form of electrode. On the other hand, in America spheres of 1 cm. diameter have been satisfactorily used for the purpose.

Moisture in the oil has a very marked influence on the dielectric strength. The curve in Fig. 5.9 is typical of this effect, and it

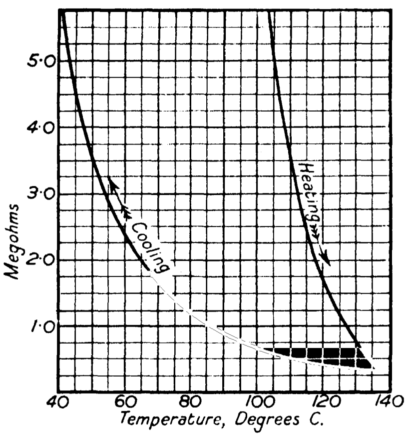


FIG. 5.8.—Specific resistance v. temperature for mineral oil.

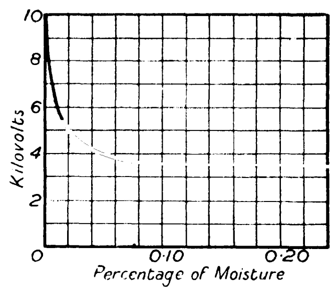


FIG. 5.9.—Effect of moisture on dielectric strength of oil.

will be seen that the addition of 0·04%, or 4 parts in 10,000 of water to the oil, is sufficient to reduce the dielectric strength of the oil by 50%, and cases have occurred in which chemical examination has failed to reveal the presence of water when it has been unfailingly shown by the electrical test. Dehydration is, therefore, always necessary, and this is effected by heating, or treatment with some dehydrating substance, such as lime, with a final filtration through dry sand, which may include an admixture of bone black and fullers' earth to clear the colour of the oil.

The dielectric strength of oils follows a law similar to that for gases, but is, for the same spark length, usually much higher (Fig. 5.10), while the effect of temperature is to increase the dielectric

strength practically according to a straight line law, and the coefficient for most oils is about 0·4 to 0·5% per degree Centigrade.

Owing to the diversity in composition it is impossible to give definite figures for the electric strength of oils, which may be anywhere between 70 kV and 170 kV per centimetre.

There is another troublesome point in the use of oils which must have attention, viz. the tendency to form deposits at moderate temperatures in the presence of air and metals. These deposits on analysis show a high percentage of carbon and oxygen, and a small percentage of hydrogen. The formation of these deposits

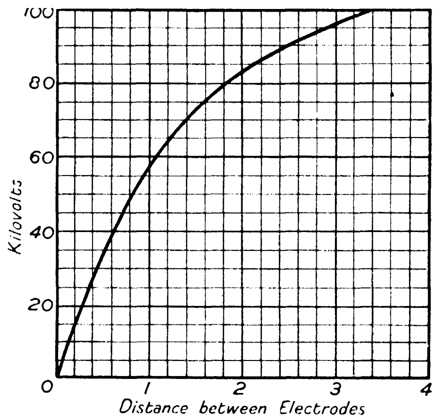


FIG. 5.10.—Dielectric strength of oil for various electrode distances.

is considerably enhanced by the presence of clean metallic surfaces, and particularly of copper. The copper does not appear to be attacked or corroded, but simply accelerates the rate at which the deposit is produced; on the other hand, lead, which is also an active metal, is corroded, and enters into the composition of the deposit, which usually takes the characteristic yellow colour of lead salts.

Solid Dielectrics

The majority of known solid dielectrics find their way into the manufacture of instruments in some way or other and can be broadly classified as follows:

Natural materials.
Synthetic materials.

Composite materials.
Ceramics.
Glass.
Varnish.

Natural Materials

The natural materials mainly used in the electrical industry are mica, asbestos, cotton, silk, wood, shellac, gums and rubber.

Mica.—Mica can roughly be divided into two classes, muscovite and phlogopite, the former being used for the majority of general purposes, whereas the phlogopite or amber mica is softer and may be used in heating elements where temperatures are high, and for commutator separators where it is necessary to build these with the separators flush with the running surface.

The majority of muscovite mica is mined in India, and is shipped to this country in the form of slab mica, which may readily be slit into a desired thickness; or in the form of mica splittings which are already split by the Indians to flakes about $\frac{1}{1000}$ in. thick and of varying sizes and shapes. The size of the splitting used naturally affects the price of the mica, the large flakes or pieces being of higher price than the smaller, since small-sized books or pads are found more frequently than the larger ones.

Mica has outstanding electrical properties which are not reproduced in any other material, in that it possesses high electric strength, low dielectric loss, chemical stability and resistance to heat ageing. Muscovite mica does not dehydrate under about 500° C., and is therefore used quite largely in the build-up of Class B insulation.

Amber mica has a somewhat higher dehydration point, and is used mainly in heating elements used in domestic irons, kettles, etc., though its occurrence in these articles is likely to be reduced owing to its high cost and the fact that the smaller grades of muscovite mica are capable of being built into micanite by the use of a suitable bonding medium, the ultimate product being effective as an insulant if the elements are properly designed.

The occurrence of mica in instrument work is limited, though it is used as a resistance former, particularly where the temperature attained by the resistance reaches a value which is higher than Class A materials can sustain.

Of more recent years, mica has been ground to a fine powder

and fused with lead borate glass to form a material known as "Mycalex," which has most of the good properties of mica, plus the fact that it can be moulded into accurate shapes and that it can be readily machined if so required. Its use is only limited by the fact that it is an inherently expensive material, though it does find many applications, particularly in high-frequency work.

Asbestos.—Asbestos is mined mainly in Canada, and in its raw state is of rock-like appearance, but when crushed becomes of a fibrous nature, and differs from most natural fibres in that the fibre itself is solid, whereas most vegetable fibres are roughly tubular in construction. In its initial form it contains considerable impurities, which may be divided roughly into two classes, magnetic and chemical. It is usually possible by processing to remove the magnetic particles almost completely, but the chemical impurities are far more difficult, and it has taken time for investigation, research and experiments to produce an asbestos which is of reasonably consistent insulation resistance. Even now there is a tendency for the tape to suffer a considerable reduction in insulation resistance in contact with moist atmospheres, and considerable care should be taken to ensure thorough impregnation and coating with insulation varnishes to maintain it in its best state.

In conjunction with suitable compounds it is often applied to conductors, and finds limited use in instrument manufacture where Class B insulation is essential.

Cotton.—Cotton is produced widely in America, Egypt and the Sea Islands, where it grows on specially-cultivated shrubs, is hand-picked, and subsequently processed by means of carding, spinning and weaving, into the forms we know it in everyday life.

The Sea Island cotton produces the finest and longest fibre, which gives the greatest mechanical strength, and which permits weaving of the finest fabrics. The output is limited, however, and the Egyptian and American cottons are used extensively in the electrical industry in the form of wire covering, woven tape and webbing, for many purposes where Class A temperatures are not exceeded. The woven material possesses excellent mechanical strength, good abrasion resistance, is readily absorptive of insulation varnish and is a reasonably cheap product which is readily accessible. It is used as an outside binding for many classes of instrument coils, and finds an extremely wide use in most electrical fields.

Silk.—Silk fibre in its initial state is the excretion of the silk-worm, which uses it as a protection for the eggs which are laid. These bunches of fibre are collected and can be readily carded, combed and spun into the threads suitable for weaving. The fibre size is extremely small and extremely strong, and in consequence very thin fabrics can be woven possessing many admirable qualities. The main source of supply is Japan, where a large industry has flourished for a number of years. Industries do, however, exist in China, India and Syria for the production of silk.

The original fibre is coated with a waxy form of lubricant, which has good insulation properties, and it is chemically stable, so that little treatment of the fibre is necessary to permit of good fabric production. The electrical properties of silk fabrics are somewhat better than those of cotton, and as its thinness permits of better space factor, it may be used in preference to cotton for technical considerations, even though it is more expensive.

Wood.—Hard woods, such as teak, beech, maple, and latterly, peroba red, are chiefly used in the electrical industry, though their popularity is in general on the wane, since densified and synthetic resin-treated woods and papers are frequently taking their place. They do, however, hold some attractions, in that they are easily worked, and if properly treated are stable and reasonably permanent.

Wood is rarely used as a primary insulation, and in fact, its use as such should be avoided, since even if it is possible to obtain timber which is felled at the right time of the year and is properly seasoned in the natural manner, there is still a likelihood that streaks of conductive materials, such as sap, sugar, etc., may be trapped in the timber itself, which will lead eventually to breakdown or excessive leakage even at low voltages.

Lac.—Lac, or shellac, as it is more popularly known, is the excretion of the lac insect, *Laccifer lacca*, belonging to the family of Cochidae, which are similar to the Cochineal insect. It is parasitic of several types of tree mainly found in India and Burma. The tiny insects or larva emerge from the female of the species and crawl to the fresh green shoots of the tree, where they fix themselves and suck the sap from the twigs. The insects cover themselves with a protective coating, and finally grow sufficiently to emerge and swarm to fresh twigs, where the propagation of the insect is recommenced. The secretion they leave behind, which usually builds itself up, layer by layer, round the twig, is known

at this stage as " Sticklac " and is a basic raw material, from which, by various refining and cleaning processes, the common shellac is made.

The material possesses thermo-plastic properties and, indeed, was one of the first truly thermo-plastic materials to be employed in the electrical industry. If properly cleaned of its impurities it is almost completely soluble in alcohol and makes a very useful varnish. It has high electric strength and other desirable properties which are very widely appreciated in the electrical field.

Resin.—Resin is an exudation from coniferous trees, and in some instances dug from the earth, where it has accummulated as natural exudation from the pine, or it may be taken from the trees by incision. The names of the various types of copal are legion, but those most used in insulating varnishes are the manila types. Resins are mixed with oils, etc., often under a comparatively high temperature, to produce the medium from which insulating varnishes are manufactured. Considerable secrecy is kept by the varnish makers as to the precise ingredients used.

Oils.—Majority of oils used in the production of electrical insulating varnishes are of the vegetable type, the main ones being linseed, tung and castor oils. Linseed oil is derived from the flax plant, found extensively in the Argentine, U.S.A., Russia, India, etc., and the oil is mechanically crushed from the seeds and subsequently processed to remove undesirable ingredients which make it unsatisfactory as a varnish constituent.

Tung oil is derived from the nuts of trees, which grow mainly in China, though there are small quantities produced in South Africa, Kenya, Australia, New Zealand, etc. It is generally assumed that the original Chinese oil is the most suitable. The oil is notable for its water resistance and for its excellent electrical properties; it also has the advantage over linseed oil in that it can be polymerized, whereas linseed oil requires to be oxydized, and therefore requires the presence of air to attain a fully dry state.

Castor oil is derived from the seeds of a plant by mechanical means or by extraction. It is a non-drying oil, and is therefore used in conjunction with other oils and resins to produce varnishes of a greater degree of flexibility than is readily obtainable with other types of oil.

Rubber.—Rubber is derived from trees by incision and allowing the sap containing the crude rubber to exude. This is subsequently collected and treated to make pure latex, which is the basic material from which the rubber compounds are made for covering wires and cables, etc., as used in the electrical industry and many other industries. It has excellent insulating properties if properly compounded, and the recent admixture of synthetic rubbers has further widened the scope of its use.

Synthetic Materials

One of the oldest and in some ways the most-used synthetic resin is phenolformaldehyde, which was discovered by Dr. Baekeland in the early part of this century. The combination of phenol and formaldehyde in the presence of a catalyst, at a temperature of 130° C., produces a hard, infusible, insoluble resin, which has good electrical insulating and water-resisting properties.

Many other resins employing similar fundamental processes have been evolved, such as aniline formaldehyde, urea formaldehyde; all have their own uses and their special properties. The majority of them found their way into the electrical industry in the form of mouldings or as rods, sheets, or boards, all of which can be machined or moulded to the desired shape, and with suitable equipment can be made of good appearance and having a long life under adverse conditions.

The vinyl resins, which are a synthesized product of coal and lime, are increasing in use and popularity, and appear in the form of polyvinylchloride-covered wires, sheets, tubes, etc. Properly blended with suitable plasticizers they are practically non-ageing, and can be used under varying conditions of temperature and humidity with considerable success.

Polythene.—This is again a product of coal and lime from which ethylene is extracted and subsequently polymerized under conditions of extremely high pressure and temperature; the result is an opaque wax-like material with exceptionally good electrical insulating properties. It can be moulded, extruded, or rolled into thin films and is easily machinable.

Nylon.—Nylon is a synthesized protein having an almost identical chemical formulation to natural silk. This material is drawn into threads, and is subsequently woven into fine fabrics with good

insulating properties, of exceptional mechanical strength. It may also be used in a liquid form as a coating for electrical conductors in much the same way that conductors are enamelled. The resulting film is considerably tougher than natural oil enamels, and is not so adversely affected by varnish solvents.

Methylmethacrylate.—More commonly known as " Perspex " in this country, this material is a thermo-plastic of very wide application. Its great clarity makes it a useful decorative material, but it is used extensively for aircraft lights and even for lenses. It is also used as " unbreakable glass " for instrument faces, etc.

Synthetic rubber.—Great strides in the production of synthetic rubbers have been made in the past few years, and there is little doubt that it is now possible to synthesize the materials which will give comparable results with compounds of the natural materials. The price, however, is often prohibitive for normal commercial applications. Where specialized treatment is required, however, the synthetic rubbers are superior to the natural, in that they can be made to possess a greater heat resistance and be more resistant to solvents, oils, etc., used in the electrical industry for impregnating and varnish treatment processes. Extensive research and development continues on this material, and there is little doubt that as time goes by, synthetic rubber will be used more extensively still in all branches of the electrical industry.

Polyvinylchloride.—The vinyl compounds have an enormous range of application in the electrical industry generally, apart from the very many domestic applications which it is certain will be seen in the future.

Properly compounded with suitable plasticizers the material is non-ageing and rubber-like in consistency. It can be varied from soft, spongy material to hard sheet or tube. It has excellent electrical properties, is resistant to most commercial solvents and chemicals and can, therefore, be used with advantage in almost any situation.

The compounded material has a usable temperature range of about 90° C. ; it can, therefore, be made suitable for operating temperatures down to − 20° C., providing the top temperature does not exceed 70° C. By the use of different plasticizers, however, this temperature can be shifted from, say, 0° C. to 90° C., and therefore the material is very flexible in its application, but care

should be given in the choice of particular grade required as determined by the conditions in which it will have to work.

Cellulose acetate.—Cellulose acetate is usually produced from pure cellulose, which is a natural material, usually taken in the form of pure cotton linters, which are processed by an acid treatment to produce a viscose liquid, which, with the addition of suitable plasticizers, can be formed into sheets and rods, etc. From this stock material can be produced thin, transparent sheets, tubes and films, which by suitable processing can be made into useful electrical insulating materials. Cellulose acetate has a high degree of resistance to water, and is used quite frequently for wrapping round coils to make them tropic-proof.

Composite Materials

Micanite.—Micanite is made up either by hand or by machine process, from mica splittings previously described, bonded together by means of a suitable varnish, such as shellac, flexible varnish, etc., and may or may not be supported by a fabric backing, dependent upon the eventual use to which the micanite is to be put. The best qualities of moulding micanite are hand-built, but good quality material is produced on suction towers or snowflake towers, which have been described in other publications or in patent specifications. By varying the bond used for obtaining adhesion between the mica flakes various types of micanite can be produced, such as moulding micanite, where shellac is usually used as the bond; flexible micanite, which employs a long oil flexible varnish; heat-resisting micanite, which may be made up with shellac or glyptal varnish, etc. Special micanites for use as commutator separators are usually made up with shellac, the solid content being maintained at about 5%, and the sheet being pressed at something approaching a ton per square inch, to ensure that the material will not subsequently slip during processing or running of the commutator.

Moulding powders.—The vast majority of plastic materials, whether these are thermo-plastic or thermo-setting, are made in the form of a moulding powder, which, dependent on the type of material in question, can be used for pressure or injection moulding. Probably the most common of these types of moulding powder is the one made up with phenolformaldehyde or cresoformaldehyde blended with suitable materials, such as asbestos,

wood flour, mica, etc., to make the familiar domestic mouldings, such as ash trays, telephones, etc., but these materials also find a wide application in instrument cases, bases and the various small parts included in the instrument.

With suitable moulds such powders are easily applicable to mass production methods, and excellent service results have been obtained with moulded instrument parts where these have been correctly designed and the correct grade and quality of material employed. It should, however, be noted that many pitfalls exist for the unwary, and fullest details of the service conditions likely to be met should be investigated before deciding on the grade of material to be used. Injection mouldings from materials like cellulose acetate, nylon, polyvinylchloride, polythene, etc., are also used extensively.

Varnished papers.—A wide range of varnished papers are made for use in the electrical industry; the majority, however, are employed in the manufacture of bakelized paper tubes and boards, where the paper is either coated or impregnated with phenol or cresol formaldehyde. These are subsequently rolled into tubes or pressed into sheets. With a wide range of electrical and mechanical properties, the designer should choose the most suitable type for the application in mind.

Other papers are coated with insulating varnish, and are used as interleaves between layers of instrument coils, though in general this practice is diminishing, since it is found that bare paper can be effectively employed where the coil is adequately sealed or impregnated subsequent to winding.

Ceramics

The manufacture of porcelains is one of the oldest industries in the world, and continues on fundamental lines as it did some thousands of years ago as practised by the Chinese civilizations of those days.

Specialized control is necessary, however, for electrical ceramics. The process of making porcelains briefly consists of grinding the basic materials, which include clays, felspar, etc., into finely-divided particles, mixing with water, forming to shape whilst the material is in a clay-like state, drying to remove excess water, and subsequently firing at a high temperature to produce the hard, close-grained porcelain which is used throughout the electrical

industry. Subsequent to this process, the exterior of the product may be glazed to give additional weather-proofing properties, but it is sometimes the practice to use unglazed porcelains for resistance bobbins and for various parts of instruments.

Glass

The art of glass making is, like ceramic production, of ancient lineage, but research and development over the past few years have produced types of glass which are particularly suited in the electrical industry, as a result of which it is not uncommon to see glass pin type insulators for transmission lines and other numerous applications, where solid glass is particularly suited in place of porcelain or other materials.

The development of a suitable glass and a suitable process for extruding that glass into fine filaments has also been developed in recent years, with a result that it is now possible to manufacture woven fabrics and tapes from about ·002 in. thick upwards. These materials possess great tensile strength, permanence and resistance to heat ageing at the highest temperatures likely to be met in electrical machinery. The yarns spun from the glass fibre, each filament of which is approximately ·0002 in. diameter, can also be applied to wires to give an excellent Class B insulated conductor. The advantages of this material are primarily its resistance to heat ageing and its inability to absorb moisture. Since suitable varnishes have been developed for use in conjunction with woven glass, it is possible to some extent to take advantage of the above properties, and since the material also has great mechanical strength, its advantages in application to electrical windings can readily be appreciated.

Varnishes

Shellac.—Shellac is one of the chief materials used in varnish production for the electrical industry generally. It is soluble in commercial alcohol, and it is in this form that it is mainly used. It can be adulterated by means of cheaper resins, but this is generally considered undesirable; it can also have some of its properties changed by the addition of plasticizers, but here again in the majority of electrical applications this is to be avoided. The process in varnish-making is exceedingly simple, since the

shellac and solvent are placed in a wooden vat and are mechanically agitated together into a solution. Varying specific gravities can be obtained to suit the application by the addition of further solvent.

Flexible varnish.—This material is made up on conventional varnish practice of heating the constituents in a varnish kettle to the necessary temperatures to ensure adequate mixing, and subsequently breaking the viscosity down by means of a volatile thinner such as can be obtained from the petroleum of coal tar extract or alcohol.

Impregnating varnish.—The procedure for manufacture is similar to that employed in the making of flexible varnish, but, of course, different constituents are used to give the desired effect of thorough drying which usually accompanies a baking operation. Quite naturally there is a tremendously wide range of such varnishes, and some care is necessary in the selection of the correct material to suit the job to be treated.

The materials briefly described above are all available from numerous sources, and it is in the engineers' interest to make sure that the best possible quality consistent with price limitations is obtained.

Numerous British Standards have been current for some time governing the requirements of the standard of some of these materials, and these are quoted below for general guidance :

B.S. 7. Rubber and P.U.C. insulated flexibles and cables.
,, 119. Clear baking oil impregnating varnish.
,, 148. Insulating oils.
,, 156. Enamelled cu. wire.
,, 216. Vulcanized fibre.
,, 234. Ebonite for electrical purposes.
,, 419. Varnished cloth and tape for electrical purposes.
,, 474. Synthetic resins for manufacture of boards.
,, 488. Moulded insulating materials.
,, 514. Baking insulating varnishes.
,, 547. Synthetic resin bonded paper sheets (Grade 1).
,, 316. Synthetic resin bonded paper sheets (general purposes).
,, 626. Micanite for commutator separators.

B.S. 633. Cotton tapes and webbings.
,, 634. Finishing air-drying insulating varnish.
,, 698. Papers (unvarnished) for electrical purposes.
,, 737. Non-ignitable and self extinguishing boards.
,, 738. Non-ignitable and self extinguishing properties of solid electrical insulating materials.
,, 771. Synthetic resin moulding materials and mouldings.
,, 934. Vulcanized fibres, rods and tubes.
,, 954. Lac.
,, 972. Synthetic resin bonded fabric sheets.

The majority of the more recent thermoplastic materials have not as yet formed the subject of British Standards, and some indication of the insulating properties to be expected are given in the following table :

TABLE XV.

Material.	Breakdown voltage v/mil.	Permit-tivity.	Power factor.	Volume resis. (ohms./cm.).	Water absorp-tion.
Methylmethacrylate	390	3·4	·05	10^{15}	·3%–·4%
Polyvinyl chloride	600	6·2	·12	10^{11}–10^{14}	·3%–1%
Nylon	..	6	·1	10^{13}	7%
Polythene	1,000	2·3	·0001	3×10^{17}	*Nil*
Cellulose acetate	1,000	6	..	5×10^{6}	..

The following table gives a general guide to the properties of some dielectrics :

TABLE XVI.

Material.	50 c.p.s. Dielectric strength (kV/mm.).	Specific resistance at 25° C. (Ω.cms.).	Approx. permittivity.
Asbestos	3–5	$1{\cdot}6 \times 10^{5}$	..
Rubber	10–25	2 to 10×10^{8}	2–3
Mica	40–150	5 to 100×10^{6}	5–8
Micanite	30	10 to $6{,}000 \times 10^{6}$	6–8
Mycalex	20	10^{10}	6–7
Porcelain	9–20	1 to $1{,}000 \times 10^{6}$	4–7
Glass (plate)	5–12	5×10^{6} to	3–8
Shellac varnish	5–20	9×10^{9}	3
Flexible and impregnating varnishes	Up to 60	Similar to shellac varnish	Similar to shellac varnish

MAGNETIC MATERIALS*

Under this heading are included only such substances as are generally classed as para- or ferro-magnetic, and these are therefore limited to the various grades of iron, steel and the alloys of these materials, with molybdenum, tungsten, silicon, vanadium, and chromium. Nickel and cobalt are also magnetic, but only feebly so compared with iron ; and the alloys of Heusler, consisting of manganese, copper and aluminium, are also ferromagnetic.

Langevin classified bodies according to their magnetic behaviour into three classes, viz. :

(i) Diamagnetic, or those whose permeability was less than unity.
(ii) Paramagnetic, or only feebly magnetic substances.
(iii) Ferro-magnetic, or strongly magnetic bodies.

In Class (ii) Langevin supposed the susceptibility to be independent of the field intensity, and inversely as the absolute temperature, whilst bodies in Class (iii) have a susceptibility which is a complicated function of both the field intensity and the temperature.

The most important magnetic materials are, of course, iron and steel, and these, as commercial products, are not simple bodies, since they contain under normal conditions carbon, both in a combined and free graphitic state, together with small quantities of sulphur, silicon, phosphorus, and manganese.

The carbon constituent has a very important influence on the magnetic behaviour of the resultant material as well as upon its mechanical properties.

Formerly it was usual to classify by their carbon content ; thus " iron " was the name reserved for that class of material whose carbon content was largely in the graphitic state, rendering the material soft and ductile. On the other hand, materials containing more than 2% of carbon, of which a large proportion was in the combined condition, and which possessed the property of hardening on quenching, were classified as " steels."

Of recent years the line of demarcation between these two classes of materials has become less pronounced ; with the introduction

* This section is based on the material which appeared in the first edition of this work. Later developments in magnet materials are dealt with in the next section.

of mild steels, which are irons with remarkable low carbon contents, the classification no longer holds.

Hardenable steels fall broadly into two great groups, viz. :

(*a*) High carbon steels, containing 0·3 to 1·5% of carbon.

(*b*) Alloy steels, which have in addition to carbon relatively high percentages of such metals as tungsten, chromium, etc., mentioned above.

All these hardenable materials possess the remarkable property of retaining permanently a large proportion of the magnetism communicated to them, or, in other words, are materials from which permanent magnets can be constructed.

Apart from the nature of the material itself, the permanence of a magnet, when once magnetized, is dependent upon the following :

(*a*) The heat-treatment of the material.

(*b*) The shape and proportions of the magnet.

(*c*) The method of magnetization.

(*d*) The maturing or ageing process.

(*e*) The subsequent treatment as regards temperature change, vibration and external fields.

If a sample of material be subjected to a magnetizing force which can be gradually increased, it will be found that the magnetism at first increases rapidly as the force increases, but after a certain stage is passed further increases in the magnetizing force produce gradually-diminishing amounts of magnetism, until, when very intense fields are reached, the increase of magnetism becomes insignificant, or the iron has become "saturated" ; if now the magnetizing force be gradually withdrawn, the magnetism does not fall as rapidly as it increased ; it lags behind the magnetizing force, and to this property Ewing gave the name of "hysteresis." Eventually it will be found that when the magnetizing force is entirely withdrawn, the sample still retains a large proportion of its initial magnetism, and the amount of this remanent magnetism measures the retentivity of the sample.

Now, from the point of view of the magnet maker, it is desirable that this retentivity should be high, but it by no means follows that because a material shows a high retentivity it is a good material for the manufacture of permanent magnets, for it is also necessary that the remanent magnetism should be tightly held by the material.

Some very soft and permeable materials may be magnetized to very high values, and on withdrawal of the magnetizing force exhibit remarkably high retentivity; but the least vibration or reversal of field will cause the whole of the retained magnetism to disappear and leave the sample practically unmagnetized. If, therefore, we wish to examine how tightly the magnetism is retained, all that is necessary is to find the magnitude of the reversed force that is required to demagnetize it, and this measures the "coercive force," and the higher the value the better the material for magnet making. Thus in the choice of a material we require two properties. Firstly, a high retentivity, and secondly, a high coercive force. Unfortunately these two quantities are somewhat opposed, for the high coercive force can only be attained by the special properties and hardness of the material, and when these are pushed to high values it becomes difficult to raise the intensity of magnetization to a very high initial value, and hence the retentivity may be comparatively low.

Although, therefore, we may make a very permanent magnet, it will not be a very powerful one, and so it is necessary to compromise and leave the steel at a lower temper in order that we may get the required strength, and sacrifice some of the coercive force.

The majority of the alloy steels are better in this respect than ordinary high carbon steel, and are therefore much more suitable for permanent magnet construction. A criterion by which a steel may be judged as to its suitability for magnet construction may be obtained by taking the product of the coercive force and the remanent intensity of magnetization, and this should be a maximum for the best material.

S. P. Thompson suggested in 1910 that the ideal material should have a value of I_{rem} 800 and H_c 80, and until quite recently no steel approached these values. More recently the cobalt steels, which exhibit a coercive force of 246, with a value of B_{rem} 10,400, surpassed the ideal. Such steels have been made available by the Cobalt Magnet Steel Co., but on account of the high price of cobalt, they are expensive. It is, however, important to remember that with these steels, in order to get the best results, it is necessary to apply much higher magnetizing forces—1,000 to 2,000 H, on account of the high reluctivity of the material. Cobalt steel magnets can be built up of a number of straight sections, thus

avoiding the need for forging, though they can be forged to any shape desired.

S. Evershed,* in a classic paper to the Institution of Electrical Engineers, attacked the theory of permanent magnets in a very complete and ingenious manner, basing his theory on the assumption that the so-called magnetic molecule behaves as an elementary electronic current ring, capable of rotation about a pivotal axis through a diameter. He thus combined the original molecular circuit hypothesis of Ampère with the Ewing molecular theory, and showed that a uniform distribution of such magnetic molecules would not really represent the conditions found in iron and other ferro-magnetic bodies, since with such a distribution there cannot exist intermediate states of equilibrium between the unmagnetized and fully magnetized conditions.

By simply modifying the distribution, however, and gathering the magnetic molecules into individual groups, more or less detached from neighbouring but similar groups, the necessary conditions can be fulfilled, providing we also suppose that these groups form tribes or companies within certain boundaries—a condition which he suggested was indicated by the crystalline nature of ferro-magnetic bodies, where each minute crystal constitutes a tribal volume.

In very soft and permeable material the gathering together is very slight, while in hardened and alloy steels it is much greater, but even in such materials it is still very far from the limit of molecular accommodation, so that it is not unlikely that if it is possible to bring about a still greater group concentration, materials of much higher coercive force would be available. But, on the other hand, it does not follow that such materials are necessarily useful for the production of permanent magnets unless at the same time they are capable of maintaining the required magnetic energy, which is an affair of the intensity of the molecular current and the number of magnetic molecules in unit volume.

The science of metallography has of recent times thrown much light on the influence of structure in the various forms of steel upon its magnetic behaviour; by micrographic means the complex constituents may be studied at the same time as the physical properties of the material. By these means the constituents

* S. Evershed, "Permanent Magnets in Theory and Practice," *Inst. Elect. Eng.*, 1920.

most influencing the magnetic qualities have been observed during their formation and transformation under heat treatment, and the importance of the carbon contents so studied.

It is interesting to note that the modern theory of ferromagnetism very closely resembles that put forward by Evershed. The modern theory attributes the ferromagnetic effects to groups of electrons called " domains," which consist of electrons spinning on their own axes. The magnetic axes of the spinning electrons in any one domain are held parallel to each other by mutual forces known as exchange forces so that each domain behaves as a single unit. These domains are, in essence, the current rings of Evershed, and account for the magneto-motive forces inherent in ferromagnetic materials. When the material is unmagnetized, the domains are arranged in various orientations so that the total magnetic effect is zero in any direction. Under the influence of an external magnetic field the magnetic axes of the domains are more or less oriented in the direction of this field, so that their effect is added to that of the applied field. When the magnetizing force is removed the orientations produced by it tend to persist, and the amount to which they persist is dependent on their distribution in space, which again depends on the internal structure of the material. This feature gives the remanent magnetism, and a reverse magnetizing force is necessary to upset this orientation and to reduce the induction to zero.

Dr. Carl Benedicks also studied the influence of the carbon contents on the coercive force, and the curve in Fig. 5.11 summarizes his observations, and clearly shows the obvious advantage of high carbon steels for permanent magnets.

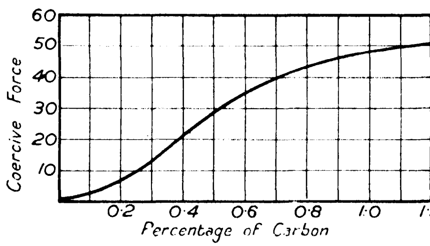


Fig. 5.11.—Influence of carbon content on coercive force.

The influence of the introduction of certain metals into the steel has already been noted, and this process is the basis of the so-called self-hardening tool steels, which remain hard even when hot and without quenching.

The behaviour of manganese as an alloying constituent is, however, peculiar. The curves in Fig. 5.12 due to Barrett, Brown and Hadfield illustrate the effect of this material, and from them

it will be observed that a steel containing 13% of manganese is practically non-magnetic, and yet the material is of extreme hardness. Several other non-magnetic steel alloys were investigated, consisting of steel with various admixtures of manganese, nickel, tungsten and chromium. On the other hand, it is possible to produce alloys of iron more magnetic than the purest commercial iron. Thus 2·5% of silicon or aluminium added to the steel produce a remarkable and highly permeable material, which, on account of its relatively high specific resistance, is valuable for the laminated cores of alternate-current magnets and transformers.

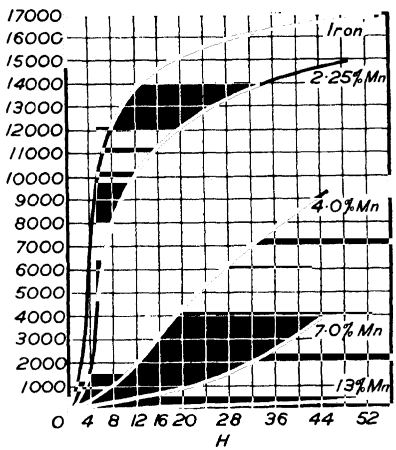


FIG. 5.12.—Effect of manganese alloy in steel.

The requisite heat treatment, in order to obtain the best magnetic results from the material selected for the construction of permanent magnets, must be considered. The first point to be noted is that for most steels when heated to a temperature above 700° C. their magnetic properties cease to exist, and they can no longer act as magnets. They, however, regain their magnetic quality when the temperature falls below 680° C. to 690° C.

Now, if we study the time temperature curve for such bodies while cooling, we shall find that the curve is not a smooth one, but that there are kinks occurring at definite temperatures, indicating a pause or halt in the fall of temperature, or even an increase of temperature. These are known as recalescence or transformation

points, and indicate a change occurring in the nature of the constituent bodies, and an evolution of the latent heat. Above the lowest recalescence point the material is non-magnetic, and in order that it may be satisfactorily hardened, it must pass as rapidly as possible through this point. Heated and quenched from a temperature below this, it will not perfectly harden and may not harden at all.

The reason of the hardening process is as yet by no means clear, but it seems to be that the most generally accepted theory is that the hardenite constituent is arrested and not allowed to transform into the softer pearlite equivalent, as it would otherwise do if allowed to slowly cool.

On heating the material the time temperature curves show similar pauses to those on the cooling curve, except that in general they are higher, and the highest of these, corresponding to the transition state between the magnetic and non-magnetic state, is the temperature to which the steel should be heated before quenching.

From what has already been said, the rate of quenching is obviously important ; of all the methods available, the water spray appears to be the most efficient, saline solutions and cold water come next, and mercury, linseed oil, boiling water and molten lead in the order named.

With regard to tempering or slightly softening the steel after hardening a compromise must be effected, and little real advantage is obtained by letting the material down too far, for although in the less hard material I_{rem} may be higher, the coercive force becomes disproportionately lower, and in general it is better not to go below a " straw tint " when great permanence is sought, whilst with tungsten and alloy steels generally there is no advantage in tempering.

According to Sir R. Hadfield, metallurgists hold that the inclusion of tungsten in steel does not of itself harden the iron, but helps to prevent the carbon present from segregating or separating as cementite, and maintains it in the condition of hardening carbon. Certainly the presence of tungsten does not allow of the formation of large crystals in the structure, and promotes a fine-grained fracture, and it is probable that chromium, manganese and vanadium act similarly.

Tungsten steels have been used by all magnet makers. In France, Allevard and Marchal steels have been exclusively used, while in Germany, Remy steels have been most popular.

The process of magnetizing for permanent magnets is invariably carried out by means of an electric current. Long bars are magnetized in a long solenoid, and short bars and horse-shoe shapes by placing them between the poles of a suitable electro-magnet. In the first case the coil should be at least twice as long as the bar to be magnetized, and in either case the value of the field strength should not be less than $H = 250$, and in the solenoid there is no difficulty in getting a field well above this value. In the case of the electro-magnet, Dr. Thompson advised that it should be so proportioned that there are not less than 500 ampere-turns per in. of magnet steel available, and the higher this value can be pushed the better. $H = 500$ or about 1,000 amp.-turns per in. should be used for tungsten steel magnets.

The duration of the magnetizing force is of very little consequence, providing time is allowed for the current to reach its maximum value in the coil, since the resulting magnetism depends upon the maximum value attained, but a slight advantage is obtained by successive short applications, and mechanical tapping during the process may also produce a slight increase; it is also better to break the current slowly, in order to obviate the demagnetizing effects of the oscillations likely to occur on the sudden rupture of a highly-inductive circuit. When the gap of the magnet is short and the pole surfaces large, considerable leakage of the magnetizing flux may occur across the gap. To minimize this, the British Thompson-Houston Co. resorted to the ingenious device of lowering a rapidly rotating copper disc into the gap during the process of magnetization; the eddy currents induced in this disc thus oppose the tendency to leakage, and drive the flux round the steel loop. After magnetization for some time the magnet is somewhat unstable, and requires ageing or maturing, in order to bring it into a steady and constant condition, and remove the sub-permanent or temporary part of its newly-acquired magnetism. The processes adopted for this purpose may be summarized as follows:

I. By repeated heating and cooling.
II. By protracted gentle heating.
III. By subjection to repeated mechanical shock.
IV. By partial demagnetization.

Barus and Strouhal examined the second method, and showed that after six hours' exposure to the temperature of steam the

remanence reached a constant value, while during the first four hours there was a steady and tolerably rapid fall. They further observed that the final remanence was the same, whether the steaming occurred before or after magnetization.

Barus found that Stubbs' steel rods lost about 10% of their hardness in three years if they were initially glass hard, and this same loss is occasioned by three hours' immersion in steam, and concluded that the effect of atmospheric temperature action for years could be artificially reproduced by steam acting for the same number of hours.

J. Coulson has shown that the ageing process may be rapidly and effectively carried out by exposing the magnet to the effect of nascent hydrogen, making it the kathode in a bath of 25% sulphuric acid. The fall in magnetic moment does not occur at the same rate in all kinds of steel, but with "Special Alloy Steel" a fall of 19% occurs after a few minutes' exposure. Heating the electrolyte accelerates the ageing, and in a bath heated to 60° C. the point of stability is reached in an hour or less.

The dimensions and shape of permanent magnets are of extreme importance. Short bars, cubes, spheres, etc., even of hardened steel, have comparatively little retentivity, since the poles of every magnet exercise a self-demagnetizing influence. In bar magnets the poles are always acting upon one another, and exercising demagnetizing forces on the material of the magnet between them. The shorter the bar, therefore, the greater the demagnetizing influence. If H_d is the demagnetizing field due to the poles, its value is proportional to the intensity of intrinsic magnetization I_m; hence,

$$H_d = K \times I_m$$

where K is the coefficient of demagnetization.

Thompson and Moss determined the value of K for various forms of cylindrical bars, and their results are shown in Fig. 5.13. The importance of the dimension ratio is, of course, very great in the construction of galvanometer magnets, where the ratio is necessarily small, and consequently the demagnetizing coefficient great, and therefore any modification in design which will allow of a greater ratio without reducing the sensitivity is of great importance.

Split rings and closed forms of magnet were shown by Du Bois

and Lehmann to have a demagnetizing effect nearly proportional to the width of the gap, and if there were no magnetic leakage it would be strictly proportional, providing the gap is small compared with the radius of the ring ; the demagnetizing coefficient is given as about 0·035 per degree width of slit for the case of a simple ring, with a saw-cut through it at right angles to its plane ; if, however, the poles are enlarged by pole pieces, the demagnetizing coefficient will be very greatly reduced. In general, if we call

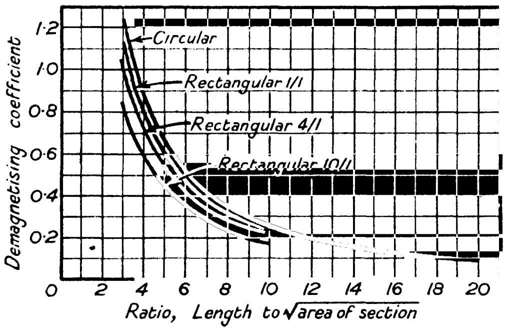


FIG. 5.13.—Demagnetization coefficient K for various forms of cylindrical bar.

l_m the length of the mean magnetic path in the magnet and l_g the length of the air-gap, A_m the area of cross section of the magnet and A_g the pole face area, then we may write the demagnetizing coefficient K,

$$K = \frac{4\pi}{v}\left\{\frac{l_g}{l_m} \cdot \frac{A_m}{A_g}\right\}$$

where v is the leakage factor of the magnet or ratio of the flux in the body of the magnet at its middle point to the useful flux in the gap.

From this it is at once obvious that if K is to be small, both l_g and A_m must be small, and l_m and A_g as large as possible, and there is also an advantage in a large leakage factor.

The ratio of the length of the magnet to its cross-section divided by the ratio of the length of the air-gap to its cross-section has been called by Heinrich and Bercovitz the " safety factor " of a magnet, and for commercial instruments this may lie between 130 and 500, the most usual value being 300 to 350 in the best modern

indicating instruments. The value for meter brake magnets is, however, usually much lower, owing to the necessity for compactness, and values between 100 and 200 are here quite usual (see Part II). With cobalt steel magnets, owing to their high coercive force, very much lower values of this factor may be employed.

Predetermination of Permanent Magnets.

S. Evershed, in his paper already referred to, has, however, shown that it is not sufficient, from the point of view of the economical design of a magnet, to be simply guided by the remanence and coercive force of the steel, but that much depends on the shape of the demagnetizing curve.

Now this curve really represents the relation between that part of the magneto-motive force per centimetre length of the steel which is not expended in overcoming its reluctance, and hence is available for producing a field externally (in S. Evershed's notation this is Φ and the induction in the steel β).

If, now, we desire to maintain a magnetic field by means of a permanent magnet, for instance, in the gap space between two pole pieces of strength H, the magnetic energy per unit volume in this space is $H^2/8\pi$ (see page 167), or, if A is the area of the pole face and g the length of the gap, the total magnetic energy is

$$\frac{H^2Ag}{8\pi} \text{ or } \frac{HA}{8\pi} \times Hg$$

Now HA is the total useful flux in the gap, and Hg is the magneto-motive force. And hence the useful energy in the gap, in 8π units, is $HA \times Hg$ ($= B_u \times V$ in Evershed's notation). Now to the useful flux in the gap we must add the leakage flux from the pole pieces, which does not pass through the gap, and the product of this total flux into the terminal magnetic potential gives the magnetic energy, which must be supplied by the magnet to be designed.

That is, if Φ_1 is this total flux, and V_m the magnetic potential, the magnetic energy is $\Phi_1 V_m$. Hence, if A is the cross section of the ideal magnet and l its length

$$\Phi_1 = \text{induction} \times A, \text{ and } V_m = \text{surplus M.M.F.} \times l,$$

or, in terms of the co-ordinates of the demagnetizing curve for the

steel, $\Phi_1 = B_r \times A$ and $V_m = H_r \times l$, where B_r and H_r are the values of these co-ordinates of the demagnetizing curve at the point selected.

The matter is, however, further complicated by two considerations. Firstly, by the leakage which naturally takes place between the limbs of the magnet, independently of the leakage already considered at the pole pieces themselves; and secondly, when this is allowed for, what shall be the correct point on the demagnetizing curve selected to give the most economical magnet ?

Now, since we have that $\Phi_1 \times Vm = B_r H_r Al$, then $Al = \frac{\Phi_1 \; V_m}{B_r \; H_r}$, and since Al is the volume of the steel in the magnet, we must seek a value of the product $B_r H_r$ which, when divided into the energy requirement of the apparatus, will make Al a minimum.

If we take the demagnetization curve for a steel, and work out the product of $B_r H_r$, and plot it against the corresponding values

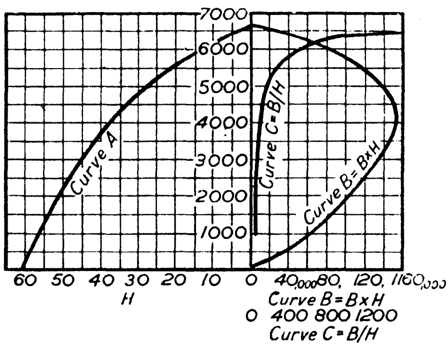


FIG. 5.14.—Curves for determining optimum design of permanent magnet (S. Evershed).

of B_r, we obtain a curve like that shown in Fig. 5.14, curve B, which has a decided maximum for some value of B_r, and therefore, in the ideal case, it would be most economical to work at the value indicated by this. Unfortunately, however, it is not possible to do this, because of the magnetic leakage of the magnet itself; and since we usually work with the steel of uniform cross-section, this leakage involves a change in the value of B as we pass from the centre of the yoke, or neutral section of the magnet, towards either

pole ; it is therefore necessary to select a value of B_r somewhat above the most economical point, so chosen that the terminal value is sufficient to maintain the energy requirement ; and at a point somewhere in the limbs the most economical value is attained, or in other words, we must work over the economical region. The most important question, therefore, to be settled is, what point of the demagnetizing curve must we select ?

In this connection there is one relation which is of interest : the ratio of $\frac{l}{A}$ represents the reluctance of the magnet itself, and if we call K_g and K_p the magnetic conductances or permeances of the gap and pole pieces respectively, then $K_g + K_p$ is the terminal permeance of the apparatus, and $1/(K_g + K_p)$ is the terminal reluctance. Now $\Phi_1/V_m = K_g + K_p$, and therefore since $\Phi_1/V_m = \frac{B_r}{H_r} \cdot \frac{A}{l}$, $\frac{B_r}{H_r} = \frac{l}{A}(K_g + K_p)$, and it follows that $\frac{B_r}{H_r} = \frac{\text{Reluctance of Magnet}}{\text{Terminal Reluctance}}$. If, therefore, for any demagnetization curve we plot the relation $\frac{B_r}{H}$ against values of B_r, we obtain a curve similar to that shown in Fig. 5.14, curve C. Now the Heinrich and Bercovitz safety factor of a magnet given on page 255 is practically this ratio ; it is not exactly, because it does not include the reluctance of the leakage paths from the pole pieces, and S. Evershed points out that for strict physical interpretation B_r should be replaced by the M.M.F. per cm. expended within the magnet, which is numerically equal to the induction, so that the ratio of the magneto-motive forces expended inside and outside the magnet is equal to the ratio of the internal and external reluctances.

In design, therefore, it is found that the average value of H_r is not very different from the most economical value indicated by the position of the maximum value of the B_rH_r curve for the particular steel employed. If, therefore, we call B_ε and H_ε the values of H and B on the demagnetizing curve, which correspond to the maximum value of the product B_rH_r, the length is at once found by putting $l = \frac{V_m}{H_\varepsilon}$, and this seldom requires any further correction ; but the determination of A is not so easy, and must be done by using a series of approximations.

For this purpose the arithmetical artifice of transferring the leakage between the limbs to the ends of the magnet is first adopted, and, calling the permeance of the leakage path thus transferred K_l, the value of the leakage flux is first assumed to be given by the relation $\frac{1}{2} V_m K_l = \Phi_l$, so that the total flux of the magnet will be $\Phi_1 + \Phi_l$ and we find that the area $A = (\Phi_1 + \Phi_l/)B_\varepsilon$. This, however, must be done by trial and error; first, taking an area $A_o = \Phi_1/B_\varepsilon$, and l as indicated above, we find a provisional value for Φ_1, the leakage flux, from the relation $\frac{1}{2} V_m K_l$. This leads to a new area A_1, and from A and l a new leakage permeance is determined, based upon these dimensions; a new area A_2 is then found, and this process is continued until an area is arrived at such that it satisfies the equation $A = \frac{\Phi_1 + \Phi_l}{B_\varepsilon}$.

The process is shortened if, after determining A_1 as above, A_2 is calculated as $\frac{\Phi_1 + 2\Phi_l}{B_\varepsilon}$; the value of A_2 will in practically all cases be too great, and A_1 will be less than proper value, and from these two cases it should not be difficult to compute the correct value of A.

Having determined the dimensions in this way, the energy output may be predetermined in two ways. In the first, the ratio equation is employed and gives approximate results. What we have actually estimated are A, l, and K_l, together with the permeance of the whole of the leakage paths associated with the pole pieces K_p, and the permeance of the actual path traversed by the useful flux to an imaginary permeance of aK_l across the ends of the magnet; the ratio equation becomes $\frac{B_r}{H_r} = \frac{l}{A}(K_g + K_p + aK_l)$, and this gives the ratio of $\frac{B_r}{H_r}$, and, having this ratio, the average value of H_r may be obtained from the demagnetization curve. Calling this H_a, we can then calculate the terminal magnetic potential $V_m = H_a l$, and the useful flux $\Phi_1 = V_m K_g = H_a l K_g$. The terminal flux delivered by the magnet is $V_m(K_g + K_p)$, and the predetermined energy will be $V^2(K_g + K_p)$.

Now Evershed points out that although in the first place l was determined from the relation $l = V_m/H_\varepsilon$, it does not necessarily follow that the value of H_ε, determined in this way, will exactly equal

H_s, and hence it may be necessary to modify the length and area in order to get values of V_m and Φ sufficiently close to the two factors of the required energy.

The accuracy of the forecast is largely dependent on the value assigned to the constant, a, in the term aKl above, and so far no simple relation between this factor and the shape of the magnet has been found, but, from experience with ordinary forms of magnet, a may have any value between 0·5 and 0·33, and by taking the larger value we shall err on the safe side.

For a more accurate and detailed predetermination he has adopted a step-by-step integration process, in which the magnet is divided up into a series of lengths or regions symmetrically disposed about the neutral section, and over each of these regions the values of B_r and H_r are treated as constant quantities; but in passing from one region to the next the values are supposed to change. The mode of subdivision is shown in Fig. 5.15.

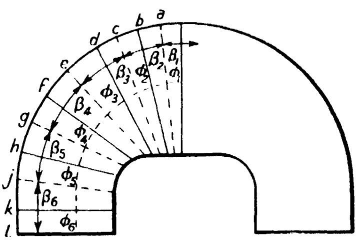


FIG. 5.15.— Division of magnet for step-by-step integration.

The process begins with an arbitrary choice of B_r for the neutral section, and from the demagnetization curve for the steel the corresponding value of H_r is obtained; this, multiplied by $\frac{1}{2}l_1$ on one side of neutral section $+ \frac{1}{2}l_1$ on the other, gives the potential difference V_a between the centres of the first pair of leakage zones. If q is the leakage permeance of the air path between the limbs per cm. width, the permeance of the first leakage path is $q_1 l_1$, and the change in flux density at the junctions between the central region and those on each side of it will be $\frac{V_a q_1 l_1}{A}$, where A is the area of the magnet, and subtracting this from the original value of B_r, chosen for the central region, a new density is obtained.

This is then treated in the same way; the value of H_r for it is obtained from the demagnetization curve, then $H_r(l_1 + l_2)$ is obtained, and $V_b = V_a + H_r(l_1 + l_2)$, and from this $\frac{V_b q_2 l_2}{A}$ is calculated, and the decrement of flux density between the first and second step obtained as above. The process is continued until the ends of the limbs are reached, and a terminal flux density $B_n \times A$ gives the terminal flux Φ_n, and a terminal difference of potential V_n.

The leakage permeance of the pole pieces and gap having been determined, the terminal flux and difference of potential must bear a ratio to one another which we will designate by the symbol K_t.

Now since the initial choice of the flux density B_r was quite arbitrary, it is very unlikely that the final values of Φ_n and V_n will be the required values. What we really require is that $\Phi_r/V_n = K_t$, and if our choice of initial flux is such that Φ_n/V_n is greater than K_t, we must perform a further integration with a lower initial density than that first chosen, and so on. Usually three such integrations are sufficient to cover a range of values, which includes that of the terminal permeance K_t, and by plotting a curve connecting the initial density and the terminal ratio from these values the required initial density can be found.

As an example of this method of design let us consider the magnet required for the moving coil instrument designed on page 324. In a practical design of moving coil instrument, the dimensions of the centre core iron, the air gap and the space available for the magnet itself are determined by conditions other than those of the magnetic design. The shape and dimensions of the magnet system decided upon for this instrument are given in Fig. 5.16. The material chosen was Alcomax, and for this method of design it is necessary to draw a very detailed demagnetization curve, and this is given in Fig. 5.17, the figures used being the average of those published by one manufacturer of this material.

It was first necessary to calculate the leakage permeances of the various parts of the magnetic circuit, and this is the biggest difficulty in magnet design. It is generally possible to estimate these permeances approximately, and Evershed has given some useful formulae which can be used for this purpose.

The first step is to calculate the permeance of the working gap,

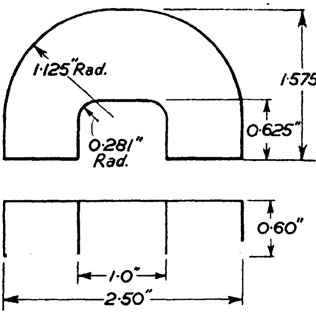


FIG. 5.16.—Dimensions of magnet used in the example.

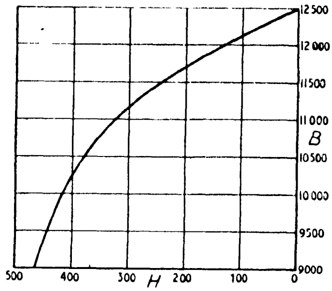


FIG. 5.17.—Demagnetization curve of Alcomax magnet.

which is shown in greater detail in Fig. 5.18. The permeance of one gap is given by the formula,

$$P_a = \frac{w\theta}{\log_e\{1 + g/a\}}$$

In the case under consideration

$$\begin{aligned} \theta &= 154^\circ = 2{\cdot}6878 \text{ radians.} \\ a &= 0{\cdot}25 \text{ in.} = 0{\cdot}635 \text{ cm.} \\ w &= 0{\cdot}6 \text{ in.} = 1{\cdot}524 \text{ cm.} \\ g &= 0{\cdot}05 \text{ in.} = 0{\cdot}127 \text{ cm.} \\ g/a &= 0{\cdot}05/0{\cdot}25 \text{ in.} = 0{\cdot}2. \end{aligned}$$

$$\text{So } P_a = \frac{1{\cdot}524 \times 2{\cdot}6878}{\log_e 1{\cdot}2} = 22{\cdot}45.$$

Since there are two gaps in series, the effective permeance = 22·45/2 = 11·23.

As regards the leakage permeances, the first to be calculated will be that between the pole tips, which are drawn in greater detail in Fig. 5.19. The permeance between one pair of tips will be sw/g, and in this case $S = 0{\cdot}108$ in. $= 0{\cdot}274$ cm., $w = 0{\cdot}6$ in. =

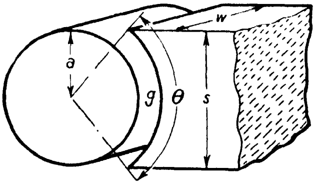


FIG. 5.18.—Gap dimensions.

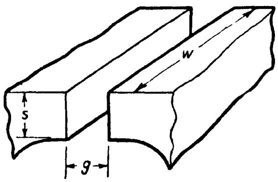


FIG. 5.19.—Detail of pole tips.

1·524 cm., and $g = 0·125$ in. $= 0·317$ cm. This gives the permeance of one gap as 1·318, and since there are two gaps in parallel the total pole tip leakage permeance is $2 \times 1·318 = 2·636$.

Next there is the fringe permeance between the pole pieces. Owing to their peculiar shape this is not easy, but an approximation can be obtained by using the formula given in Evershed's paper for the fringe leakage between two cylinders as shown in Fig. 5.20. The equations for the fringe permeance depend on an auxiliary quantity v given by the expression

$$v = a \log_e \left[1 + 2 \left\{ \frac{x + \sqrt{x^2 + xg}}{g} \right\} \right]$$

The leakage permeance then is

$$P_F = \frac{\pi \sqrt{v^2 - x^2}}{\cos^{-1}(x/v)} \qquad \text{if } v > x$$

$$P_F = \pi \qquad \text{if } v = x$$

$$P_F = \frac{\pi \sqrt{x^2 - v^2}}{\cosh^{-1}(x/v)} \qquad \text{if } v < x$$

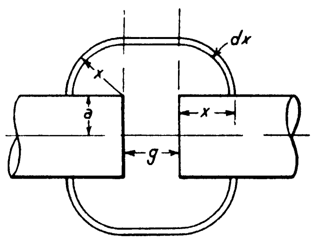


FIG. 5.20.—Fringe leakage between two cylinders.

The actual pole pieces are shown in Fig. 5.21, and it becomes necessary to determine values for the various quantities which

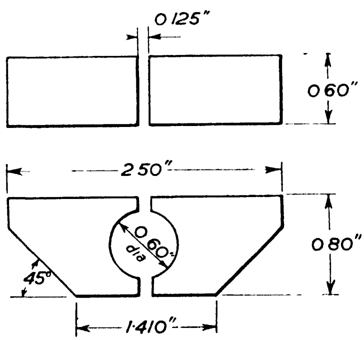


FIG. 5.21.—Dimensions of pole pieces.

appear in the above formulae. The gap g can be taken to a sufficient degree of approximation as half the maximum gap of 0·6 in., i.e. $g = 0·3$ in. $= 0·762$ cm. The fringe can be taken as extending

from the edge of the effective gap to half-way across the magnet poles, so that $x = 0{\cdot}575$ in. $= 1{\cdot}46$ cm. To determine a it is necessary to replace the rectangular cross-section by a cylinder having the same periphery. Thus

$$2\pi a = 2(0{\cdot}8 + 0{\cdot}6)$$

$$\text{So } a = \left(\frac{1{\cdot}4}{\pi}\right)\text{in.} = \frac{1{\cdot}4 \times 2{\cdot}54}{\pi}\text{ cm.} = 1{\cdot}13\text{ cm.}$$

The quantity $v =$

$$1{\cdot}13 \log_e \left[1 + 2\left\{\frac{1{\cdot}46 + \sqrt{1{\cdot}46^2 + 1{\cdot}46 \times 0{\cdot}762}}{0{\cdot}762}\right\}\right] = 2{\cdot}55$$

$$\text{Since } v > x,\ P_F = \frac{\pi\sqrt{2{\cdot}55^2 - 1{\cdot}46^2}}{\cos^{-1}(1{\cdot}46/2{\cdot}55)} = 6{\cdot}83.$$

The total terminal permeance $= 11{\cdot}33 + 2{\cdot}64 + 6{\cdot}83 = 20{\cdot}8$.

The magnet itself is shown in Fig. 5·16, and for the purpose of this investigation is divided into eight parts. The leakage permeance is calculated by the spherical pole formula given by Evershed. If S is the surface area of the leakage surface then the leakage permeance is $1{\cdot}77\sqrt{S}$. The leakage permeances, lengths and mean areas of each part or zone are next calculated, and these are given in Table XVII.

TABLE XVII.

Zone.	Length (cm.).	Area (sq. cm.).	Leakage permeance.
1	1·2	3·85	5·23
2	1·2	3·36	5·09
3	1·2	2·97	5·08
4	1·2	2·9	3·54

The method now requires an assumption to be made of the flux density in the neutral section. For a first attempt this will be taken as 11,000 lines per sq. cm., and the value of H corresponding to this flux density is from Fig. 5.17, 320 oersteds. The M.M.F. available at the ends of the first zone will be $1{\cdot}2 \times 320 = 384$. The leakage flux from the first zone is then $384 \times 5{\cdot}23 = 2{,}005$ lines. The flux in the neutral section $= 11{,}000 \times 3{\cdot}85 = 42{,}300$ lines, a density of $40{,}295/3{\cdot}36 = 12{,}000$ lines. The value of H

corresponding to this is 135, and the increase in M.M.F. due to zone 2 is $2 \times 1{\cdot}2 \times 135 = 324$. Note there are two zones 2, one on either side of the neutral section. The total M.M.F. at the ends of zone 2 is $384 + 324 = 708$. The leakage flux from zone 2 is $708 \times 5{\cdot}09 = 3{,}600$ lines, so the flux passed on to zone 3 is $40{,}295 - 3{,}600 = 36{,}695$ lines. This calculation is continued until the poles are reached, when the ratio of the total flux to the terminal M.M.F. should be equal to the terminal permeance. If this is not the correct value, a fresh start must be made with a new value of neutral flux density. The calculations relating to three values of neutral flux density are given in Table XVIII.

In no case does the terminal permeance equal the desired value of 20·7, but by plotting curves of the figures concerned it is found that the correct terminal M.M.F. is 1,360. Since the gap permeance is 11·23, the total gap flux $= 1{,}360 \times 11{\cdot}23 = 15{,}290$ lines. The effective area of the gap can be calculated from length × permeance, i.e. $0{\cdot}127 \times 22{\cdot}45 = 2{\cdot}855$ sq. cm. So the gap flux density $= 15{,}290/2{\cdot}855 = 5{,}350$ lines per sq. cm.

From the formula for the moving coil instrument the working flux is B multiplied by the area of the working coil. In this case the area is 1·68 sq. cm., and consequently the working flux is $1{\cdot}68 \times 5{,}350 = 9{,}000$ lines.

TABLE XVIII.

Zone.	B.	Total flux.	H.	Length.	Increase in M.M.F.	M.M.F.	Leakage P.	Leakage flux.	Area.
1	11,000	42,300	320	1·2	..	384	5·23	2,005	3·85
2	12,000	40,295	135	1·2	324	708	5·09	3,600	3·36
3	12,360	36,695	39	1·2	93·6	801·6	5·08	4,065	2·97
4	11,270	32,630	279	1·2	669	1,470·6	3·54	5,210	2·9
Poles	..	27,420	..	..	..	..	..	..	..
Terminal P equals 27,420/1,470·6 equals 18·62.									
1	11,050	42,500	313	1·2	..	376	5·23	1,962	3·85
2	12,060	40,538	120	1·2	288	664	5·09	3,380	3·36
3	12,500	37,158	0	1·2	0	664	5·08	3,370	2·97
4	11,630	33,788	216	1·2	518	1,182	3·54	4,180	2·90
Poles	..	29,608	..	..	..	..	..	..	..
Terminal P equals 29,608/1·182 equals 25·05.									
1	11,020	42,400	317·5	1·2	..	380·5	5·23	1,992	3.85
2	12,020	40,408	131	1·2	314·5	695	5·09	3,540	3.36
3	12,410	36,868	33	1·2	80	775	5·08	3,935	2.97
4	11,370	32,933	263	1·2	631	1,406	3·54	4,980	2.9
Terminal P equals 27,953/1,406 equals 19·89.									

The results of tests on one hundred magnets gave an average working flux of 9,400 lines, some 4½% higher than the calculated value, which is quite a good agreement.

Modern Magnet Materials

The previous section gave a picture of the art of producing magnet materials up to about 1920. Just prior to this it was found that an addition of cobalt and tungsten to carbon steel produced a large and useful change in the magnetic properties, particularly in the value of coercive force. The exact values of coercive force and remanence obtained depend on the composition of the steel. Thus the well-known 35% cobalt steel has a coercive force of about 250 oersteds and a remanence of approximately 10,000 gauss, while a steel with a lower percentage of cobalt, say 9%, has a coercive force of about 150 oersteds and a remanence of about 7,500 gauss. The cobalt steels fall into the same general class of quench-hardening steels as the tungsten and chromium magnet steels. The immediate advantage of these steels was that any desired magnetic flux could be obtained with a smaller volume of magnet material than with the older tungsten and chromium steels. These steels can be machined in the soft state, and afterwards hardened to produce the desired magnetic properties.

Within the last ten years or so a considerable amount of work has been done in another class of materials, known as dispersion-hardening alloys. These alloys do not contain carbon, except as a possible impurity, and so are not properly called steels. The hardening procedure causes a dispersion of one of the two phases of which they are composed in the other phase, and hence their name. The first of these alloys, that of aluminium, nickel and iron, was investigated more or less simultaneously in America, Germany and Japan. The remanence of these alloys was somewhat low, but the coercive force was high, about 475 oersteds, so that new applications of permanent magnets become possible. Owing to their very hard and brittle nature, these alloys cannot be formed by forging and machining, but are either cast or moulded in powder form under pressure and then sintered. Any final finishing required is done by grinding.

The next step in the development of these alloys was the addition of cobalt to the aluminium-nickel-iron alloy, which produced a

further improvement in the magnetic properties. It was, however, found difficult to obtain these higher propérties except in thin cast sections, until it was discovered that an addition of copper enabled the best results to be obtained. As a result of continuous experiment the alloy known as Alnico was finally developed, and the approximate composition of British Alnico is nickel 18%, aluminium 10%, cobalt 12%, copper 6% and iron 54%. The coercive force of Alnico is about 500 oersteds, and the remanence about 7,500 gauss.

A still later discovery was that if an alloy of this type was subjected to a magnetizing force during the process of heat treatment, a further improvement can be obtained in the direction in which this force is applied, while the magnetic properties in the directions at right-angles to this force are worsened. These anisotropic alloys are available in this country under names Alcomax and Ticonal, and in America under the name Alnico V. These alloys have a coercive force between 500 and 600 oersteds, and a remanence between 12,000 and 13,000 gauss.

Typical demagnetization curves for these modern magnet materials are given in Fig. 5.22.

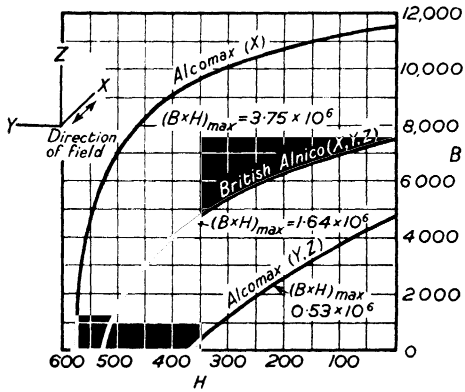


FIG. 5.22.—Demagnetization curves for some modern magnetic materials.

Further Note on the Design of Permanent Magnets.

A development in the method of the design of magnets for instruments which really follows on from Evershed's ratio method, and which will be found to be very useful, is explained in the following.

Let p_t = the total permeance of the magnetic circuit external to the magnet itself, including the permeance of all leakage paths as well as that carrying the working flux.

p_g = the permeance of the working gap, or path carrying the useful flux.

p_L = the permeance of all the leakage paths.

l = effective length of magnet.

a = effective cross-sectional area of magnet.

H, B = co-ordinates of the point on the demagnetization curve, at which the magnet will operate with these permances.

B_R = remanent value of the magnetic material.

H_c = coercive force of the magnetic material.

Φ = total flux provided by magnet.

Φ_g = flux through working gap.

Then $p_t = p_g + p_L$.
The total flux $\Phi = Hl.p_t$.
The total flux is also equal to $B.a$ and so

$$Ba = Hlp,$$

i.e. $$\frac{B}{H} = \frac{l}{a} \cdot p_t.$$

The quantity on the right-hand side of this expression has been called by some writers the " unit permeance " of the magnetic circuit, and is represented by p. Now in Fig. 5.23 if A is the

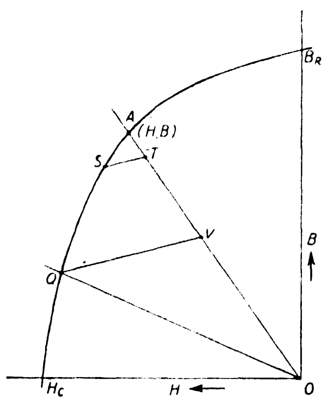


FIG. 5.23.—Diagram to determine working point on demagnetization curve.

operating point on the curve, then the slope of a line drawn through this point and the zero is B/H, which is the unit permeance p. So, knowing the unit permeance, the working point can be determined from the curve.

It can also be calculated in the following manner: Desmond has shown that the demagnetization curve can be represented approximately by the formula

$$kBH + B_R H_c = BH_c + B_R H.$$

where k is a quantity which is a function of the curve factor γ where

$$\gamma = \frac{(BH)_{max}}{B_R H_c} \text{ and } K = \frac{2\sqrt{\gamma} - 1.}{\gamma}$$

Since $B = pH$ this can be substituted in the above equation and gives the following equation for H:

$$H^2 - H\left\{\frac{B_R}{kp} + \frac{H_c}{k}\right\} + \frac{B_R H_c}{kp} = 0.$$

This equation has been solved for various values of p and various magnetic materials and the results are plotted in the form of curves in Fig. 5.24.

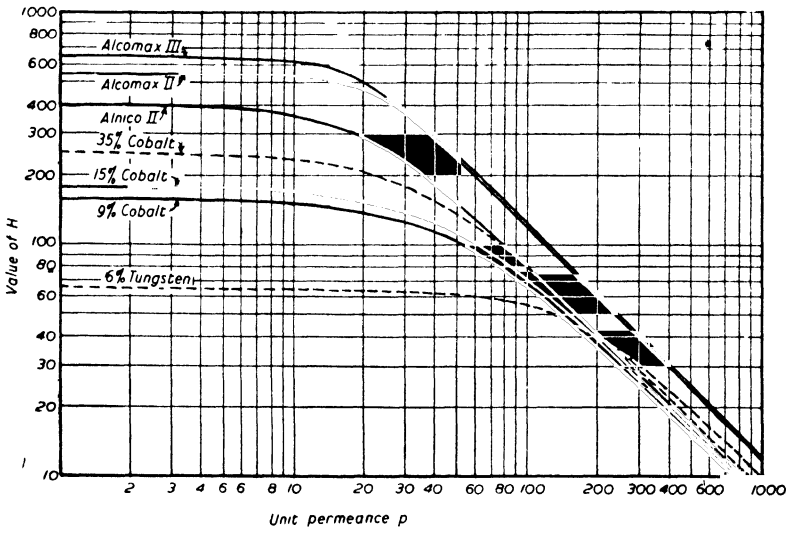


FIG. 5.24.—Values of H and permeance for various magnetic materials.

In addition to the normal cast and rolled magnets considerable use is made nowadays of sintered magnets, particularly for very small sizes. The magnetic properties of sintered magnets are slightly inferior to the cast magnets, and the curves giving values of H for various values of p for sintered Alnico and Alcomax are given in Fig. 5.25.

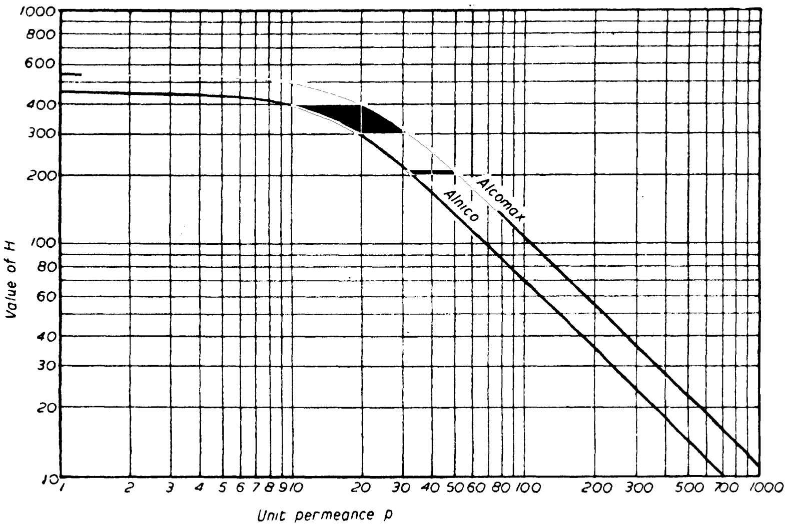


FIG. 5.25.—Values of H and p for Alcomax and Alnico.

Thus for a given magnet the point A can be determined. Now if the magnet is subjected to a demagnetization force for the purpose of ageing or stabilising, the working point will move down to a point such as S, and on removal of the demagnetizing force, it will not move back along the Curve SA, but along a minor hysteresis loop represented by the line ST, to the point T on the line OA, and this point becomes the new working point with a small reduction in the M.M.F. available from the magnet.

An important point to be noted here is that in many cases a magnet cannot be magnetized when connected to its working gap, and must be magnetized on some other circuit and then transferred to the working gap. Again, in moving-coil instruments, it is very desirable where the construction permits to be able to remove the movement to enable repairs or minor adjustments to be made.

It will be seen in a later chapter that in those cases where this can be done, the centre iron used in the magnet gap is removed with the movement and so the working gap is removed and replaced by one much larger. The magnet is now working into a permeance slightly higher than the leakage permeance, and for all practical purposes it may be considered as working into its leakage permeance.

In Fig. 5.23 the slope of the line OQ is the " unit leakage permeance " and is equal to lp_1/a, and the point Q is that to which the working point moves under the conditions outlined above. When the working gap is replaced, the working point moves along the minor hysteresis loop QV to the point V on the unit permeance line, and this becomes the new working point. .It is evident that, under these conditions, the M.M.F. from the magnet may be very appreciably reduced, and the interesting point to note is that a small leakage permeance is by no means the best in this case. In fact it is advisable to have a fairly large leakage and to adjust the dimensions of the magnet to supply both the desired working flux and also all the leakage fluxes.

To an instrument designer the problem of magnet design often presents itself in this manner. Within a certain space it is desired to place a magnet to obtain the maximum possible flux in a given gap, with the condition that the working gap may be removed and replaced without affecting the gap flux.

Many methods of calculation have been proposed for this but nearly all suffer from the disadvantage that they assume that the leakage permeances do not change when the magnet dimensions are changed, and in many cases, particularly in instruments, this is not so, and changes in leakage permeance must be taken into account.

Referring again to Fig. 5.23, if H_2 is the value of H at V, and H_1, B_1 are the co-ordinates of Q, obtained from the curves of Figs. 5.24 and 5.25, then it can be shown that

$$H_2 = H_1 \left\{ \frac{\mu_r + p_1}{\mu_r + p} \right\}$$

where μ_r is the slope of the line QV and is known by various names such as "recoil permeability " " reversible permeability," etc. Approximate values for this " recoil permeability," μ_r, are usually

now given by magnet makers when giving the general magnetic properties of their various magnets materials. Typical values are :

9% cobalt steel .	16
35% cobalt steel .	11
Alnico III . .	3·3
Alcomax II . .	2·0
Alcomax III . .	3·3

The method of design which the revising author has found most suitable can be summarized as follows : The maximum dimensions of the magnet are governed by the space available and a rough estimate of the possible dimensions is made. The various permeances are calculated and the position of the point V is determined. From the corresponding value H_2, the length of the magnet and the permeance of the working gap, the total flux in the latter can be calculated. The length of the magnet is then altered and the calculations repeated. It will be found that there is an optimum length which gives the maximum flux in the gap for a given magnet area. The area can now be modified and a new optimum length and maximum flux obtained. A few calculations of this sort soon enable the designer to decide on the best dimensions of the magnet to give the maximum flux in the working gap.

Even when there is no limitation of space this method is still satisfactory as there will be found optimum values of length and area beyond which it is inadvisable to go as any increase due to dimensions is offset by increased leakage etc.

Ageing and Stabilization

It is essential for magnets which are used in electrical measuring instruments to have the greatest possible constancy of flux. In moving coil instruments, for example, the sensitivity and therefore the accuracy of the instrument is directly proportional to the value of the magnet flux. In integrating watt-hour-meters, the braking torque, and therefore the speed, is directly proportional to the strength of the brake magnet, and extreme constancy is required in both these cases. The magnet strength of any magnet may deteriorate with time, heat, the application of mechanical shocks or vibration, or the subjection of it to external magnetic influences.

There is a tendency with the quench-hardening steels both to change their magnetic characteristics with time, the amount

of this change and the length of time necessary to obtain constancy desired depending on the composition or heat treatment of the material, and on the dimensions of the magnet and magnetic circuit. The dispersion hardening materials do not seem to suffer in the same way from this change with time. Not much information is at present available on this point, as presumably sufficient time has not yet elapsed to enable the necessary data to be collected.

The effect of mechanical shocks or vibration is again very slight in the case of the dispersion hardening alloys. In the case of the old steels, mechanical shocks or vibration did produce a drop in the magnetic strength. Generally speaking, the higher the coercive forces of the magnet, the less it is affected by the mechanical disturbances.

The flux produced by a magnet is affected by temperature, and appears to drop slightly as the temperature is increased. Experiments show that this temperature coefficient is approximately the same for all permanent magnet materials, and is about 0·02% per degree Centigrade.

The magnets are usually subject to some form of artificial ageing processes before their final use in instruments in order to reduce, as far as possible, their susceptibility to change, due to various external influences. Some manufacturers store their magnets for a considerable time before use; others subject them to various temperature cycles. A quite common practice nowadays with the high coercive force materials is to demagnetize the magnets a few per cent. by an externally-applied alternating field. In this connection it may be noted that when a magnet is demagnetized, and this is sometimes done by open-circuiting the magnet deliberately, the working point does not travel round the normal demagnetization curve, but passes along what is called a minor hysteresis loop, as illustrated in Fig. 5.26. These minor hysteresis loops are very thin, and in fact are usually considered as straight lines. Many investigators such as Hornfasch and Edgar, Desmond, etc., have investigated theoretically the conditions of operation of a magnet when the working point is on one of these minor loops. It should

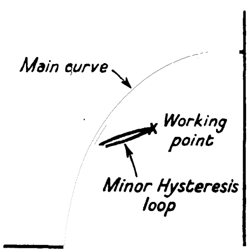


FIG. 5.26.—The minor hysteresis loop.

be noted that the working point is then actually inside the normal demagnetization curve.

The necessity for artificial ageing to obtain a high degree of constancy in the magnetic flux cannot be too much stressed, particularly when the instrument in which the magnet is to be used is to be of a high degree of accuracy, and a lot of trouble is very often taken over this particular point.

Temperature Coefficients of Magnets

The temperature coefficient for permanent magnets has been studied by Ashworth and others. It varies very greatly in different materials, and is negative. Ashworth's values are given below;

TABLE XIX.—*Temperature Coefficients of Magnets.*

Material.	a.	Dimension ratio l/d.
Carbon magnet steel hardened	−0·00137	8
Musket-steel hardened	−0·00069	12
,, ,, ,,	−0·00097	17
Cast-iron ,,	−0·00018	11·8
,, ,,	−0·00016	13·1
Pianoforte wire glass hard	−0·000603	16
,, ,,	−0·000228	48
,, ,,	−0·000097	80
,, ,,	−0·000055	96

Whipple puts the average temperature coefficient of magnets at − 0·00029, and Cancani has found for glass hard "English steel" the value − 0·000436, and for the same material annealed soft − 0·002635.

There appears to be some doubt about the temperature coefficients of the modern magnet materials; some sources indicate that the coefficients are negligible, and others that the coefficients are as high as with the older materials. Further research on this point seems to be very necessary.

Electro-Magnets

Where iron-cored electro-magnets are employed in instrument construction high permeability and low coercive force become

of primary importance, and materials possessing these qualities must be sought.

Pure iron or electrolytic iron together with certain alloy irons probably head the list of such materials, but soft charcoal and Swedish irons are also very generally used. C. F. Burgess and J. Aston examined the properties of electrolytic iron and alloys with arsenic and bismuth, and showed that the latter alloys gave material very comparable in magnetic quality with the pure iron. The alloys of Barrett and Hadfield, in which aluminium and silicon are used, have been already referred to.

Professor Pierre Weiss in 1912 discovered an alloy of iron and cobalt in which the magnetic qualities seemed to reach very high values, and Yensen, in America, has given some very instructive figures for similar alloys. From his observations he concluded that the iron cobalt alloy has a saturation value of magnetization 13% higher than pure iron, melting *in vacuo* giving 3% higher values than corresponding ordinary meltings. When melted *in vacuo* its maximum permeability is above 13,000 at $B = 8{,}000$ gauss, and while this value is considerably lower than pure iron melted *in vacuo*, its permeability in medium fields, such as $H = 50$ to $H = 200$, is 25% higher than pure iron or commercial grades of iron. Its hysteresis loss at $B = 10{,}000$ or below is considerably less than the best commercial grades of transformer iron, and at 15,000 or above it is about the same as transformer iron. Its specific resistance is about 10×10^{-6} ohms—about the same value as pure iron.

Table XX, page 276, gives Yensen's comparisons for this alloy and other irons. Mechanically it will be seen that the alloy, although somewhat brittle, is fairly strong, it being about the same tensile strength as pure iron when in the annealed condition, but when forged it is more than twice as strong. Yensen recommends it for high densities, but points out that the price of cobalt prohibits its general use.

Yensen and Gatwood have also investigated iron aluminium alloys containing the latter constituent up to 8%. They find the best alloy magnetically is one containing 0·4% of aluminium, and this annealed at 1,100° C. has a maximum permeability of 35,000, while the hysteresis loss at B_{max} 10,000 is 450 ergs per c.c., and at B_{max} 15,000, 1,000 ergs per c.c. The alloys containing higher aluminium content are not so good magnetically, but even

TABLE XX.—*Magnetic Properties of Steel Alloys.*

Material.	Composition.	Values of B. $H = 10$.	$H = 20$.	$H = 50$.	$H = 100$.
Kreusler annealed dynamo steel	..	15,680	..	17,100	18,280
Barrett's aluminium steel	2·25 Al	15,300	17,000	18,000	..
Burgess and Aston arsenic alloy	3·86 As	15,050	16,300	17,550	18,650
„ „ arsenicbismuth alloy	2·0 Bi	13,250	16,100	18,350	19,750
„ „ arsenic alloy	3·56 As	14,200	16,100	17,550	18,750
Sankey's Lohys sheet	..	14,150	16,100	17,050	18,250
Burgess and Aston electrolytic iron	..	10,000	15,950	17,700	18,850
„ „ arsenic iron	4·14 As	14,200	15,900	17,250	18,450
Hadfield's magnet steel	..	12,150	15,900	17,350	18,200
Barrett's pure iron	..	14,000	15,900	17,200	..
Burgess and Aston arsenic alloy	1·81 As	11,850	15,750	17,450	18,750
„ „ „	0·29 As	13,200	15,550	17,250	18,550
Allen's cast steel	..	12,100	15,050	17,200	18,450
Barrett's silicon alloy	2·5 Si	14,000	15,000	16,600	..
High grade sheet	..	13,100	14,800	16,300	17,600
Transformer iron alloyed with arsenic	5·0 As	9,550	14,750	16,750	18,000

with 3% Al the specific loss is only half that of a 3·5% Si iron. The specific resistance increases by 12 micro-ohms for each 1% of Al up to 3%, above which the increase is more gradual. The high aluminium alloys are less brittle than silicon alloys, and vacuum treatment of the 0·4% Al iron yields a remarkable magnetic substance.

Dr. Guggenheim has investigated the effect of silicon as an alloying constituent in Oehler iron, and finds that the initial values of the permeability decrease at first with an increase in the percentage of silicon, but afterwards increase together. The maximum values of the permeability behave in the same way, and at inductions above 14,000 for a given field strength the permeability decreases more quickly with an increase in the percentage of silicon. The two curves in Fig. 5.27 illustrate the variation.

Burgess and Aston have obtained similar results with electrolytic iron alloyed with silicon, but they point out that the silicon effects greater improvement in commercial iron than in their pure iron.

The relative magnetic behaviour of the various grades of iron are shown in the curves in Fig. 5.28.

The chief interest in electro-magnets from the instrument maker's point of view, however, centres round their behaviour when subjected

TABLE XXI.—*Properties of Iron Alloys.*

Material.	Max. μ.	Hysteresis loss (ergs per c.c. per v.). B = 10,000	B = 15,000	Coercive force, Gilbert's. B = 10,000	B = 15,000	Retentivity gausses. B = 10,000	B = 15,000	B at μ_{max}.	Heat treatment.
Pure iron melted *in vacuo*	22,800	820	1,700	0·27	0·33	9,350	14,000	10,000	Annealed at 900° C.
Fe_2Co alloy melted *in vacuo*	13,200	1,460	3,200	0·48	0·65	9,100	12,000	8,000	" "
Standard transformer steel	3,850	3,320	5,910	1·2	1·33	7,700	9,900	7,000	Received manufacturers' standard treatment.
4% silicon steel	3,400	2,260	3,030	0·88	0·88	5,400	5,400	4,000	
Swedish charcoal iron	4,850	2,490	4,530	0·88	0·95	6,900	8,000	6,500	
Pure iron melted *in vacuo*	24,300	686	1,655	0·22	0·26	9,300	13,000	8,500	Annealed at 1,100° C.
Fe_2Co alloy melted *in vacuo*	8,800	2,230	4,400	0·75	1	9,300	12,300	8,000	" "

Material.	Specific resist. in 10^{-6} ohms at 20° C.	As forged. Stress at yield points (lb. per sq. in.). (*a*)	As forged. Ultimate strength (lb. per sq. in.). (*b*)	Elongation per cent. 1 (*a*)	Elongation per cent. 2 (*b*)	Reduction in area per cent. (*e*)	Annealed. (*a*)	Annealed. (*b*)	Annealed. (*c*)	Annealed. (*d*)	Annealed. (*e*)
Pure iron melted *in vacuo*	9·85	40,600	42,630	5·5	32	71	16,100	35,500	25	49	78
Fe_2Co alloy melted *in vacuo*	9·72–10·5	73,400	97,000	<1	3	3	30,800 29,450	30,800 29,450	<1 <1	<1 <1	<1 <1
Standard transformer steel	11	..	..	..	..	..	..	..	..	..	..
4% silicon steel	51	..	..	..	..	..	..	..	..	..	..
Swedish charcoal iron	10·57	..	40,000	..	36	57	..	..	..	..	..
Pure iron melted *in vacuo*	..	..	..	..	..	..	..	..	..	..	..
Fe_2Co alloy melted *in vacuo*	10·1–9·6	..	..	..	..	..	..	..	..	..	..

to an alternating magnetic force. When an alternating current circulates in the magnetizing coils, the core reverses its polarity at each half-period, and the magnetism follows the well-known magnetic cycle or hysteresis loop.

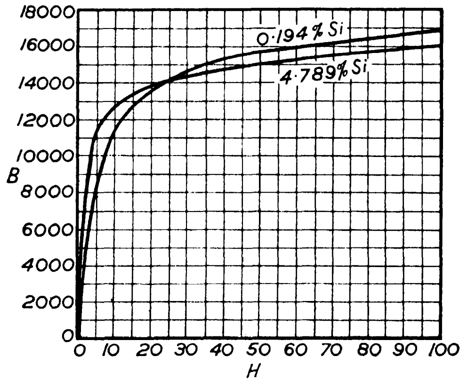


FIG. 5.27.—Variation of permeability with silicon content.

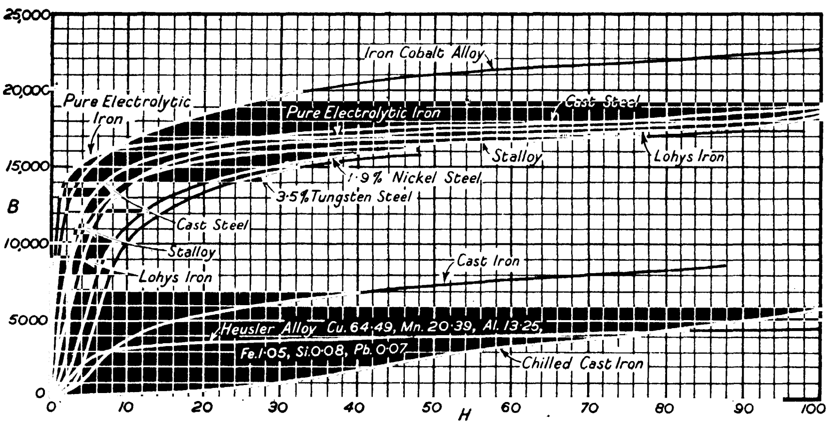


FIG. 5.28.—Magnetic behaviour of various grades of iron.

Consider a ring of magnetic material, with a mean magnetic path of l cm., and having a magnetic cross-section of A sq. cm. Let this ring be overwound with a magnetizing coil of N turns. If a current I amperes be sent through it, the magnetic force acting

on the iron is $\frac{4\pi}{10}\frac{NI}{l}$. If now at any instant the P.D. at the ends of coil is V, and the resistance of the coil is R ohms, and B is the value of the flux density in the iron, then the total number of lines linked with the coil is BNA. If the potential now changes from V to $V + dV$, and the current increases from I to $I + dI$ and B to $B + dB$ in the time dt, the time rate of change of induction is in the limit dB/dt, and for the whole number of lines linked $NA\frac{dB}{dt}$. This is numerically equal to the E.M.F. set up in the circuit by the change of flux, or in volts:

$$-\frac{NA}{10^8}\frac{dB}{dt}$$

This E.M.F. will be opposed to the impressed voltage V, and hence I at any instant must be given by the resultant E.M.F. divided by the resistance of the circuit:

$$I = \frac{V - \frac{NA}{10^8}\frac{dB}{dt}}{R}$$

Hence, multiplying throughout by I and dt, we have:

$$VIdt = RI^2\,dt + \frac{NA}{10^8}\,IdB.$$

But the magnetizing force, as we have seen above at the instant, is $\frac{4\pi}{10}NI = H$, hence:

$$VIdt = RI^2dt + \frac{A}{4\pi \times 10^7}\,HdB$$

Now $VIdt$ is the whole energy given to the coil and core in the time dt, and RI^2dt is the energy wasted in ohmic heating of the coil, and, therefore, $\frac{A}{4\pi \times 10^7}HdB$ represents the energy expended in the change of magnetism of the core.

Hence it is not difficult to see that the integral

$$\frac{1}{4\pi}HdB$$

taken between any limits must represent the whole energy expended in the iron per unit volume, in ergs per cubic cm. of iron per cycle.

This integral *HdB*, taken over the whole limit of the magnetic cycle, is the enclosed area in magnetic units of the hysteresis loop, and no matter how fast or slow the changes are produced, the loss of energy is the same per cycle between the same limits. It is therefore at once obvious that if a specimen be subjected to a rapidly alternating field, due to an alternating current circulating in its magnetizing coils, it will sweep round this curve a number of times corresponding to the frequency of the current, and the energy loss of each cycle being additive, the total loss become very appreciable indeed.

Dr. Steinmetz endeavoured to establish the relation between the maximum induction density and the energy loss in the relation

$$E_h = \eta \, B_{\max}^{1 \cdot 6} f$$

where E_h is the iron loss per cubic centimetre, f the frequency, and η a constant depending upon the nature of the magnetic material.

The value of the exponent 1·6 is an average, and the curve in Fig. 5.29 due to Dr. Lloyd for various transformer irons indicates

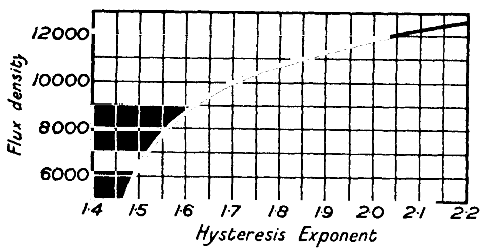


FIG. 5.29.—Values of hysteresis exponent for transformer iron.

the general trend with change in induction density. Dr. Sumpner, Professor Wilson, Mr. Wild and others have also shown that at very low inductions the energy loss is more nearly represented by B^2 than $B^{1\cdot6}$.

For example, the authors found that for inductions up to $B_{\max} = 500$, the hysteresis loss in ergs per c.c. per cycle is as follows:

Sankey's Lohys iron $E_h = 0{\cdot}00011 \; B^2_{\max}$.
,, Stalloy ,, $E_h = 0{\cdot}000057 \; B^2_{\max}$.

Table XXII gives some of the accepted values of η for various irons and other magnetic materials.

In many cases the material, when continually subjected to the

TABLE XXII.—*Hysteresis Coefficients.*

Material.	Condition at test.	Per cent alloy.	η.	Authority.
Silicon steel	..	?	0·001	Steinmetz.
British silicon steel	..	3·1 Si	0·00094 0·00102	Lloyd and Fisher.
American ,, ,,	..	3·4 Si	0·00089	Ditto.
,, ,, ,,	..	3·5 Si	0·00086	,,
,, ,, ,,	..	2·8 Si	0·00077 0·00084	,,
,, ,, ,,	..	3·9 Si	0·00086	,,
,, ,, ,,	..	3·2 Si	0·00062	,,
German ,, ,,	..	3·9 Si	0·00078	,,
,, ,, ,,	..	3·8 Si	0·00080	,,
,, ,, ,,	..	3·4 Si	0·00061	,,
American transformer steel	Unannealed	..	0·0049 0·00312	,,
Ditto	Annealed	..	0·00227 0·00174 0·00105	,,
German transformer steel	,,	..	0·00129 0·00122	,,
British transformer steel	,,	..	0·00129 0·00118	,,
Average sheet steel	..	..	0·002	Steinmetz.
Wrought iron, sheet iron, sheet steel	..	..	0·0012– 0·0055	,,
Cast-iron	..	..	0·011– 0·016	,,
Soft cast steel	..	..	0·0032– 0·012	,,
Hard cast steel	..	..	0·028	,,
Forged steel	..	..	0·015– 0·025	,,
Magnetic iron ore	..	..	0·02– 0·024	,,
Nickel	..	..	0·013– 0·039	,,
Cobalt	..	..	0·012	,,
Sankey's aluminium iron	..	..	0·0012	S. P. Thompson.
Barrett's ,, ,,	..	..	0·00068	,,
Soft Whitworth steel	Annealed	0·090 C.	0·00257	,,
,, ,,	,,	0·320 C.	0·00598	,,
,, ,,	,,	0·80 C.	0·00786	,,
Siemens steel	Forged	3·44 Si	0·00937	,,
Manganese steel	,,	4·78 Mn	0·05963	,,
,, ,,	Annealed	4·78 Mn	0·04146	,,
,, ,,	,,	8·74 Mn	0·08184	,,
Chrome steel	Forged	0·620 Cr	0·01179	,,
,, ,,	Annealed	0·620 Cr	0·00897	,,
,, ,,	Forged	1·20 Cr	0·01851	,,
Soft Bessemer steel	Annealed	0·0450 C.	0·00262	,,

alternating cycle, shows a tendency to increase the iron loss, and to obviate this artificial ageing has to be resorted to. This usually takes the form of prolonged heating. The German rules recommend 600 hours at 100° C. Clinker has, however, found considerable variation in the behaviour of irons under this process, and while ageing for 1,000 hours increases the hysteresis loss by 130% in some samples, in others there was but little effect.

The true iron loss in an alternating field is always accompanied by a second source of loss, due to the eddy currents induced in the iron by the change of induction through it. In order to reduce this to a minimum, it is usual to build the cores of thin laminations electrically insulated from one another by coating them with a thin paper or a layer of an insulating varnish, japan, or some similar composition. The magnitude of the loss will depend upon the nature of the material, and vary as the square of the induction density, the thickness of the plates, and the frequency—that is,

$$W_e = \frac{\pi^2}{6\rho} x^2 f^2 B^2 \times 10^{-16}$$

where x is the thickness of the laminations and ρ the specific resistance of the iron. In practice, however, it is found that this expression invariably leads to values which are too low, because of the uncertainty of the value of ρ and imperfection of the insulation between the laminations, and it is not unusual to increase the values calculated by the above expression by 50%.

It is at once obvious that thin laminations with a high specific resistance will reduce this loss, and in this respect the alloy irons are valuable. Thus Burgess and Aston have shown that the specific resistance of pure iron rose according to a straight-line law from 12 to 68 micro-ohms per centimetre cube by successive additions of silicon up to 5%. In the same way arsenic increases the specific resistance of electrolytic iron when used to alloy it, 3·86% of arsenic raising the specific resistance from 12·1 to 37·1 micro-ohms. The effect of alloying on the hysteresis loss is also interesting. Guggenheim found that in silicon irons the loss increased to a maximum with additions of silicon up to 2·8%, after which the loss gradually diminished as the percentage of silicon is increased up to 4·8%; at this composition the total iron loss was 1·23 watts per kilo (Fig. 5.30). Hitherto we have left out of consideration the form factor of the alternating voltage

employed. According to Roessler, the total iron losses at constant temperature may be represented by the expressions

$$W_i = 10^{-7} (\eta f B_{\max}^{1\cdot 6} + \chi f^2 F^2 B^2_{\max})$$

where η and χ are the hysteresis and eddy-current coefficients respectively for a given type of lamination, and F is the form factor V/V_{mean}, V being the effective value of the induced voltage and V_{mean} its mean value.

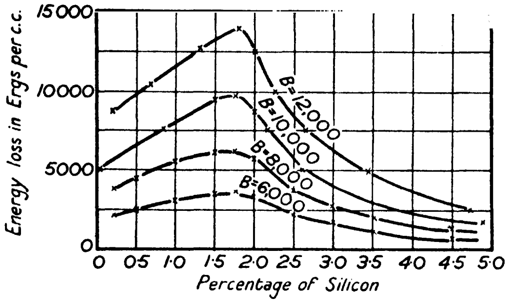


FIG. 5.30.—Effect of silicon on energy loss.

Campbell verified this law experimentally in several materials, but found that in some there is a tendency for the hysteresis loss to increase with frequency. Wild has also experimentally examined the effect of wave form on iron loss, using various forms of wave. With two waves, having amplitude factors of 1·57 and 1·295 respectively, he found the following :

TABLE XXIII.—*Variation of Iron Loss with Wave Form.*

$B_{\max}$ at 50.	Armature iron W/lb. Wave 1·57.	Wave 1·295.	% increase.	Lohys iron W/lb. Wave 1·57.	Wave 1·295.	% increase.	Stalloy W/lb. Wave 1·57.	Wave 1·295.	% increase.
2,500	0·122	0·131	7·4	0·113	0·122	7·9	0·065	0·0705	8·5
5,000	0·386	0·415	7·5	0·373	0·392	5·1	0·2145	0·232	8·2
7,500	0·753	0·81	7·6	0·741	0·787	6·2	0·42	0·457	8·8
10,000	1·22	1·31	7·4	1·205	1·285	6·6	0·685	0·743	8·5
12,500	1·79	1·952	9·1	1·787	1·944	8·8	1·018	1·12	10
15,000	2·54	2·78	9·5	2·57	2·84	10·5	1·41	1·56	10·6

Lloyd* has given a very complete discussion of the effects of wave form on iron losses, together with experimental results, and

* *Bull. Bureau of Standards*, Washington, vol. vi, No. 4.

he concludes that by adjusting the phase of a considerable component, the iron losses may be considerably reduced, the third harmonic in reversed phase giving probably the best wave form.

In the same way Wild investigated the percentage of eddy-current loss in the material which was dependent upon the higher power of the frequency. The Table XXIV summarizes his results for a nominal B_{max} 10,000 :

Campbell, in his research, also examined the effect of thicker laminations and results in agreement with those obtained theoretically from the formula given by J. J. Thomson, viz. that the eddy-current loss varies as :

$$\frac{\text{Sinh } 2ma - \sin 2ma}{\text{Cosh } 2ma + \cos 2ma},$$

where $2a =$ the thickness of the lamination and $m = 2\pi\sqrt{\mu f/\rho}$, where f is the frequency, μ the permeability (assumed constant), and ρ the resistivity of the material in absolute units.

From what has been said it will be seen that the calculation of the total iron loss is by no means an easy matter, even when samples have been tested in the laboratory and the constants so determined; it must be remembered that the iron is always sensitive to the ordinary workshop processes, particularly in respect to the eddy-current component, for it is difficult to ensure complete insulation of the plates, and anything in the nature of tooling or filing after assembly may lead to a very considerable increase of this component ; while cutting, punching, shearing or compression will increase the hysteresis component by intermediate amounts.

The effect of temperature on the magnetic qualities of iron has formed the subject of several important researches. In general it will be found that under weak magnetizing forces, the permeability rises enormously as the transformation point of the iron is approached, and at this point it falls suddenly to practically zero value.

Some of the nickel steels are, however, an exception, since their permeability decreases with increasing temperature, and this property is made use of for correcting the temperature coefficient of magnets in meters, etc., by means of a magnetic shunt of the material.

Hopkinson found the transformation point of soft iron to be 786° C., with a carbon content of 0·01% for mild steel. With

TABLE XXIV.—*Variation of Iron Loss with Frequency and Wave Form.*

Frequency.	Armature iron plates 13·6 mil. Total loss (joules per lb. per cycle).		Armature iron plates 13·6 mil. Per cent. eddies in total loss.		Lohys iron plates 14·2 mil. Total loss (joules per lb. per cycle).		Lohys iron plates 14·2 mil. Per cent. eddies in total loss.		Stalloy iron plates 14·8 mil. Total loss (joules per lb. per cycle.)		Stalloy iron plates 14·8 mil. Per cent. eddies in total loss.	
	1·57	1·295	1·57	1·295	1·57	1·295	1·57	1·295	1·57	1·295	1·57	1·295
25	0·02125	0·02305	14·8	13·6	0·02	0·0216	25·2	18·6	0·0125	0·01365	9·6	8·8
50	0·0244	0·0262	25·8	24	0·0241	0·0257	33·5	31·4	0·0137	0·0137	17·5	16·3
75	0·02755	0·02935	33·2	31·2	0·0281	0·0297	43·1	40·8	0·01485	0·01485	23·9	22·2
100	0·0307	0·0325	41	38·8	0·0321	0·0337	50·3	47·6	0·01605	0·0172	29·6	27·6

0·126% carbon it was 743° C., and for hard steel containing 0·962% carbon 695° C.

D. K. Morris, working on an iron containing 0·08% carbon, found the value 780° C., and the permeability reached 14,600 in a field of $H = 0{\cdot}2$; and Wills found the value of μ to rise to 17,200 in a field $H = 0{\cdot}172$ for wrought iron. Terry, in a research on electrolytic iron, found the transformation point occurred in that substance at 785° C. His results upon this material are shown in the following table of observations on one sample annealed :

TABLE XXV.—*Variation of Magnetic Properties of Electrolytic Iron with Temperatures.*

Temp. °C.	Energy loss.	Retentivity.	H_c.	Max. induction.	Max. μ.
23	21,300	7,940	3·8	17,100	1,040
780	11,280	8,450	1·3	17,300	3,070
1,000	5,060	14,100	0·75	17,280	9,080
1,100	4,900	12,800	0·53	17,600	11,000
1,200	5,600	13,000	0·85	17,400	8,750
1,300	7,160	13,800	0·97	17,400	7,120

The general trend of the energy loss is to fall, as the temperature rises, to a minimum. Drs. Beattie and Gerrard, working at the other end of the temperature scale, found that there was a definite increase in the hysteresis loss at the temperature of liquid air. Thus for soft iron they put the increase at 8% in an annealed specimen and 9% in an unannealed one. Steel showed an increase of 25%, nickel 44% and cobalt 20%.

In 1903 Heusler discovered that it was possible to make alloys from non-magnetic elements which had decided ferro-magnetic properties. Thus an alloy consisting of 62% copper, 12·5% aluminium and 25% manganese has a permeability approximating to that of cast-iron. It was found that simple alloys containing no copper, like MnAl and MnZn, also were ferro-magnetic, and the magnetic properties of such alloys were entirely ascribed to the presence of manganese.

Weiss also prepared elementary manganese by the electrolytic reduction of the chloride of the metal, and this in its amorphous condition was paramagnetic, but when melted in the electric furnace it acted as a ferro-magnetic body. It also has an enormous coercive force, about 670 C.G.S. units, and a retentivity about

0·01% that of iron. Ross and Gray, however, from their experiments on these and kindred alloys, believe that the presence of copper in the alloy plays an important part in their magnetic behaviour, and does not act, as is usually supposed, merely as a solvent to the other constituents.

Wedekind has extended the list of manganese compounds which exhibit magnetic properties, and has shown that aluminium may be replaced by boron, antimony, phosphorus, arsenic, bismuth, etc. The boron compound has a permeability about one-sixth that of iron, and the antimony compound is capable of magnetization to within 30% that of iron. According to Fleming, the hysteresis loss follows the law $W_h = \eta B^{2 \cdot 25}$, η having values 0·000549 for a 60% copper alloy and 0·00077 for a 68% copper mixture. The transformation temperature lies between 250° C. and 450° C. for the various alloys.

Modern Electrical Sheet Steels

During the last 20 years there have been considerable developments in electrical sheet steel, particularly for instruments and transformers; materials have been developed of which the permeabilities are very considerably higher than those of the steels previously used, and also such that the total watt loss has been very considerably reduced. These materials are nickel iron alloys and are known by various trade names, such as "Mumetal," "Permalloy," etc. They are used quite extensively for the cores on instrument transformers, and by their use phase angle errors have been reduced to very small values, and the size of the transformers themselves have also been reduced. In many cases it has been possible with current transformers to replace what would have been a multi-turns primary by one of the straight through or bar pattern.

In moving iron instruments the moving parts, when made of this new nickel alloy, give, in general, a greater torque, and owing to their lower hysteresis, much higher accuracies, when used on A.C. and D.C. Again, it is often necessary to shield delicate instruments from external magnetic influences, and here again the nickel iron alloys have been found extremely useful in providing very efficient shunts.

These materials are usually obtainable in strip and sheet form, and can be punched out into the desired form. It is, however,

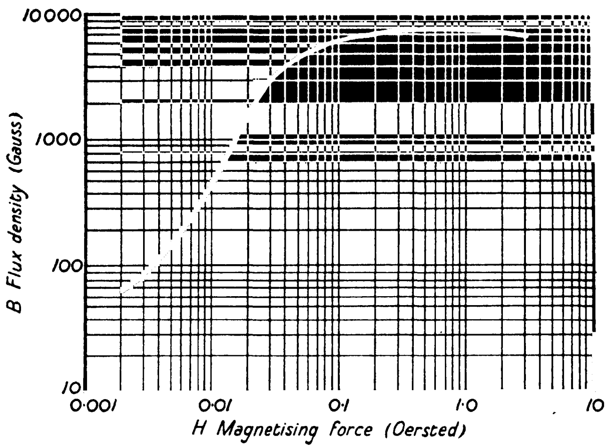


FIG. 5.31.—Magnetic properties of mumetal.

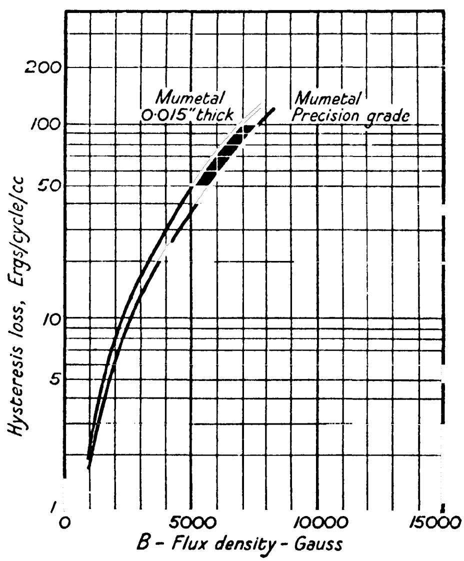


FIG. 5.32.—Hysteresis loss in mumetal.

necessary to subject them to a special heat treatment after all the machining processes have been completed, in order to obtain the maximum magnetic properties.

As an illustration of the magnetic properties of this material, Fig. 5.31 gives a normal D.C. magnetization curve of "Mumetal." The saturation value of this material is about 8,000 gauss, and it reaches its maximum permeability of about 130,000 at a magnetizing force of 0·03 oersteds. The low value of the hysteresis loss is illustrated by the curve in Fig. 5.32, which gives the values again for "Mumetal." The resistivity of this material is high, and consequently the eddy current loss is low.

CHAPTER 6

PERMANENT MAGNET MOVING COIL INSTRUMENTS

THIS type of instrument is by far the most accurate and generally useful for direct current measurements. In fact, were it not for the need of other instruments for alternate current testing, it may be doubted whether any other type of deflectional instrument would have survived except where the limit of cheapness is necessary, in which case the simple "soft iron" instrument holds its place. In comparison with soft iron instruments the moving-coil instrument has the disadvantage of somewhat more costly and delicate construction, and of needing a greater power consumption when measuring heavy currents, but it has, however, the following good features:

Considerable permanence if the magnet is properly designed and aged. Very low power consumption in the instrument. Good torque and ratio of torque to weight. Very even scale of considerable length if desired. Universal application as voltmeter or ammeter of any range when used with suitable series or shunt resistances. Independence of position when properly balanced. Perfect damping. Absence of hysteresis errors and low temperature variation. Is but little affected by stray magnetic fields.

As indicated in the first chapter, the permanent magnet moving coil instrument is an adaptation from the Kelvin syphon recorder and d'Arsonval galvanometer, and it was first brought into practical service as a voltmeter in 1888 by Dr. Weston, and the Weston Electrical Instrument Co. has continued to construct some of the finest instruments of this type. The first Weston voltmeter, of which the modern instruments are merely modifications in details, is diagrammatically shown in Fig. 6.1. A strong, permanent magnet of horse-shoe form is fixed horizontally on a wooden base, and is provided with pole pieces bored cylindrically as in a dynamo machine. In the centre of the space between the poles is a cylindrical core of soft iron which serves the double purpose of concentrating and preserving the magnetic field, and of making it nearly uniform and radial, so that a very even scale is produced. A very light moving coil of rectangular shape is pivoted so as to turn freely in the gap, the current being led to and out of it by

two non-magnetic spiral springs, which also provide the controlling torque. The pointer is a light aluminium tube flattened at one end into the form of a vertical knife edge which moves over the scale beneath, errors of parallax being avoided by the use of a mirror also beneath the pointer.

By making the metal former on which the coil is wound of low resistance, and by keeping the inertia of the coil sufficiently small, the eddy currents generated in the former by its rotation in the magnetic field suffice to damp the instrument perfectly.

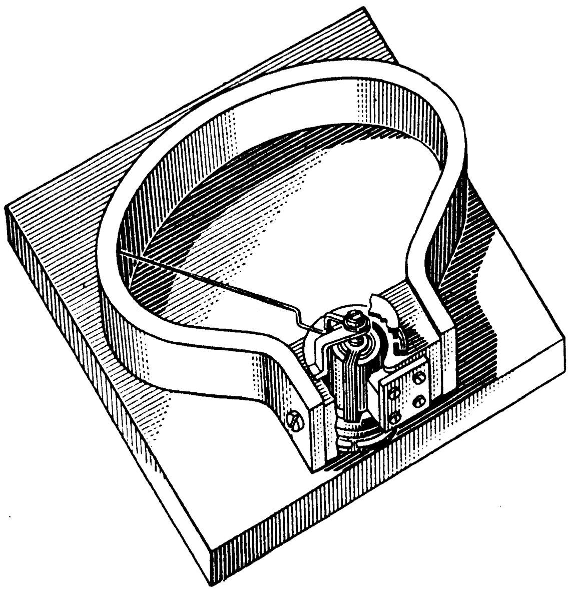


FIG. 6.1.—Early form of Weston moving coil instrument.

These features are preserved in all good modern moving-coil instruments, and by careful design and good workmanship instruments may be constructed of sufficient precision to serve as sub-standards on direct current circuits. Such an instrument is shown in Fig. 6.2, which illustrates the construction of a Weston Laboratory Standard. For use as a voltmeter the instrument is simply connected in series with a high resistance made of low temperature coefficient alloy (Weston invented the alloy, afterwards known as Manganin, for such purposes), and brought out to terminals to which the P.D. to be measured is applied (Fig. 6.3).

To adapt the instrument as an ammeter a resistance also of low temperature coefficient material of comparatively small value is connected in series with the moving coil so that a full-scale deflection is obtained with a fraction (say about $\frac{1}{10}$ of a volt). The current to be measured is then passed through a low resistance or "shunt" producing a drop of potential proportional to the current, which is indicated by the instrument when connected to its two potential terminals P.P. in Fig. 6.3.

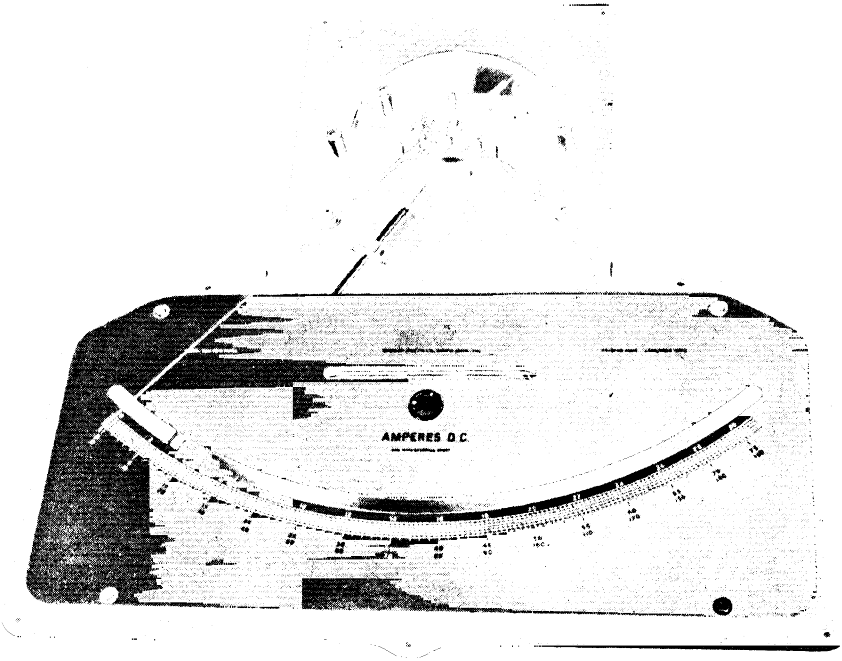


FIG. 6.2.—Weston Lab. Standard.

From the manufacturing standpoint the chief objections are the cost of the permanent magnet and the delicacy of construction of the moving system. If the instruments are made in large quantities by suitable machinery, however, the cost can be very considerably reduced. The main points which have been aimed at in the most recent developments of this type of instrument are the constancy and uniformity of the permanent magnets, the increase of the field strength and reduction of pivot friction, so that the drop of potential allowed across the shunt may be reduced

to a minimum, while still preserving sufficient resistance in series with the moving coil to " swamp " its own temperature variation. According to Janus, the ratio of torque to weight should not be less than 0·167, while Heinrich and Bercovitz put the ratio at 0·05. The figures in Table XXXI show that this lower figure is seldom reached in modern practice, and it is certainly desirable to keep the ratio as high as possible. Another aim has been to standardize the form of the magnetic circuit and to provide a suitable and convenient adjustment to the controlling springs, etc., so that the instrument may be set to read correctly on a predetermined printed scale.

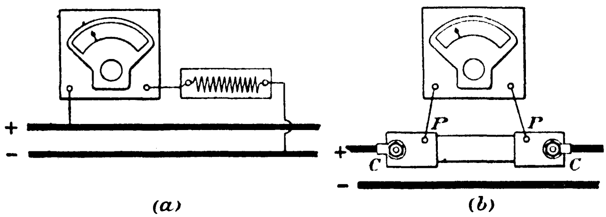


FIG. 6.3.—Method of connecting moving-coil instrument, (a) as voltmeter and (b) as ammeter.

The modern tendency has been to adopt 75 millivolts and 15 milliamperes as the working P.D. and current in the " millivoltmeter " of the moving coil with its swamping resistance, which means that the total resistance is 5 ohms ; of this not more than 1 ohm should be in the coil and springs and 4 ohms at least in the swamping resistance, thus reducing the temperature variation of the combination to 0·4/5 or 0·08% per 1° C. The power consumption in the instrument coil and swamping resistance is then $0{\cdot}075 \times 0{\cdot}015 = 0{\cdot}001125$ watt at full-scale deflection.

As a voltmeter the resistance is usually 100 ohms per volt at the normal working P.D. and the total power consumption only 1 watt for each 100 volts, although with portable instruments nowadays the tendency is to use 500 or 1,000 ohms per volt. But as an ammeter with 75 millivolts drop, this means that with 150 amperes 11·25 watts are lost in the shunt, or with 1,500 amperes 112·5 watts, and so on, which is much higher than in soft iron instruments. This loss of power is not serious in itself, but it needs a sufficiently heavy and bulky shunt to dissipate the loss without undue heating.

CONSTRUCTIONAL DETAILS OF MOVING COIL INSTRUMENTS. MAGNETIC SYSTEMS

In practice by far the greater number of manufacturers adopt the simple U-shaped magnet (Fig. 6.4*a*), and in most cases the limbs are made long in comparison with the distance between them. This is not so, however, when the new high coercive fine alloys are used. Table XXVI gives a few typical dimensions for magnets and coils.

The advantages of a long magnet have already been pointed out in Chapter 5, and in this respect the expanded horseshoe type (Fig. 6.4*b*) has definite advantages, but it is, of course, more costly to produce. The same is true of the circular form (Fig. 6.4*c*) which is adopted by some instrument makers, and also of the Siemens and Halske form (Fig. 6.4*d*).

Instances of laminated magnets are sometimes found in miniature type instruments, and although it is now well known that lamination does not contribute anything to the perfection of the magnet, it is

TABLE XXVI.—*Dimensions*

Maker.	Total length of magnet (mm. l).	Distance between limbs (d).	Ratio $\frac{l}{d}$.	Width of limb.	Depth of limb.	Sectional area of magnet (sq. mm.).
Reiniger, Gibbert & Schall	300	41	7·32	15	31	465
Abrahamson	270	40	6·75	14·5	25	362·5
Hartmann & Braun	260	40	6·5	8·5	30	255
Keiser & Schmidt	290	34	8·54	9·3	33·5	311·5
Nalder Bros. & Thompson	260	47	5·83	9	24	216
Crompton & Co.	320	50	6·4	12·5	25	312·5
Nadir	265	42	6·32	9	34	306
Carpentier	250	30	8·33	10	18·5	185
Siemens & Halske	250	41	6·1	11·5	30	345
A.E.G.	280	43	6·5	10	31	310
Weston	260	40	6·5	12	30	360
Everett, Edgcumbe	250	40	6·25	11	31	341
Compagnie Compteur	260	34	9·12	10	31	310
Weston	310	41	7·57	10	31	310
Chauvin & Arnoux	190	..	..	25	14	350
Record	25	400	6·25	11	28	308
Record	435	41	10·85	11	42·5	469
Record	315	35	9	13	35	455
White Elect. Inst. Co.	232	30	7·75	9·5	19	180
Crompton & Co.	275	50	5·5	12	27	324

found convenient to obtain the steel in the form of punchings which can be hardened and magnetized and then assembled to the required depth. On the whole, however, such magnets are very inferior to those forged from the solid metal.

Single gap magnets have been employed by several makers, like the Meylan type shown in Fig. 6.4*e*, which was the type employed by the Compagnie des Compteur and also by Gaiffe. In such magnets the polar extensions curve over one another as shown, and the coil is pivoted on an axis parallel to and nearly coinciding with one of its long sides, so that only the other long side is in the field and is effective in producing torque.

Westinghouse have also employed a modification of this type in which the curved pole pieces are vertically above one another, and the active coil side is then radial from the axis of rotation.

In Fig. 6.4*f* is shown an early form of Davies magnet, which was, however, abandoned as not giving sufficient induction in the gap and not being well designed for permanence. It was replaced by a form which employed two large circular magnets to provide the

f Magnets and Coils.

Diameter of pole bore.	Length of single air-gap (mm.).	Permanence safety factor.	Width of coil (mm.).	Length of coil (mm.).	Ratio $\frac{\text{length}}{\text{width}}$.	Winding breadth (mm.).	Type of magnet.
28	1·5	419	25	35	1·4	9	Ordinary U.
26	2	253	24·5	30	1·225	7·1	,,
26	1·75	474	25	29	1·16	8·5	,,
28	1·5	610	25	33	1·32	9	,,
22·3	1·9	354	26	29	1·45	6·5	,,
29	2	389	27	35	1·3	10	,,
28·5	1·25	700	26	36·5	1·4	8·75	,,
24	2	290	23	21	0·915	6	,,
27·7	1·6	395	26·7	33	1·235	8·5	,,
29	3	252	27	38	1·4	8	,,
27	1·25	490	..	..	..	8	,,
27·5	2·25	292	25	40	1·6	9·5	,,
26	2·5	284	16	30	1·875	6·5	Meylan.
26·5	1	865	25	37	1·48	8·5	Expanded horseshoe.
22	..	..	..	..	..	..	Circular.
26	2·25	321	23	30	1·3	7	Ordinary U.
..	1·75	577	20	11·5	0·575	8·5	Cirscale magnet.
..	1·25	304	14·5	7·4	0·51	7	Small cirscale
21·8	1·9	322	20	22·5	1·125	6	Ordinary U.
25	2·5	274	22	26	1·18	8	,,

flux to the concentric pole system in order to obtain the long scale which characterized these instruments.

The Record Cirscale instruments employ the form of magnet shown in Fig. 6.4*g*. In this the vertical gap flux is linked with the two horizontal sides of the moving coil which embraces the central pole, so that the two gaps above and below this pole may be regarded as in parallel.

Another magnet system of this type was employed by the Whitney Co. in America. In this case an ordinary-shaped magnet is turned on its side so that one limb is vertically over the other;

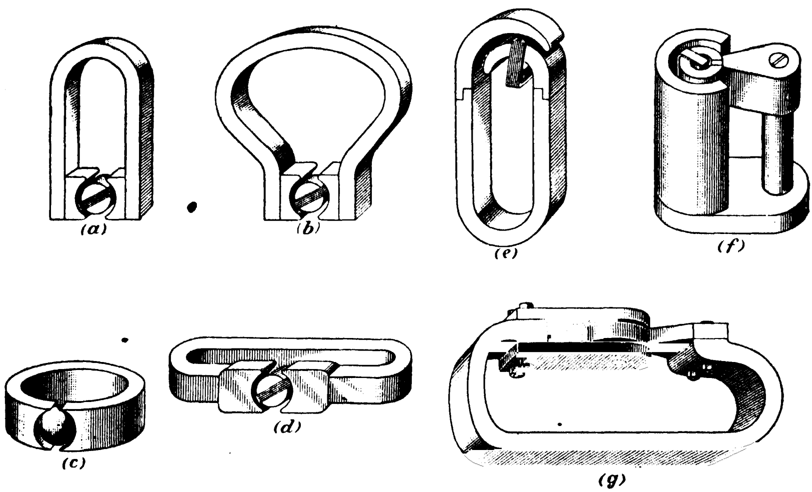


FIG. 6.4.—Types of permanent magnet.

the upper limb is shortened and provided with a ring-shaped pole piece, while the lower is fitted with a trough-shaped polar extension which embraces the outer portion of the ring so that a vertical flux traverses the gap space above and below the ring pole. The coil is threaded on this latter so that three of its sides are in the field.

One of the advantages claimed for these homopolar-type magnets is the low reluctance of the air-gap owing to the large pole face area and the shortness of the gap, which, at most, is only equivalent to half that in the bipolar type.

The faces of the magnet limbs are ground true after hardening for the reception of the pole pieces, which are usually screwed to them by steel screws passing right through the limb. These

are of soft, permeable material, and in some designs are cast in solid with a non-magnetic alloy, so that the pole bore may be truly cut and faced in the solid block of material before it is inserted between the polar extremities of the magnet (Fig. 6.5). On the other hand, many manufacturers screw the pole pieces to thick front and back plates of brass or some non-magnetic metal, one of these plates often supporting the central iron core as a fixed part of the

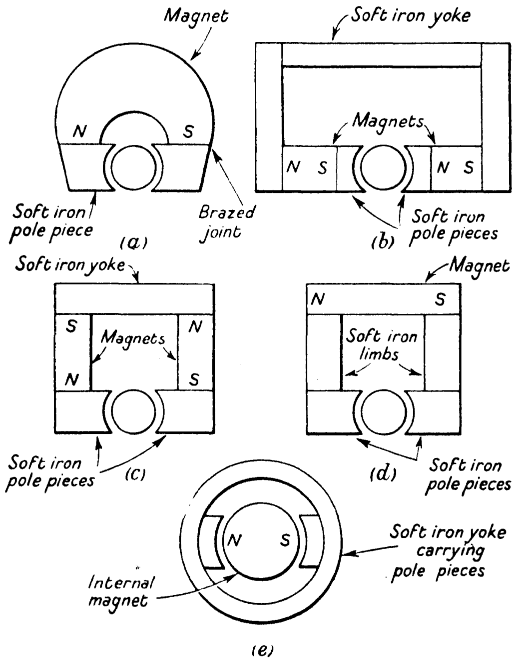


FIG. 6.5.—Methods of mounting magnets and pole pieces.

magnetic system, as, for example, in the Weston and Nalder instruments.

The introduction of the latest forms of magnetic alloys, which are characterized by a greatly increased coercive force, has modified somewhat the methods of construction of the magnet system. The higher coercive force has entailed the use of shorter magnets, and the hardness of the material, which makes machining very difficult, has caused various methods of assembly to be introduced. These shorter magnets also enable the physical dimensions of the

instrument to be reduced, and small, circular instruments have been produced as small as $1\frac{1}{2}$ in. in diameter. The materials are usually cast, and in very small sizes, sintered and are usually held by some form of clamp on to the pole pieces and supporting plates. Generally soft iron pole pieces are used, although occasionally a magnet is shaped so as to form its own poles. The disadvantages of this latter method are the necessity of machining the bore in the pole pieces to an accurate dimension, and the possibility of slight non-homogeneity in the material at the surfaces of the poles resulting in an uneven distribution of flux round the gap, and consequently a departure from a true linear scale.

Diagrammatic arrangements of the magnets and pole pieces are given in Fig. 6.5. In (*a*) the magnet is cast to a horse-shoe shape, the pole faces ground and the pole pieces brazed to it. The pole pieces are prepared in one piece and bored and cut after brazing. Care must be exercised in carrying out the brazing operation to ensure that the temperature of the magnet is not raised too high, or its magnetic properties may be impaired. A temperature of 600° C. for a short time does not seem to have any deleterious effect on Alnico. The method of construction has the advantage that no supporting plates or clamps are necessary, the mounting of the magnetic system being carried out by screwing the pole pieces directly to supports mounted on the base of the instrument. This is quite important in miniature instruments, for example, where space is limited. Since quite short magnets can be used a construction such as shown at (*b*) can be used, and magnetically this is perhaps the most efficient arrangement, since the magnetic leakage flux is kept to a minimum, and the largest possible fraction of the total magnet flux is used in the working gap. Although short magnets can be used the dimension across the pole pieces may still be too high to enable the construction (*b*) to be used in circular-cased instruments, and so that shown at (*c*) is often used. This is not so efficient as arrangement (*a*) as the leakage flux is greater, but it is rather more compact. A variation of (*c*) is to incline the magnets at an angle, giving a construction which is a compromise between (*b*) and (*c*), and one which lends itself readily to assembly in a circular case. Another form of construction which is frequently used is given at (*d*). Here one magnet is used linked to the pole pieces by soft iron limbs. This is the least efficient of the three (*b*), (*c*) and (*d*) magnetically, but owing to the

large total flux which can be obtained from such a magnet, it is still possible, even with the greatly increased leakage, to obtain a working flux in the air-gap considerably higher than with the older magnet steels, and yet have a smaller and more compact magnet system. All three arrangements (*b*), (*c*) and (*d*) have constructional difficulties, since, in general, plates and clamps are necessary to hold the various components together, so that some of the advantage of reduced magnet size is lost in the additional space required by the components necessary to hold the arrangement together.

An interesting development which has only been made possible with these new alloys is that which is known as the internal magnet instrument, shown diagrammatically in Fig. 6.5*e*. In these the conventional centre core inside the moving coil becomes the permanent magnet, and this and the moving coil are surrounded by a circular yoke. This construction, which is used, for example, in the Nalder-Lipman instrument (Fig. 6.6), gives a very compact movement, and permits the construction of instruments down to $1\frac{1}{2}$ in. diameter. A similar construction is used in some modern forms of moving coil relay. In this case the moving coil is large and may be so much as 2 in. square. This again is pivoted and swings round a central core, which forms the magnet, the whole being surrounded by an iron return ring. One of the most important features required in this relay, as in all contact-making devices, is a very high torque, and with the large moving coil used and the powerful magnet it is possible to obtain a torque amounting to 1,000 gramme-cm. or more.

An example of the complete cast magnet without separate pole pieces is given in Fig. 6.7, which shows the arrangement of a miniature instrument made by Everett, Edgcumbe & Co., Ltd. The form shown in Fig. 6.5*a* is used by the Salford Instrument Co. as shown in Fig. 6.8 in one of their instruments. An interesting example of the construction of Fig. 6.5*b* is shown in Fig. 6.9, which shows a magnetic system used by the Metropolitan Vickers Electrical Co., Ltd. This system consists of two pole pieces, two small magnets and an outer return ring all held together by a die-casting process. This design has the advantage that it is possible to obtain various values of gap flux without changing the process of manufacture by using magnets of different materials. The magnets are totally covered by the die-cast material. Two

recesses in the horizontal diameter are provided to accommodate spigots on the movement frame to position the core centrally. The magnet is finally bored after die-casting, and is located from

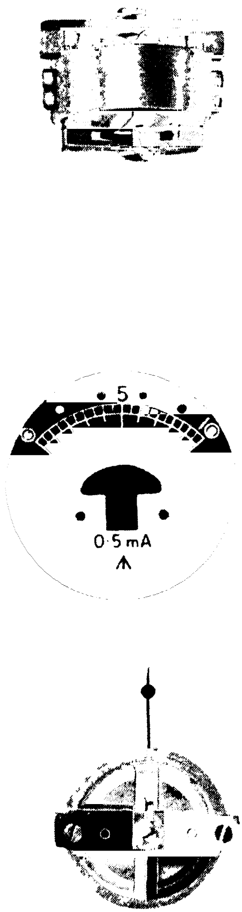


FIG. 6.6.—Miniature moving-coil meter with yoke surrounding coil. (Nalder Bros. & Thompson).

the two spigot holes during this operation. A magnetic shunt for fine adjustment is incorporated in the assembly. This acts as a shunt across one pole gap, and consists of two discs with a spring in between so that it is self-locking on the screwed stud on which

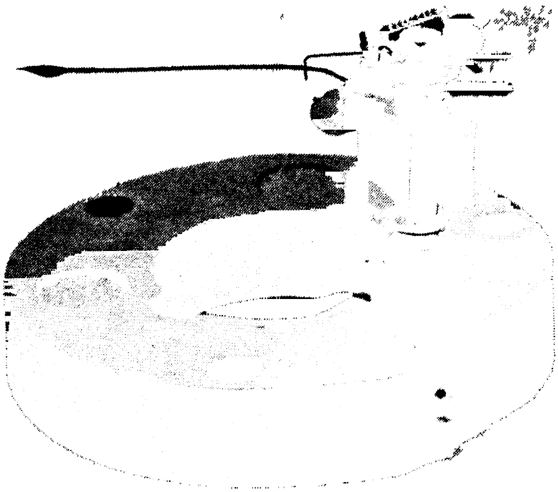


FIG. 6.7.—Example of complete cast magnet without separate pole pieces.

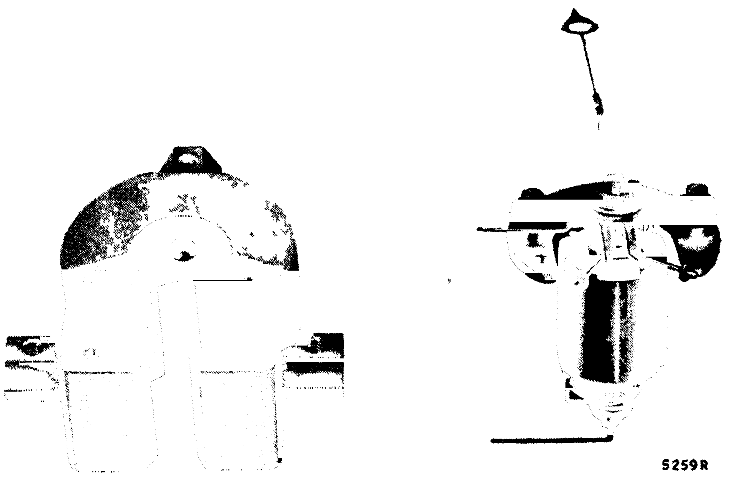


FIG. 6.8.—Movement of the form shown in Fig. 6.5*a* (Salford Instrument Company).

they rotate. A practical example of the construction of Fig. 6.5*d* is shown in Fig. 6.10, which shows a system used by Everett, Edgcumbe & Co., Ltd. and in which the block magnet and the side limbs together with the necessary clamping bolts can be seen. Hartmann & Braun, Siemens & Halske and others overcame the difficulty by supporting the core in a rectangular frame of non-magnetic metal, the sides of which slide in the interpolar spaces between the pole shoes, and the lower portion of the frame forms the support for the bottom jewel.

In instruments constructed by Messrs. Everett, Edgcumbe the pole pieces are cast in solid, and the filling material projects inwards between the polar horns, and is there machined off so as to form a pair of slides which engage with the slides of the iron core and centralize it.

Messrs. Crompton originally guided the core into a continuous pole bore by providing it with two sets of non-magnetic studs which project from either side of it by a gap length, these schemes being rendered possible by employing inwardly-projecting pivots in the first case and pivots inserted in the stationary core in the second.

In modern instruments the separate assembly of movement frame, central core and movement is almost universal. One example of this is shown in Fig. 6.11, which shows a movement made by the Metropolitan Vickers Electrical Co., Ltd., and used with the magnet of Fig. 6.9. The core is cast integral with the frame to obtain consistent accurate location, and also to avoid drilling and tapping small holes and subsequent assembly operations. The rib along the front of the core and the rear limb of the bracket are arranged with suitable grooves to engage the pole tips of the magnet, so that the movement can be inserted into a magnetized magnet without any risk of damage to the coil due to attraction of the core across the poles. Two members extended from the platform are suitably arranged with the rib on the core to form a three-point seating for standing the movement on the bench. A groove on the rear limb and the underside of the platform conveniently houses the lead from the rear control spring for connection at the front of the instrument. Other complete movement assemblies ready to slip into the magnet are shown in the Everett, Edgcumbe instrument (Fig. 6.7) and in the Salford instrument in Fig. 6.8.

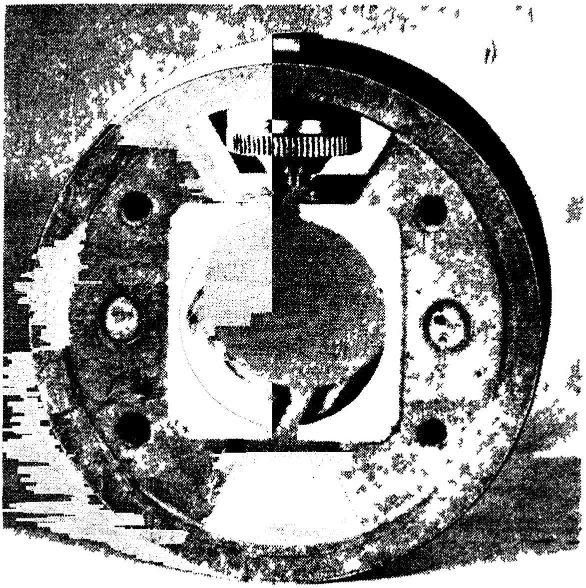


FIG. 6.9.—Magnetic system held in a die casting. (Metropolitan Vickers Co.).

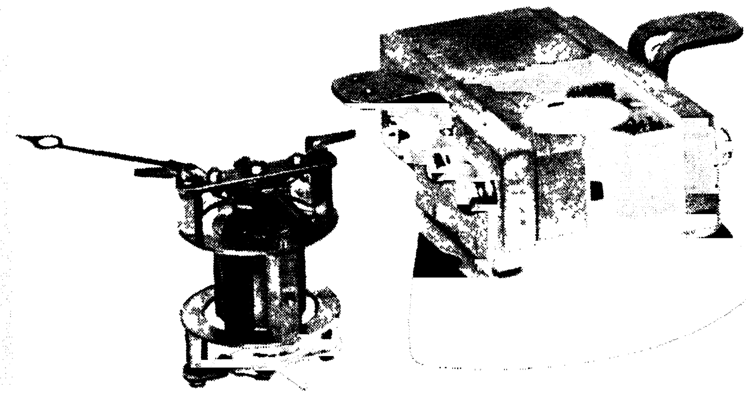


FIG. 6.10.—A practical example of the construction of Fig. 6.5*d*. (Everett Edgecumbe & Co.).

The most usual forms of pole tip are shown in Fig. 6.12, and the interpolar space is seldom less than 8 mm. or more than 10 mm., giving a pole arc of 130° to 150°.

The cylindrical core with well-rounded edges is almost universal, except in instruments by Chauvin & Arnoux, Franke, and the Cambridge Instrument Co., where a spherical core and circular coil is employed.

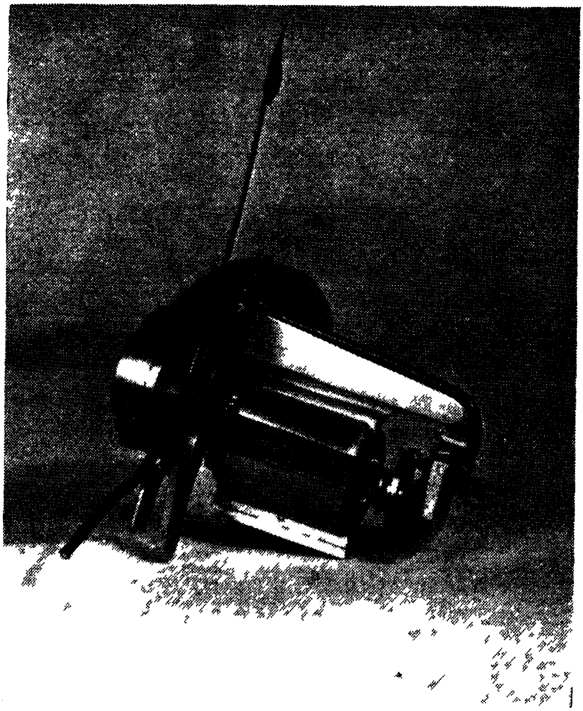


FIG. 6.11.—Assembly of Movement as 1 unit, the core being cast integral with the frame. (Metropolitan Vickers Co.).

The core is finished to a fine surface, and should then be efficiently protected against oxidation by lacquering with a hard-setting smooth lacquer, a thin stove enamel, tinning, electroplating, or by calorizing, it being very essential to prevent oxidation from taking place, since the coil usually fits the core with a minimum amount of clearance at the sides, and therefore the slightest increase in diameter may produce a sticky movement. Meyer, Kelvin,

Gans and Goldschmidt, and Carpentier flattened the core cylinder at the sides opposite the interpolar spaces (Fig. 6.12*b* and *f*), and Weston cut a shallow groove with a view to assisting the uniformity of the gap flux by preventing it from spreading at the pole horn; at the same time this allows the coil to be slipped over the core without the risk of deforming or distorting it. With independent pivots the core is left in the form of a solid cylinder, but in the early Evershed and Scholler and some Crompton instruments

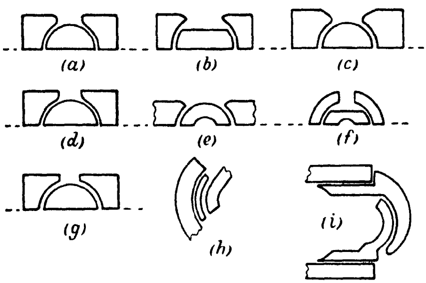


FIG. 6.12.—Types of pole piece and core for moving coil instruments.

a complete spindle is adopted. Evershed drilled the core completely through, using an open-ended coil (Fig. 6.14*g*), while Scholler and Crompton also slit the core, so that the coil can be slipped in place over it before it is finally fixed to its support.

The degree to which the steel is magnetized seems to vary very widely. The value of B in the magnet may be as high as 6,000 to 7,000, but, on the other hand, values as low as 700 to 800 are occasionally met with. The leakage factor will depend upon the dimensions and general form of the magnet, but even with magnets of similar shape very diverse values are found; thus in the case of U-shaped magnets factors varying from 4 to 2·2 have been obtained.

The Moving Coil and Attachments

The wide rectangular coil is almost universal, the form being such that the ratio of length to breadth is between 1·3 to 1·5, these figures giving about the best compromise between high weight efficiency and high torque efficiency. The winding width measured across the former is usually kept as narrow as possible, 6 to 10 mm. being the limits in practice, and the broader the coil is in this

direction the more likely is trouble to occur from fouling the poles, and the greater is the difficulty of getting it into place over the iron core without serious distortion.

Formers of aluminium or copper are universal, and as already mentioned, these should be pressed up from sheet metal without joints, and the long sides curved to a radius so that the conductors are concentric with the core to allow of the use of the shortest possible gap. Aluminium is probably the better material both on account of weight and inertia, but many manufacturers employ copper, although there is usually no difficulty with careful design to obtain the required damping with aluminium, and in some instances it is even necessary to employ materials like phosphor bronze or German silver in order to prevent over-damping. Some of the Chauvin instruments are unique in that they have an outer copper damping ring over the winding of the circular coil.

The former is lightly insulated by lacquering or serving it with a thin layer of insulation before winding, and the conductors are laid over this as evenly as possible, great care being taken to regulate the tension so as to prevent as far as possible the tendency to buckle or distort the former.

Copper wire, insulated with enamel, silk, nylon, Formex, etc. is very generally employed, but of recent years, as mentioned in Chapter 5, aluminium wire is being substituted in order to reduce weight and inertia.

The ampere turns in different instruments vary over a wide range ; the minimum value seems to be about 0·2 and the maximum 1·5, but the majority of modern instruments have a value between 0·4 and 0·7. Milli-voltmeters have usually a single layer winding, which not infrequently consists of two or more sections connected in parallel, and a smaller number of ampere turns are employed so that the resistance shall be kept down and a smaller P.D. will be required to produce the full-scale deflection. The torque will therefore be somewhat less than in the case of voltmeters, and lighter springs are necessary, and as these may form a considerable portion of the whole resistance, special alloys of high conductivity are sometimes employed, but usually such materials have elastic properties which are inferior to that of phosphor bronze.

In some instruments the difficulty is surmounted by using two broad, thin ligaments of gold foil at the lower end of the coil (Fig. 6.13), arranged in such a way that they exercise practically

no control on the movement. A single phosphor bronze spring is then sufficient to provide the necessary control.

The various forms of coil and methods of pivoting are shown in Fig. 6.14. The attachment of the pivots has already been

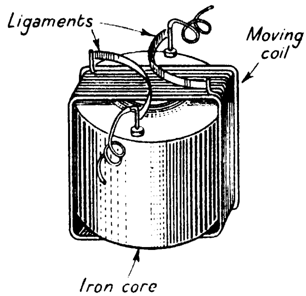


FIG. 6.13.—Coil with leading-in ligaments.

discussed in Chapter 2. In some instances inside pivots are attached to the undersides of the former, as in Fig. 6.14*e*. The jewel mounts are then inserted in holes drilled in the axis of the iron core, and one or sometimes both are urged outwards by spiral

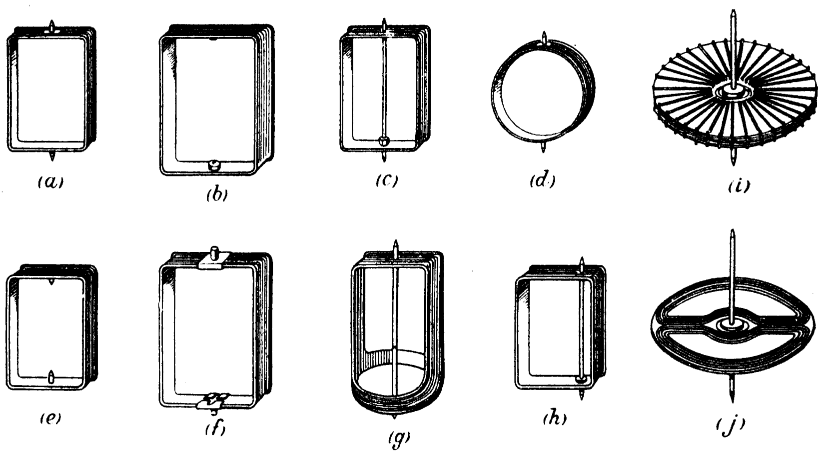
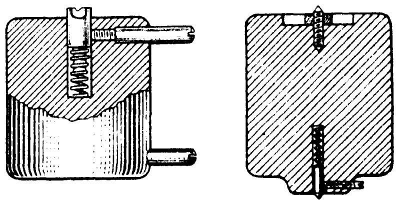


FIG. 6.14.—Types of moving coils.

springs which are inserted in the holes below the jewels; after adjustment with the coil in place they are secured by screws passing at right angles through the core, as in Fig. 6.15*a*. In some continental instruments employing this form of pivoting no jewel

adjustment is provided and the very objectionable system of springing the coil into place is adopted, and this is liable to damage both pivot and jewel besides tending to distort the coil.

The early Crompton instruments had the inverse of this arrangement, and in several respects were a departure from the usual practice. In these instruments the pivots are of comparatively large dimensions and are inserted in the iron core, the bearings being carried on the coil former. The lower pivot is spring-seated for purposes of adjustment. This arrangement of pivoting is an important improvement, for if the pivots rotate and are more than of microscopic dimensions, they may become laterally magnetized and cause changes of zero. The method, however, necessitates the employment of sharp pivots and phosphor bronze cups, since it is not possible to balance the movement perfectly so as to prevent



(a) (b)

FIG. 6.15.—Methods of inserting jewels or pivots in core.

rolling in hard jewels. The core and coil were then attached by two screws to a ring-piece, which in turn is fastened to the magnet system, so that the whole movement can be withdrawn from the face of the instrument. One spring only is used for control, and is actuated at the front end, its outer end being held in a clamp mounted on a milled ring rotating on the core-frame and providing the zero adjustment. The gold ligament arrangement mentioned above is employed to convey current to the coil so that the spring is not included in the electric circuit.

Coils with a continuous spindle (Fig. 6.14*c* and *g*) are rarely employed, despite the advantage mentioned in Chapter 2.

The open-ended coil (*g*) was employed by Evershed, but is, of course, more expensive to construct and has a higher inertia than the simple rectangle. On the other hand, it provides for the complete removal of the movement without disturbing any part

of the magnetic system, and therefore reduces the liability to change to field strength which otherwise might occur.

In some portable instruments by the Whitney Co. in America pivots are dispensed with altogether, and the coil is fitted with two "jewel holes" and threaded on a fine, tightly-stretched non-magnetic wire which passes through a central hole in the core, and serves to define the axis of rotation while the coil is kept vertically central by the upper and lower helical control springs. It is claimed that this system of mounting enables the instrument to withstand very rough usage.

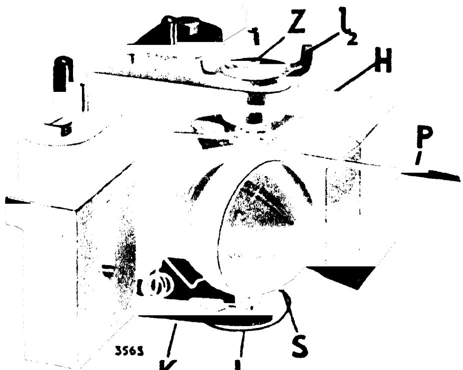


FIG. 6.16.—Movement of "Unipivot" Instrument.
(R. W. Paul and the Cambridge Instrument Co.).

Robt. W. Paul successfully developed a sensitive moving coil instrument in which the coil was supported on a single pivot, practically at the centre of gravity of the moving system. The circular coil is wound on a brass or German silver former to which is attached the single pivot staff. A spring jewel is supported in the centre of the spherical iron core, which is made in two halves and bolted together. A single helical spring of phosphor bronze above the coil serves as a control and current lead, the other end of the winding terminating in a ligament below the coil (Fig. 6.16). Such instruments are, however, essentially for horizontal use and work with a very low torque. In most instruments two springs are employed which serve the double purpose of control and current leads; generally these are situated one above and the other below the coil, but occasionally, as, for instance, in the Hartmann &

Braun instruments, they are both situated at the front end of the coil.

In the Record instruments (Fig. 6.17), the two springs are situated below the coil, and are wound in opposite directions to compensate for temperature change.

Usually the outer end of the spring is fixed to the spring tongue on the jewel bridge, while the inner end is attached to a collet on the axis, but in the Chauvin and Carpentier instruments this

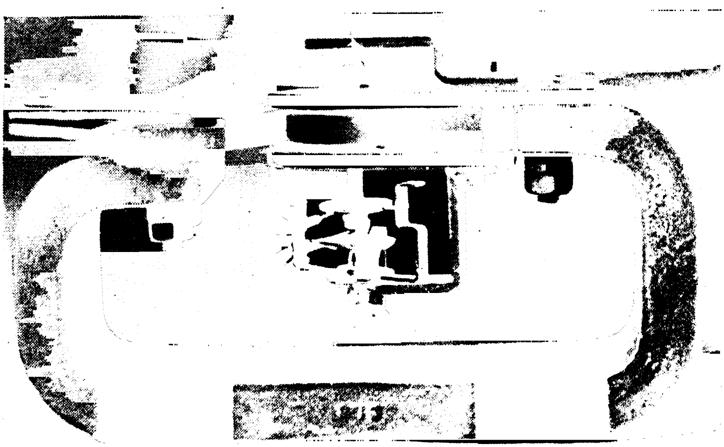


FIG. 6.17.—System with springs fitted below the coil. (Record Electrical Instrument Co.)

arrangement is reversed, the outer end of the spring being soldered to an arm which is really a prolongation of the pointer backwards and thus moves with it, while the inner end is attached to a collet clamped under the locknut of the jewel screw.

With single-gap magnets, as already mentioned, the pivot axis is placed near to one side of the coil, and this really constitutes one of the defects of the system, since it leaves the greater portion of the mass of the coil to be counterbalanced on the other side of the axis. In most of the instruments of this class the counter-balance is partly effected by the pointer; but this is not always sufficient, and additional counter-weights are necessary. This means a heavy movement and a considerable moment of inertia.

The Whitney instrument mentioned on page 309 suffers particularly in this respect, for the coil is actually carried on a short

arm from the axis of rotation, and the inertia is so high that there is considerable difficulty in obtaining adequate damping.

In the Record Cirscale instruments (Fig. 6.17) the axis practically coincides with the coil side, and this, together with the very narrow coil (which makes the quantity of inactive material at the maximum radius small), reduces the amount of counter-balance required very considerably so that the inertia is not excessive,

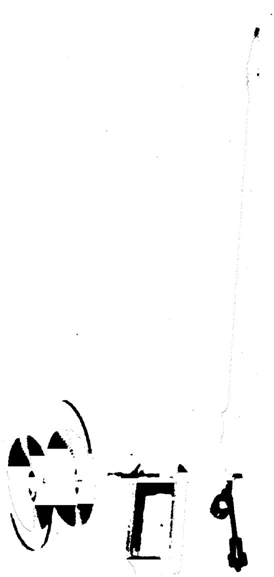


FIG. 6.18.—Arrangement of coil and pointer on the spindle of the record instrument.

and, since both sides of the coil former are effective, good damping is readily obtained ; moreover, the narrow coil has the advantage of reducing the length of inactive conductor, and therefore permits of greater swamping resistance in milli-voltmeters than is usual in the ordinary type of instrument. Fig. 6.18 shows the arrangement of the coil and pointer on the spindle. As seen in Fig. 6.17, the control springs are below within the arch of the magnet, and are therefore freely accessible. The jewels are carried by angle brackets screwed to the upper and lower pole pieces, and the central pole is slit along the principal magnetic length to facilitate the intro-

duction of the coil, and the joint made in this way does not interfere with the flux distribution.

A very long scale (300 degrees) is obtained with this form of instrument with practically even divisions from end to end (Fig. 6.19,) and in both this and the Whitney instrument cited above the arrangement of field is practically astatic, so that the influence of external fields is greatly reduced without any form of shielding.

Another great convenience in this arrangement is that of being able to examine the clearance between coil and pole pieces at all times without in any way interfering with the assembly, for it is

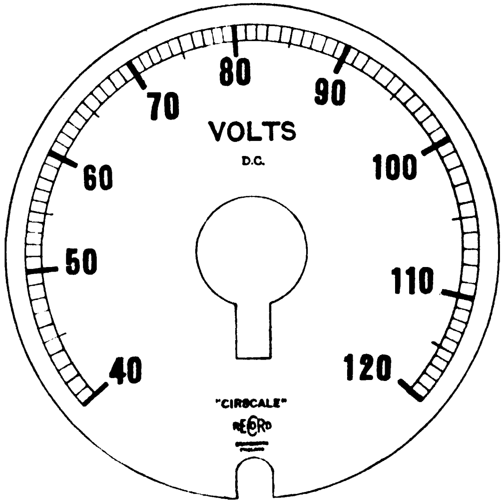


FIG. 6.19.—Scale of Record instrument.

only necessary to look through the gap at a brightly illuminated surface and the presence of any foreign matter likely to impede the movements of the coil is thus easily detected and removed.

Discoidal forms of coil (Fig. 6.14*i* and *j*) have been employed by Kennelly and also by Thompson. In the Kennelly instrument a single gap magnet is employed in which the pole pieces are one over the other, and in the horizontal gap so produced the disc coil moves. This consists of an aluminium disc on which the active conductors are disposed radially so that they pass outward from the centre to the periphery, and are then carried round half the circumference and brought radially inwards from the other end of a diameter which is not in the magnetic field ; a similar path is

traversed by each turn, but displaced angularly from that of the last until the whole disc is covered, and the two ends of the winding are then terminated in two spiral control springs on the spindle above the coil in the usual way. A similar instrument was constructed on the Continent by Feussner and Fordermann.

The Thompson instrument is similar in principle, but instead of employing conductors distributed uniformly over the disc they were concentrated into two **D**-shaped coils on the aluminium disc, the straight sides being along a diameter. Two permanent magnets embraced the disc, one on each side of the spindle, the direction of the fields through the disc being opposite. The arrangement is therefore astatic, and practically independent of external fields.

The chief defects of such discoidal coils are: (*a*) The costliness of manufacture; (*b*) the relatively large weight on the pivots; (*c*) the large amount of inactive conductor which has to be grouped at the maximum radius and thus considerably increases the inertia of the moving system; (*d*) they usually require a longer gap in the magnetic circuit.

In the case of most other types of instrument the gap seldom exceeds 2 mm., or for mechanical reasons falls below 1 mm. The Weston instruments working with a gap of this length, which is the shortest of any examined, demand very perfect workmanship and materials. The torque in this type of instrument is high, in most cases being of the order of 400 to 600 dyne cm. If the constants of the instrument are known, this may be easily calculated as indicated in the following design section; otherwise, since practically all instruments are spring controlled, it may be experimentally determined by hanging weights upon the pointer and turning the instrument until, when in the fully-deflected position, the pointer is horizontal, then the weight multiplied by the radius at which it is supported gives the total torque. The torque of a moving coil instrument can also be determined electro-magnetically by the aid of a fluxmeter or ballistic galvanometer, as shown in Fig. 6.20, provided that the moving coil instrument is not of high resistance and is not shunted.

Since $T_k = \frac{1}{10} I \frac{d\Phi}{d\theta}$ (see formula 73, Chapter 4), where I is the current in amperes and Φ the number of linkages of the coil with the field, it is only necessary to measure the number of linkages for various positions of the coil corresponding to different known

currents passing through it to be able to determine its torque without opening the instrument or interfering in any way with its mechanism. In Fig. 6.20 the moving coil instrument is connected to a battery and regulating rheostat, and then to the upper contact of a key, so that the instrument is normally deflected and the current can be regulated to any desired value. The bottom contact of the key is connected to a fluxmeter, so that by suddenly depressing the key the moving coil instrument is disconnected from the battery and instantly joined to the fluxmeter before the coil has had time to move appreciably. As it goes back to its zero position the flux-

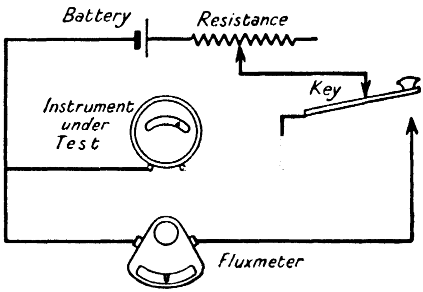


FIG. 6.20.—Circuit for measuring torque of M.C. instrument.

meter directly indicates all the linkages irrespective of the strength of the magnet and the number of turns of the moving coil. By putting a transparent scale of degrees over the dial the values of Φ and I can be obtained for various values of the deflection. If we then plot curves of Φ and I against the angular deflection, the values of $\frac{\delta\Phi}{\delta\theta}$ can be obtained by drawing tangents to the curve of Φ. In the case of moving coil permanent magnet instruments, however, the proportionality of the current and deflection is usually so close that $\frac{d\Phi}{\delta\theta}$ is practically constant, and it is therefore generally sufficient to obtain a reading at one point of the scale. The torque is then given by the formula $T_k = 5{\cdot}73\,\frac{I\Phi}{\theta}$, where I is in amperes, Φ the number of linkages for the deflection of θ degrees.

As an example, a moving-coil instrument having a resistance of 18·8 ohms was found to give a deflection of 55° for a current of 7·5 milliamperes. On discharging it through a fluxmeter having a

constant of 13,500 maxwells per division, a deflection of 17·7 divisions was obtained, corresponding to $13{,}500 \times 17{\cdot}7 = 2{\cdot}39 \times 10^5$ linkages.

Thus by the above formula, torque $= \dfrac{5{\cdot}73 \times 2{\cdot}39 \times 10^5 \times 7{\cdot}5}{55 \times 10^3}$
$= 187$ dyne cm.

As a check on this measurement the instrument was opened up, and a weight of 26 milligrams on the pointer at a radius of 7·5 cm. gave exactly the same deflection. The mechanical torque was therefore $26 \times 7{\cdot}5 \times {\cdot}981 = 191$ dyne cm., which agrees fairly with the 187 dyne cm. obtained electrically. The small discrepancy is due to the resistance of the instrument, as it was found that on adding 100 ohms to it the fluxmeter deflection fell to 14·5 divisions, or 3·2 divisions less. This implies that if the instrument had no resistance the fluxmeter deflection would have given 192 dyne cm., agreeing almost exactly with the mechanical measurement.

The final adjustment of the calibration of the completed instrument is usually done on the series resistance, but in some cases a magnetic shunt is employed for the purpose which produces a

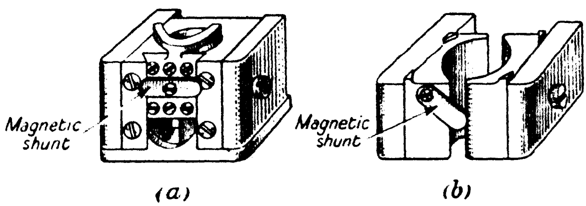


FIG. 6.21.—Types of magnetic shunt.

very slight change in the magnetic field. Fig. 6.21*b* and *c* shows the way this is done in the Nadir and Siemens & Halske instruments, and Fig. 6.21*a* the method employed by the Compagnie des Compteurs. A form of magnetic shunt has also been described in connection with Fig. 6.9.

DESIGN OF PERMANENT MAGNET MOVING COIL INSTRUMENTS

This type of instrument is almost the only one which lends itself to fairly accurate design. A number of factors such as physical dimensions, appearance, etc., are settled by considerations

other than the magnetic and electrical characteristics of the instrument, and have their effect on the ultimate design. Thus the physical dimensions determine very largely the space available for the magnet, the length of scale and design of pointer, so that the angular deflection and the moment of inertia of the pointer are settled. Consideration of the conditions for giving the best torque/weight ratio shows that this is obtained with a square coil, and considerations of speedy response also show that the dimensions of the coil must not be too great. From these the dimensions of the moving coil and pole pieces are determined, and since the magnet size is governed by the space available, it follows that the working flux in the air-gap can be calculated (see Chapter 5). In approaching any design, therefore, a number of factors are already known.

In designing a particular instrument a number of factors, which may not be immediately apparent, must be taken into account. In a milliammeter, for example, the customer will specify the range and he may state the maximum resistance. Occasionally he will also state the damping requirements, but often these are left to the designer, and it is therefore his task to see that the instrument he designs has satisfactory damping, and reaches its final steady reading in a reasonable time. To carry out a complete design it is necessary to know or stipulate the full-scale sensitivity, the resistance, the damping requirements, and any special conditions of the circuit in which the instrument is to be used.

The full-scale current must, of course, be known. In the case of a milliammeter, which is to have a certain range, it is the maximum value of this range. In the case of a shunted ammeter, it is usual to make the full-scale sensitivity 5 or 10 milliamps, depending on which value leads to the more suitable design. In the case of a voltmeter it is decided by the sensitivity demanded in ohms per volt. If, for instance, the sensitivity is to be 200 ohms per volt, then the full-scale current sensitivity is 5 milliamps.

The resistance is sometimes specified for milliammeters, but in other instruments it is often determined by considerations of temperature error. As an example in a shunted ammeter operating from a 75-millivolt shunt and having a full-scale sensitivity of 5 milliamps., the resistance of the instrument circuit is 15 ohms. To keep the temperature error within reasonable limits, a resistance of zero temperature coefficient must be placed in series with the instrument coil, this resistance being at least four times that of

the coil. This makes the maximum resistance of the coil three ohms.

The damping requirements specified may only consist of a statement that the overswing must not exceed a certain percentage of the final reading. In this case the minimum permissible damping factor can be calculated, and the minimum value of the damping ratio can be obtained from the curve in Fig. 3.8. To complete the damping requirements it is necessary also to state the natural time period, and this can be determined by considerations of the torque/weight ratio.

A useful criterion of a well-designed instrument is the ratio of the full-scale torque to the total weight of the moving system (see also Chapter 2). A convenient way of writing this ratio is :

$$\text{Torque/weight ratio} = \beta = \frac{\text{Full-scale torque in dyne-cm.}}{\text{Movement weight in grammes.}}$$

It has been found by experience that in the switchboard-type of instrument, i.e. one which operates with the axis horizontal, a satisfactory instrument is obtained if this ratio is 100 or more. By satisfactory is meant an instrument which will not give trouble due to friction between the pivots and jewels. For the portable type of instrument with a vertical axis, the ratio can be reduced to 40 and still give a good instrument.

The natural period of vibration of the moving system is

$$T_\lambda = 2\pi \sqrt{\frac{K}{S}}$$

where K is the moment of inertia of the moving system in gm.-cm.^{2} and S is the control torque in dyne-cm./radian.

Now $S = T/\Theta$, where T is the full-scale torque and Θ is the full-scale deflection, and $T = \beta W$, where W is the total weight of the moving system.

Hence

$$T_\lambda^2 = 4\pi^2 \frac{K\Theta}{\beta W} \text{ or } \beta T_\lambda^2 = \frac{4\pi^2 K\Theta}{W}$$

For any given instrument Θ is a constant, and it is found that the moment of inertia K and the weight W do not vary greatly from instrument to instrument. So it can be stated that

$$\beta T_\lambda^2 = \text{constant, approximately.}$$

It is possible always to make a rough estimate of K and W for any given type of instrument, so that the constant can be determined. Thus if the torque/weight ratio is not to be less than 100, there is a maximum value of the natural period which should not be exceeded.

It sometimes happens that in addition to a specification of the overswing a further statement may be made as to speed of response. A typical statement should be:

> "The pointer must have reached 50% of its final deflection in not more than 200 milliseconds from the time of switching on."

With a known damping ratio the curves in Fig. 3.4 enable the required natural period to be determined to meet this condition.

It should be noted that if too low a value of natural period is demanded, the torque/weight ratio may become so high as to be impracticable. It is advisable always to design for a value of natural period less than that required, and a value of damping ratio which is slightly higher.

In designing an instrument it is necessary to assume that it will be used in the most general type of circuit. This means a circuit as shown in Fig. 6.22. The moving coil R is in series with a resist-

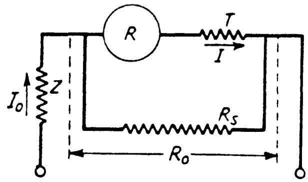


FIG. 6.22.—General circuit arrangement of moving coil meter.

ance T, which may consist only of the control springs, and the combination is shunted by a resistance R_s. The whole is in series with a resistance Z. In the case of a voltmeter or of a shunted ammeter, the resistance Z is the swamp or series resistance, while in a milliammeter Z is zero. The shunt resistance R_s is an aid to obtaining the correct damping requirements, and to this end also the coil may be wound on a frame of resistance R_F.

From the type of instrument demanded, the values of the total working flux and the full-scale deflection Θ are known. Also, as stated previously, the value of K does not vary considerably

from instrument to instrument of a given type and size, so that an approximate value can be calculated from the design. The circuit conditions of the instrument enable the quantities I_o, R_o and z in Fig. 6.22 to be determined, and the damping requirements give the values of the natural period T_λ and the damping ratio n.

From these known quantities it is possible to calculate the number of turns on, and the size of wire to be used in, the moving coil, and all other required values follow from these.

Derivation of Design Equations

The fundamental equations on which the design is based are:

$$S\Theta = \Phi I N \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

$$T_\lambda = 2\pi\sqrt{\frac{K}{S}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

$$n = \frac{\Phi^2}{2\sqrt{KS}}\left[\frac{N^2}{R_D} + \frac{1}{R_F}\right]$$

$$= \frac{\Phi^2}{2\sqrt{KS}}\left[\frac{N^2}{R_D} + F\right]. \quad . \quad . \quad . \quad . \quad (3)$$

where $F = 1/R_F$ = conductance of the frame.

From equation (2)

$$S = \frac{4\pi^2 K}{T_\lambda^2}$$

Also from Fig. 6.22,

$$I = \frac{I_o R_o}{R+T} = \frac{I_o R_o}{\gamma} \text{ say, where } \gamma = R + T$$

Substituting in equation (1)

$$\frac{4\pi^2 K\Theta}{T_\lambda^2} = \Phi\frac{I_o R_o N}{\gamma}$$

or

$$\frac{I_o R_o N}{\gamma} = \frac{4\pi^2 K\Theta}{\Phi T_\lambda^2} = k \text{ say} \quad . \quad . \quad . \quad (4)$$

Again from Fig. 6.22,

$$R_D = R + T + \frac{R_s z}{R_s + z} = \gamma + \frac{R_s z}{R_s + z}$$

and

$$R_o = \frac{\gamma R_s}{\gamma + R_s} \therefore R_s = \frac{\gamma R_o}{\gamma - R_o}$$

Hence

$$R_D = \gamma + \frac{\dfrac{\gamma z R_o}{\gamma - R_o}}{\dfrac{\gamma R_o}{\gamma - R_o} + z} = \frac{\gamma^2 P}{\gamma P - z R_o} \quad . \quad . \quad (5)$$

where

$$P = z + R_o$$

From (3)

$$\frac{N^2}{R_D} + F = \frac{2\sqrt{KSn}}{\Phi^2} = \alpha \text{ say}$$

$$\therefore \frac{N^2}{R_D} = \alpha - F$$

Therefore

$$\frac{N^2(\gamma P - ZR_o)}{\gamma^2 P} = \alpha - F \quad . \quad . \quad . \quad (6)$$

From (4)

$$\frac{N^2}{\gamma^2} = \frac{k^2}{I_o^2 R_o^2}$$

Substituting in (6)

$$\frac{k^2}{I_o^2 R_o^2}\left\{\frac{\gamma P - zR_o}{P}\right\} = \alpha - F$$

$$\therefore \gamma = (\alpha - F)\frac{I_o^2 R_o^2}{k^2} + \frac{zR_o}{P} \quad . \quad . \quad (7)$$

From (4) $\qquad N = \dfrac{k\gamma}{I_o R_o} = \dfrac{(\alpha - F)\, I_o R_o}{k} + \dfrac{kz}{I_o P} \quad . \quad . \quad (8)$

The coil resistance $R = \gamma - T = (\alpha - F)\dfrac{I_o^2 R_o^2}{k} - T.$

Hence resistance per turn $= \dfrac{R}{N} = \dfrac{\gamma - T}{N} = \dfrac{I_o R_o}{R} - \dfrac{T}{N}$ $\quad . \quad (9)$

Now $\qquad k = \dfrac{4\pi^2 K \Theta}{\Phi T_\lambda^2}$ and $\alpha = \dfrac{2n\sqrt{KS}}{\Phi^2} = \dfrac{4\pi K n}{\Phi^2 T_\lambda}$

$$\therefore \frac{\alpha}{R} = \frac{4\pi k n}{\Phi^2 T_\lambda} \cdot \frac{\Phi T_\gamma^2}{4\pi K \Theta} = \frac{n T_\lambda}{\pi \Phi \Theta}$$

Hence from equation (8)

$$N = \frac{I_o R_o n T_\lambda}{\pi \Phi \Theta} + \frac{4\pi^2 K \Theta z}{\Theta T_\lambda^2 I_o P} - \frac{F I_o R_o \Phi T_\lambda^2}{4\pi^2 K \Theta} \quad . \quad (10)$$

and the resistance per turn from (9) becomes

$$r = \frac{I_o R_o \Phi T_\lambda^2}{4\pi^2 K \Theta} - \frac{T}{N} \quad . \quad . \quad . \quad (11)$$

Practical Form of Design Equations

In equations (10) and (11) all quantities are in absolute units. For purposes of design it is more convenient to convert them into practical units.

If I_o is in milliamps, R_o, z, P, T, r are in ohms. and Θ is in degrees. Then the equations become

$$N = 1{\cdot}83 \times 10^6 \frac{I_o R_o n T_\lambda}{\Phi \Theta} + \frac{6{,}890\, K \Theta z}{\Phi T_\lambda^2 I_o P} - \frac{F I_o R_o \Phi T_\lambda^2}{6{,}890\, K \Theta} \quad . \quad (12)$$

$$r = \frac{I_o R_o \Phi T_\lambda^2}{6{,}890\, K \Theta} - \frac{T}{N} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (13)$$

Considerations of Torque/Weight Ratio

It is evident from equations (12) and (13) that the number of turns and the size of wire used are dependent on the frame used, and if a number of frames are available, then a choice should be made of the frame which will give the highest torque/weight ratio. This can be determined as follows:

For a given type of instrument we have

$$\beta T_\lambda^2 \propto \frac{\text{Moment of Inertia}}{\text{Total weight}}$$

Since the design is being prepared for a given natural time period, this becomes

$$\beta \propto \frac{\text{Moment of Inertia}}{\text{Total weight}}$$

In Fig. 6.23, which represents a cross-section of the frame, let

w = available winding width.
g = total depth of winding plus frame.
t = thickness of frame.

δ_1 = density of wire material.
δ_2 = ,, frame ,,
N = number of turns.
l = length of mean turn.
a = cross-sectional area of wire.
k_1 = space factor of the winding.
k_3 = factor by which the weight of the coil and frame have to be multiplied to give the moment of inertia.
P = moment of inertia of the pointer.
D = dead weight including the frame.

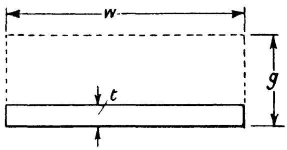


FIG. 6.23.—Cross section of moving-coil frame.

Then the weight of N turns $= N\delta_1 la$ and $N = \dfrac{k_1(g - t)w}{a}$.

$\therefore$ Weight of N turns $= \delta_1 lw(g - t)k_1$.
Weight of frame $= \delta_2 ltw$.
$\therefore$ Total weight $= \delta_1 lw(g - t)k_1 + \delta_2 ltw = f(t)$ say.
The total moment of inertia $= P + k_3 f(t)$.

Therefore $$\beta \propto \frac{P + k_3 f(t)}{D + f(t)}$$

This will be a maximum when $d\beta/dt = O$ or

$$\{D + f(t)\}\, k_3 f'(t) = \{P + k_3 f(t)\}\, f'(t)$$

$$\therefore f'(t) = O \text{ or } P = k_3 D.$$

The second condition is unlikely, and in any case appears to give a minimum. So the required condition is $f'(t) = O$.

i.e. $$-\delta_1 lwk_1 + \delta_2 lw = O$$

i.e. $$\frac{\delta_2}{\delta_1} = k_1 \quad . \quad . \quad . \quad . \quad (14)$$

Now since k_1 is always less than unity, this means that the density of the frame material must always be less than that of the

wire material for optimum conditions, e.g. copper wire on an aluminium frame, but not aluminium wire on a copper frame.

Referring back to equations (10) and (11), it is clear that unless N is small and/or T is large

$$r \simeq \frac{I_o R_o \Phi T_\lambda^2}{6{,}890\, K\Theta}$$

and so it is possible to write

$$N = X - Fr \quad . \quad . \quad . \quad . \quad (15)$$

The area of the frame $= wt$.

$\therefore\ F = \frac{wt}{\rho_2 l}$, where ρ_2 = specific resistance of the frame material.

Also $r = \frac{\rho_1 l}{a}$, where ρ_1 = specific resistance of the wire material.

$$\therefore \quad Fr = \frac{wt}{\rho_2 l} \cdot \frac{\rho_1 l}{a} = \frac{\rho_1}{\rho_2}\frac{wt}{a}$$

Also $\qquad N = \frac{k_1(g-t)w}{a}$ and from (14) $k_1 = \frac{\delta_2}{\delta_1}$

Hence $\qquad \frac{\delta_2}{\delta_1} \cdot \frac{w(g-t)}{a} = X - \frac{\rho_1}{\rho_2}\frac{wt}{a}$

$$\therefore \quad X = \frac{\delta_2}{\delta_1}\frac{w(g-t)}{a} + \frac{\rho_1}{\rho_2} \cdot \frac{wt}{a}$$

$$= \frac{w}{a}\left[\frac{\delta_2}{\delta_1} g + \left(\frac{\rho_1}{\rho_2} - \frac{\delta_2}{\delta_1}\right) t\right] \quad . \quad . \quad (16)$$

For various frames and sizes of wire these values can be calculated and tabulated. When X has been established in the course of a design, a reference to the table will show the frame which will give most nearly the desired damping conditions and the highest torque/weight ratio.

If the wire and frame are of the same material, then equation (16) simplifies to $X = wg/a$ and is independent of the thickness of the frame.

A design according to equations (12), (13) and (16) is a theoretical one which will exactly meet the requirements. This is not necessarily a practical design, and it must be adjusted so that a standard wire size can be used. Again, it may require modification in that

the number of turns demanded may exceed the number which can be accommodated. In such a case it is usually necessary to adjust the design requirements.

Example of Design

Suppose it is desired to design a multi-range voltmeter of the portable type, having a sensitivity of 1,000 ohms per volt.

Before the design can be commenced it is necessary to collect together the preliminary information already discussed. To illustrate this, suppose that the required instrument is to be a 6-in. scale portable instrument with a moving coil of the dimensions

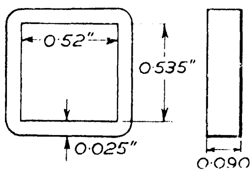


FIG. 6.24.—Moving coil dimensions used in design example.

given in Fig. 6.24. For this instrument also the following information is available :

Total working flux $= \Phi = 9{,}000$ lines.

Full scale $= \Theta = 97^\circ$.

Moment of inertia of knife edge pointer $= P = 0{\cdot}528$ gm.-cm.^{2}

Weight of pointer $= 0{\cdot}135$ gm.

Spring resistance $= T = 1$ ohm.

It is necessary to calculate—

(*a*) The factor by which the weight of the moving coil must be multiplied to give the moment of inertia.

(*b*) The resistance per turn for each gauge of wire.

(*c*) The weight per turn for each gauge of wire.

(*d*) Approximate weight of fully-wound coil for each gauge of wire.

(*e*) Approximate moment of inertia of fully-wound coil for each gauge of wire.

(*f*) Number of turns per layer, maximum number of layers and hence maximum number of turns for each gauge of wire and each available frame thickness.

The moment of inertia factor (a) can be calculated from the following formula

$$\text{Moment of inertia} = K = \frac{Wd^2}{4}\left\{\frac{l + \frac{d}{3}}{l + d}\right\}$$

where W = weight of the moving coil in grammes.

l = the length of the moving coil in centimetres.

d = the width of the moving coil in centimetres.

For the moving coil of Fig. 6.24 this formula gives $K = 0{\cdot}318\,W$.

The values required under (b) — (f) are given in Table XXVII.

The figures in Table XXVII are calculated for enamel-covered copper wire. Also the figures apply only to a frameless coil, since no frames are used in this instrument. In calculating the turns, an allowance of 0·005 in. has been made for the varnish. The winding width taken is therefore 0·085 in. and the depth 0·020 in.

The average weight of a fully-wound coil from the table is 0·252 gm., and the average moment of inertia is 0·0768 gm.-cm.^{2} An increase of 10% is allowed on this figure for varnish, etc., giving a value of 0·0845 gm -cm.^{2} Since the moment of inertia of the pointer is 0·528 gm.-cm.^{2}, the value of K to be used for the design is 0·528 + 0·085 = 0·613 gm.-cm.^{2}

In this case no damping requirements have been specified, so that these particulars are left to the designer. Dealing first with the damping ratio, it is advisable that the overswing, while readable, should be small. Reference to the curve in Fig. 3.13 shows that a damping ratio of 0·8 should be satisfactory.

The natural period can be determined from a consideration of the torque/weight ratio. The weight of the movement is approximately 0·252 + 0·135 gm. = 0·387 gm. With a torque/weight ratio of 100, this means a torque of 38·7 dyne-cm., and since this is for 97° = 1·692 radians, the value of S, the spring torque per radian is 38·7/1·692 = 22·8.

$$\text{Hence } T_\lambda = 2\pi\sqrt{\frac{0{\cdot}613}{22{\cdot}8}} = 1{\cdot}05 \text{ seconds.}$$

An attempt should be made to design to a time period somewhat less than this, say 0·9 second.

The only quantity which has not yet been settled is the resistance

TABLE XXVII.

Gauge of wire.	Overall diameter (in.).	Resis. per turn (ohms).	Wt. per turn (m-grm.).	Turns per layer.	No. of layers.	Max. turns.	Max. weight (g.).	Max. M.I.
50	0·0013	1·87	0·254	65	15	975	0·248	0·0789
49	0·0015	1·305	0·366	56	13	728	0·266	0·0846
48	0·002	0·733	0·651	42	10	420	0·274	0·0871
47	0·0026	0·469	1·018	32	7	224	0·228	0·0726
46	0·003	0·326	1·462	28	6	168	0·206	0·0781
45	0·0035	0·239	1·992	24	5	120	0·239	0·0761
44	0·0039	0·183	2·605	21	5	105	0·274	0·0871
43	0·0044	0·145	3·295	19	4	76	0·25	0·0795
42	0·0048	0·117	4·06	17	4	68	0·276	0·0878
41	0·0052	0·0969	4·91	16	3	48	0·236	0·0751
40	0·0056	0·0813	5·85	15	3	45	0·264	0·084
39	0·0061	0·0695	6·86	13	3	39	0·268	0·0853
38	0·0069	0·0521	9·15	12	2	24	0·22	0·07
37	0·0078	0·0405	11·74	10	2	20	0·235	0·084
36	0·0086	0·0325	14·68	9	2	18	0·264	0·0913
35	0·0095	0·0266	17·92	8	2	16	0·287	0·0547

R_o. This can be done by considerations of temperature error. Suppose the minimum range required is 0–150 millivolts, then to keep the temperature error down to a reasonable value, the volt drop across the shunted movement can be set at 30 millivolts. Since the full-scale sensitivity is 1 milliamp, this makes $R_o = 30$ ohms.

Substituting all the known values in the design equations gives the following results :

$$N = \frac{1{\cdot}83 \times 10^6 \times 30 \times 0{\cdot}8 \times 0{\cdot}9}{9{,}000 \times 97} + \frac{6{,}890 \times 0{\cdot}613 \times 97}{9{,}000 \times 0{\cdot}9 \times 1}$$

$= 101{\cdot}4$. Note $z/\rho = 1$ and $F = 0$.

$$r = \frac{30 \times 9{,}000 \times 0{\cdot}81}{6{,}890 \times 0{\cdot}613 \times 97} - \frac{1}{101{\cdot}4} = 0{\cdot}524 \text{ ohms.}$$

From Table XXVII the nearest gauge to this is No. 47 with a value of $r = 0{\cdot}469$ ohms. Taking this value of r is equivalent to reducing I_oR_o to 26·4, and this in turn gives a revised value for N of 96 turns. From Table XXVII this corresponds to a coil of three layers each of 32 turns. The remainder of the design is as follows :

The coil resistance $= R = 96 \times 0{\cdot}469 = 44{\cdot}9$ ohms.
Coil circuit resistance $= 44{\cdot}9 + 1{\cdot}0 = 45{\cdot}9$ ohms.
Coil current $= 26{\cdot}4/45{\cdot}9 = 0{\cdot}575$ mA.
Shunt current $= 1{\cdot}0 - 0{\cdot}575 = 0{\cdot}425$ mA.
Shunt resistance $= R_s = 26{\cdot}4/0{\cdot}425 = 62{\cdot}1$ ohms.

$$\text{Torque} = \frac{9{,}000 \times 0{\cdot}575 \times 96}{10^4} = 49{\cdot}7 \text{ dyne-cm.}$$

S = torque/radian $= 49{\cdot}7/1{\cdot}692 = 29{\cdot}4$ dyne-cm.

The following calculations are made as a check on the design :

Weight of moving coil $= 96 \times 1{\cdot}018$ milligrammes $=$ 0·0976 grammes.

Moment of inertia of moving coil $= 0{\cdot}318 \times 0{\cdot}0976 =$ 0·0311 gm.-cm.^{2}

Total weight $= 0{\cdot}0976 + 0{\cdot}135 = 0{\cdot}233$ gm.

Total moment of inertia $= 0{\cdot}528 + 0{\cdot}031 = 0{\cdot}559$ gm.-cm.^{2}

Torque/weight ratio $= 49{\cdot}7/0{\cdot}233 = 213$. This is very good.

$$\text{Natural time period} = T_\lambda = 2\pi\sqrt{\frac{0{\cdot}559}{29{\cdot}4}} = 0{\cdot}865 \text{ sec.}$$

Damping resistance $= 45{\cdot}9 + 62{\cdot}1 = 108$ ohms.

$$\text{Damping ratio} = n = \frac{9{,}000^2}{2\sqrt{0{\cdot}559 \times 29{\cdot}4}}\ \frac{96^2}{108 \times 10^9} = 0{\cdot}858.$$

The check figures are quite satisfactory. It is very unlikely that the exact damping values specified will be obtained due to the necessity of making a practical coil. In this case the natural period is less than, and the damping ratio greater than, the required values, so that the instrument should be entirely satisfactory.

The various ranges required in the instrument are obtained by placing suitable resistance in series with the shunted movement.

The Design of Permanent Magnet Moving Coil Instruments for Various Forms of Scale

The above design is for the standard type of instrument with a linear scale. There are frequently demands, however, for special scale shapes, and the general principles underlying these will be considered.

The fundamental formula for such instruments is $T = i(dN/d\theta)$, where $dN/d\theta = nAB$, n being the number of turns and A the area of the coil, and B the magnetic induction density in the gap. Hence $T = nABi$.

Consequently

$$K = \frac{dT'}{d\theta} = \frac{\partial T}{\partial \theta} + \frac{\partial T}{\partial i} \cdot \frac{diq}{d\theta} = nA\left(\frac{B}{\delta'} + i\frac{dB}{d\theta}\right)$$

where $\delta' = d\theta/di$ and $K' = a\, nAB/\delta'$.

With spring control, which is practically universal in such instruments, K is constant so that

$$B = \frac{K}{nA} \cdot \frac{\theta}{i} \text{ and } \frac{B}{\delta'} + i\frac{dB}{d\theta} = \frac{K}{nA}.$$

(1) *Evenly Divided or Linear Scale.*—In this case $\delta' = d\theta/di$ is constant, so that $\theta = ai$, where a is constant, and $B = (kA)/(nA)$. Hence B must be constant, which is obtained by employing a cylindrical core and pole-faces. Since $dB/d\theta = O$, $K' = K$, and $\tau_1 = \tau$, so that the period time $\tau = 2\pi\sqrt{(I/K)}$. The total flux Φ required to deflect the instrument over its whole scale $= AB\theta_m$.

(2) *Logarithmic Scale.*—The general expression for such a scale is $\theta = p \log_e qi$, where p and q are constants. Such a scale cannot be carried down to zero, as $\log qi = -\infty$ when i is zero, so we must abandon it and accept an evenly-divided scale below some deflection θ_1 and current i_1. In order that this shall occur without

discontinuity in the size of the divisions, $d\theta/di$ must have the same value for the linear portion as for the first part of the logarithmic scale.

For the linear portion $d\theta/di = \theta_1/i_1$, and for the logarithmic scale $d\theta/di = p/i$, or p/i_1 when $i = i_1$.

Hence p/i_1 must equal θ_1/i_1, from which $p = \theta_1$ and $\theta = \theta_1 \log_e qi$.

Evidently $\log_e qi$ must be unity when $i = i_1$, so that $q = e/i_1$, and

$$\theta = \theta_1\{1 + \log_e(i/i_1)\},$$

from which $i = i_1 e^{(\theta/\theta_1 - 1)}$ for the logarithmic part of the scale, and

$$\theta_m = \theta_1 \ \{1 + \log_e(i_m/i_1)\} = \theta_m \ (1 + \log_e f),$$

where $f = i_m/i_1$.

The instrument will then have constant percentage accuracy over an f-fold range of current.

For spring control of constant K, $\theta = nABi/K$, from which:

$$B = (K/nA)$$
$$(\theta/i) = K/(nAi_1)\theta e^{-(\theta/\theta_1 - 1)}$$
$$\text{and } B/B_1 = (\theta/\theta_1)e^{-(\theta/\theta_1 - 1)}$$
$$= 2{\cdot}718 \ f'/\text{antilog } 0{\cdot}4343\,(f' - 1),$$

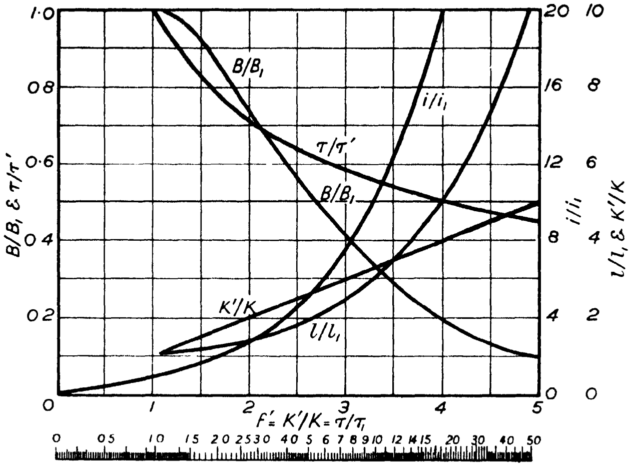
where $f' = \theta/\theta_1$.

Also $K = nABi/\theta$, and $K' = nABi/\theta_1 = K\theta/\theta_1 = f'K$, so that $\tau/\tau_1 = 1/\sqrt{f'}$.

From these formulae we can calculate Table XXVIII :

The values of l/l_1, or the ratio of the gap at any angle to that for the first linear part of the scale, are calculated on the assumption that the gap induction B is inversely proportional to the gap, which will be fairly nearly true if the permeability of the pole-shoes and core is so high that we may assume uniform magnetic potential over their faces.

Fig. 6.25 shows the calibration curve and scale for such an instrument, together with the variation of i, B, K', and τ'. Since the value of K' increases rapidly as the deflection increases, this means that if the instrument is critically damped for the lower linear part of the scale it will become more and more under-damped as the deflection rises, especially as the weakening of the field reduces instead of increases the damping constant, if the instrument is magnetically damped. But if the instrument is air-damped or oil-filled and is critically damped over the linear



Form of Scale

FIG. 6.25.—Permanent magnet moving coil logarithmic scale instruments.

TABLE XXVIII.

$f' = \theta/\theta_1 = K'/K$.	$i/i_1 = e(f'-1)$.	B/B_1.	$l/l_1 = B_1/B$.	$\tau'/\tau = 1/f$.
1	1	1	1	1
1·2	1·221	0·982	1·018	0·914
1·4	1·492	0·938	1·066	0·846
1·6	1·822	0·878	1·139	0·791
1·8	2·225	0·798	1·253	0·746
2	2·718	0·736	1·359	0·707
2·2	3·32	0·665	1·504	0·675
2·4	4·055	0·596	1·679	0·646
2·6	4·953	0·525	1·905	0·62
2·8	6·049	0·459	2·18	0·598
3	7·389	0·406	2·463	0·578
3·2	9·026	0·355	2·82	0·56
3·4	11·023	0·308	3·244	0·543
3·6	13·464	0·267	3·74	0·528
3·8	16·44	0·231	4·33	0·513
4	20·09	0·199	5·02	0·5
4·2	24·53	0·171	5·84	0·488
4·4	29·96	0·147	6·81	0·477
4·6	36·6	0·127	7·96	0·466
4·8	44·7	0·107	9·31	0·456
5	54·6	0·098	10·2	0·447

portion, the under-damping at the higher readings will not matter, as the deflection will always be within $1/2\%$ of its final value in a time equal to the initial periodic time τ.

As regards the total magnetic flux Φ, we have

$$\Phi - \Phi_1 = A \int_{\theta_1}^{\theta} B d\theta = \frac{Ke}{n} \cdot \frac{\theta^2{}_1}{i_1} \cdot \int_1^{f'} f' e^{-f'df'}$$

$$= \frac{K\theta^2{}_1}{ni_1} \{2 - (1 + f')e^{-(f-1)}\}$$

and $$\Phi_1 - \Phi_o = \frac{K\theta^2{}_1}{ni_1}$$

so that the total change of flux over the whole scale

$$\Phi - \Phi_o = \frac{K\theta^2{}_1}{ni_1} \{ 3 - (1+f')\, e^{-(f'-1)}\}$$

where $f' = \theta_m/\theta_1$, which gives us the necessary information for designing the permanent magnet.

(3) *Engine-Room Voltmeter Scale.*—In this case we require an open, evenly-divided scale between two large values of the current i_1 and i_m. The general equation for such a scale is $\theta = a(i - i_1) + \theta_1$, where θ_1 is the deflection when $i = i_1$. When $i = 0$,

$$\theta_o = ai_1 + \theta_1 = -(ai_1 - \theta_1),$$

which means that the zero of the pointer would be an angle $\theta_o = ai_1 - \theta_1$ behind the zero of the scale, or that the zero of the scale is " set up " by this angle. We can therefore get such a scale by employing a long spring with its natural zero at $-\theta_o$ and putting a stop to prevent the pointer from deflecting behind the lowest point of the scale, in which caee there will be no movement of the pointer until the current is sufficiently high to overcome the initial torque. There are two objections to this :

(*a*) The spring is always under strain, and changes of zero cannot be detected, and

(*b*) The instrument gives no indication below a certain large current or P.D.

It would therefore be an advantage to have a close, nearly uniform scale up to θ_1, changing to an open uniform scale from θ_1 to θ_m.

For the lower part of the scale, $\theta = (\theta_1/i_1)i$, and for spring control $T = nABi = K\theta$,

hence
$$B = \frac{K}{nA} \cdot \frac{\theta}{i} = \frac{Ki}{nA} \cdot \frac{\theta_1}{i_1}$$

which is constant. For the upper part
$$B = \frac{K\theta}{nAi} = \frac{Ka}{nA} \cdot \left\{\frac{\theta}{\theta + \theta_0}\right\}$$

and rises almost proportionally to $\theta - \theta_0$ if θ_0 is large, as shown in Fig. 6.26. The ratio of effective to spring control $K'/K =$

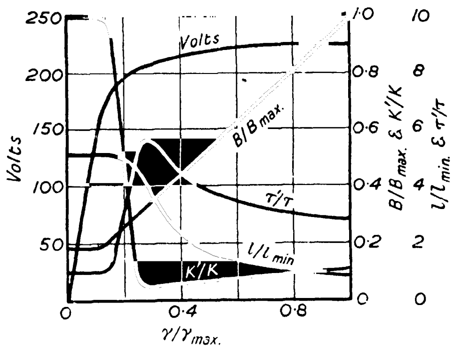


FIG. 6.26.—Variation of induction gap effective control and periodic time for permanent-magnet moving coil engine-room voltmeter.

(θ/i) $(d\theta/di)$ is then $\theta/(\theta + \theta_0)$, which is very low when θ is small, so that the periodic time, which is normal over the lower part of the scale, suddenly increases very considerably as soon as the open scale commences, and gets smaller again, although still retaining a high value, on approaching the upper end of the scale. To restore the periodic time to its normal value, K' must equal $4\pi^2/\tau^2$, and K must be $\{(\theta + \theta_0/\theta\}K'$, which means that the torque due to the spring at any point must be the same as if it had the original constant K and zero at $-\theta_0$, as in the set-up-zero instrument. This variation of control cannot easily be obtained mechanically, but it can be secured with magnetic control.

SHUNTS

For the measurement of currents beyond the few milliamperes taken by the moving coil itself, permanent magnet instruments are always used with "shunts." These devices, which are simply

standard resistances of low value and of moderate precision (an accuracy of 0·1% being sufficient for all but exceptionally refined instruments), are constructed of strips or tubes of manganin or constantan soldered to more or less heavy copper blocks, or lugs, to which the terminals are attached.

As with all resistance standards of low value, each shunt has four terminals, two (C C, Fig. 6.3*b*) for attachment of the current-

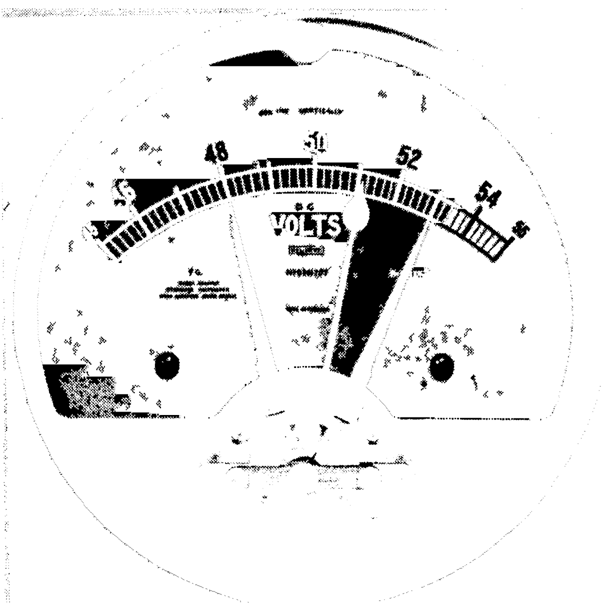


FIG. 6.27.—Shows a type of switchboard voltmeter with set-up zero which is also fitted with maximum and minimum limit indicators.

carrying leads, and two (P P) for connection to the moving-coil instrument or milli-voltmeter, which may be considered as measuring the drop of potential across the shunt in the case where the resistance of the latter is very small in comparison with that of the instrument. If, for example, the milli-voltmeter has a maximum reading of 0·075 volt, which is becoming a more or less standard value, then for an ammeter reading 15 amperes the shunt should have a resistance of $0{\cdot}075/15 = 0{\cdot}005$ ohm approximately, so that if the instrument has been calibrated once for all as a milli-voltmeter,

it is only necessary to select the correct shunt and figure the scale accordingly.

As the milli-voltmeter affects the parallel resistance of the combination, however, Table XXIX (p. 335) shows the correct values of the shunts for a milli-voltmeter having a resistance of 5 ohms.

It will be seen that for currents of 75 amperes and higher the shunting effect of the milli-voltmeter may be neglected, and the shunts are simply low resistance standards, such as are used for potentiometer work.

It will be further noticed that at the higher currents a considerable expenditure of power takes place in the shunt, with consequent heating. This not only necessitates a sufficient cooling surface to avoid risk of injury to the shunt, but gives rise to two sources of error. The first is that unless the material of the shunt is of very low temperature coefficient, its resistance will change to the extent of seriously affecting the indications of the milli-voltmeter. Secondly, unless the material forming the shunt has a very low thermo-electric power with respect to copper, E.M.F.'s will be set up at the junctions, which may likewise affect the readings of the instrument and also its zero when the current is switched off and the shunt is still warm. For example, the constantan or copper-nickel alloys usually have a thermo-E.M.F. to copper of about 40 micro-volts per 1° C., and if the shunt rises in temperature by 70° C., there will be an E.M.F. of 2,800 microvolts or 2·8 millivolts at the junction corresponding to 5·6 divisions of a 75-millivolt instrument, with a scale of 150 divisions. Fortunately the E.M.F.'s at the two junctions oppose one another, so that there is rarely an error of anything like this amount, but local cooling may cause the difference between the E.M.F.'s to be quite appreciable. Furthermore, we have the Peltier effect or unequal heating of the two junctions due to the fact that the current is in the same direction as the thermo-E.M.F. at one junction, and opposite to it at the other, causing the first junction to be relatively cold and the second relatively hot, apart from any external inequalities of cooling. It is therefore of considerable importance to eliminate or compensate these thermo-E.M.F.'s, and the simplest method is to use manganin, which is the only alloy which combines an extremely low temperature coefficient with low thermo-E.M.F. (about 1·4 micro-volts per 1° C.) to copper.

As manganin is somewhat readily oxidizable, however, and therefore requires somewhat careful treatment, some makers prefer constantan shunts, and a few have special devices for eliminating or compensating the thermo-electric troubles. In America an alloy under the trade name of Therlo has been employed for shunts, while in Germany, Kalmiz or Achenrain alloy has been used for the same purpose. Both have properties similar to those of manganin.

TABLE XXIX.—*Shunt Resistances for Milli-Voltmeter having Maximum P.D. of* 75 *Millivolts and Resistance* (r) *of* 5 *ohms.*

Current range (amp.).	Parallel resistance shunt and resistance (R) (ohm).	Shunt resistance $\frac{rR}{r-R}$ (ohm).	Power loss in shunt (watts).
0·75	0·1	0·10204	0·055
1·5	0·05	0·0505	0·111
3	0·025	0·02512	0·224
7·5	0·01	0·01002	0·56
15	0·005	0·005005	1·12
30	0·0025	0·002501	2·25
75	0·001	0·001	5·53
150	0·0005	0·0005	10·13
300	0·00025	0·00025	22·5
750	0·0001	0·0001	56·3
1,500	0·00005	0·00005	112·5
3,000	0·000025	0·000025	225
7,500	0·00001	0·00001	562·5

The following are the fundamental requirements of a satisfactory shunt :

1. Simple and permanent construction.
2. Adequate and well-designed current and potential terminals.
3. Low temperature coefficient.
4. Low or compensated thermo-E.M.F. and Peltier effect.
5. Abequate cooling surface of good radiating power.

Types of Shunts.

There are three types of shunts in general use, which may be classified as follows: (*a*) Flat plate, (*b*) parallel rod, (*c*) parallel tube.

Of these, the first (*a*) is by far the most generally adopted, since it is probably cheaper than the other two to construct. Fig. 6.28 shows some examples, and it will be seen that current enters and leaves the resistance plates by means of massive copper terminal blocks provided with suitable attachments for the cables. These terminals should be designed as far as possible to ensure

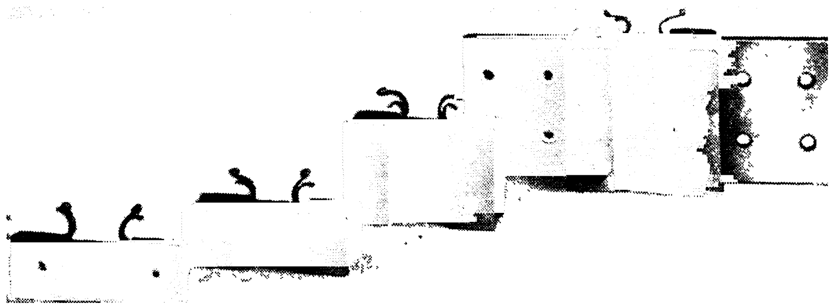


FIG. 6.28.—Types of plate shunt.

an even distribution of current over the plate. If the current enters and leaves the plate locally, then under certain circumstances the apparent resistance of the plate may vary with the method of connection due to changes in the form of the equipotential surfaces in the plate, and the indicating instrument must then be calibrated with its shunt throughout its range, and the shunts would not be interchangeable.

H. Fletcher Moulton has shown theoretically that for a square plate conductor of unit specific resistance, if the electrodes are reduced to one-fifth of the width of the side of the plate, then its apparent resistance will be represented by the numbers 1·745, 2·408, 2·589 and 3·027 for positions of the electrodes represented in Fig. 6.29*a*, *b*, *c* and *d* respectively.

If, however, the potential terminals are taken off well inside the plate, as is done in some precision resistances, changes in con-

nection of the leads will have but little effect, since the form of the equipotential surfaces will not then be appreciably affected, but with broad terminal blocks this is a refinement only necessary where an accuracy greater than that which is usually obtainable with an indicating instrument is required, and in the majority of cases the potential terminals are taken off the copper terminal blocks themselves.

A good and uniform contact must be obtained between the current terminal and the various resistance strips, and it is usual, therefore, to slit the terminal blocks along the contact faces and insert the strips in the slits and carefully solder in position ; any imperfection in the joints will lead to local heating, and disturb the uniform distribution of current between the component plates.

The Ediswan Company cast the terminals round the manganin resistance, and thus claimed to obtain a perfect joint without the use of solder at all.

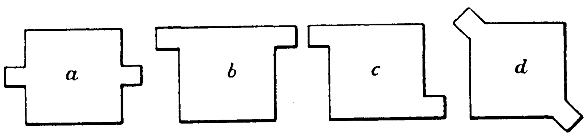


FIG. 6.29.—Alternative arrangement of shunt terminals.

The design of the terminal to which the cables are attached is of considerable importance, so that the current distribution in them shall always be the same, no matter how the size of cable or method of connection is varied—a matter which is of the greatest importance in portable instruments.

For switchboard instruments permanent connection is usually made either with broad copper strap or cables terminating in sockets, thimbles, or solid ends, and under these circumstances the current distribution is constant, and the form of terminal simplifies into a plain copper block provided with holes or studs for bolting to the connecting straps, while for very heavy currents the terminal may be laminated so as to provide a large contact area. Fig. 6.30 shows in outline some of the forms of terminal that have been employed. The form is adapted to take stranded cable and strap of moderate width, while small leads may be clamped under the milled head, and the terminal post providing the screw is arranged just to clear the bottom of the rectangular hole when these surfaces

are in contact. Fig. 6.30*b* is a form of French terminal. The bar through the screw head, more convenient for tightening than the milled head in the Paul terminal (31*c*) which has a screwed post of large diameter. This is slotted longitudinally with a broad slot, into which the cable end is clamped by means of a milled ring running on the screw. It is claimed for this that a stranded wire is not squeezed out as it is when clamped between flat surfaces, and a fairly large range of sizes can be accommodated. The slot is, however, a weakness, and a tendency of the cable to force the

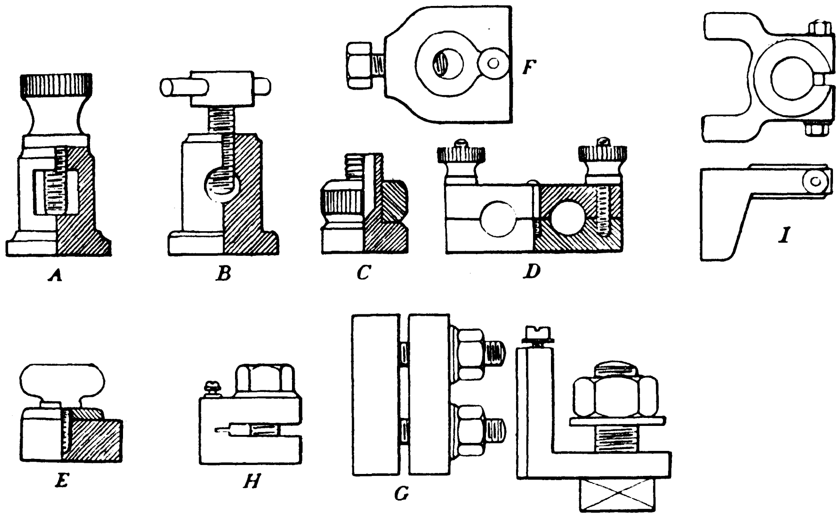


FIG. 6.30.—Forms of main terminal for shunt resistances.

cylindrical sides of the screw out of parallel causes the ring to jam, and may lead to stripping of the threads.

D is a terminal intended to give satisfactory connection with both cable and straps over a fairly wide range, and E is a terminal adopted by the Weston Electrical Instrument Co., for their small cheap shunts, and is not altogether satisfactory for stranded cables. F is a form employed by Messrs. Nalder Bros. for some of their portable instruments, and although in most respects a good form, the employment of a hexagon-headed screw is a defect, since it necessitates the employment of a separate spanner for tightening; and, apart from the additional weight to be carried, the wear and

tear at the hands of only semi-skilled operators soon make their effects felt. G is the form employed by Chauvin & Arnoux, and H the usual Weston switchboard form. In some of the Crompton instruments a forked terminal is employed, the current entering the plate at two or more symmetrical points.

The potential terminals should be symmetrical with respect to the current lugs, and should preferably be on the shunt side of the points at which the current enters and leaves the resistance; and on the centre line of the shunt. For instruments to be permanently installed the instrument leads are brought to screws and secured firmly beneath a suitable washer, or an even better plan is to sweat them permanently to strips of the same material as the shunt itself, as advocated by Mr. Evershed. Under any circumstances the contacts must be as perfect as possible, and there should be no chance of the contact resistance rising to any appreciable extent.

In portable instruments the milled head is better for these terminals, as it avoids the use of the screw-driver. Paul and the Compagnie des Compteurs provide the instrument with leads terminating in two coned plugs, which fit in corresponding holes in the current terminals. This is a convenient arrangement providing the plugs and holes are worked to a good "resistance-box" fit, but otherwise they may give rise to variable contacts and consequent error.

In some shunts the current terminals are grooved round between the point to which the cable is attached and the potential terminal, with a view to directing the current flow with respect to the point from which the instrument leads are taken off, and such a device certainly conduces to greater accuracy.

The dimensions of the actual resistance plates, rods or tubes are to a great extent governed by the permissible temperature rise, and this again will depend upon the thermo-electric properties of the alloy employed. In general the proportions of the shunt may be calculated in the following manner:

Let I be the maximum current the shunt has to carry and V the drop required for full-scale deflection of the instrument. Then $R = \frac{V}{I}$, but $R = \frac{\rho l}{A}$, where ρ is the specific resistance of the material and l and A its length and cross-section respectively. If b is the breadth of the plate and x its thickness, $A = bx$, and therefore

$R = \frac{\rho l}{bx}$. The heat generated in this resistance is $0{\cdot}24I^2R$ per sec., and the volume of the metal is lbx. If q is the specific heat of the material and Δ_x its density, then, assuming no heat to be lost by radiation, convection, or conduction, the temperature θ would be $\frac{0{\cdot}24I^2R}{lbxq\,\Delta_x}$. Loss of heat, however, always occurs. Let Q_H be the quantity lost by radiation and convection, then we have

$$Q_H = Us(\theta - \theta_1) \text{ per second,}$$

where U is the emissivity of the plate and s its complete surface, and θ_1 the temperature of the surroundings. For a steady temperature, therefore, the heat generated must be equal to the heat lost, or

$$0{\cdot}24I^2R = Us(\theta - \theta_1)$$

$$\theta - \theta_1 = \frac{0{\cdot}24I^2R}{Us}$$

If now there are n plates in parallel, the total cross-section is $n(bx)$ and the total surface is $2n(bl)$ if we neglect the edges of the plates. Therefore we may write

$$\theta - \theta_1 = \frac{0{\cdot}24I^2\rho l}{U\,.\,2n(bl) \times n(bx)} = \frac{0{\cdot}12I^2\rho}{Un^2b^2x}.$$

This expression takes no account of the loss of heat by conduction, and on the other hand it assumes that the whole surface of the plates is equally effective as radiators, which of course can hardly be the case unless the spacing were more generous than can be usually afforded, so that on the whole the two effects tend to cancel.

The chief uncertainty is the value of U, the emissivity; and since by far the greater amount of heat is lost by convection and the convective coefficient varies with the form and position in which the plates are mounted, it is difficult to settle on any particular value, apart from the fact that experience shows that U varies with the temperature, and is also dependent on the extent of the surface, the value for the same temperature rising very rapidly as the surface diminishes.

Dr. T. Barratt showed that for a round rod or wire of bare metal 97·5% of the heat lost is due to convection and only 2·5% is radia-

tion, and for a dull black varnished surface his figures were 87·4 and 12·6 respectively for temperatures between 17° and 23° C.

The increase of emission due to blackening is thus about 11·5%, and is the same for all metals and independent of the diameter of the rod within the limits examined. For temperatures between 100° and 110° C. the convection for a bar metal surface was 95·6% and the radiation 4·4%, and the corresponding numbers for the blackened surface 79·3 and 20·7%.

The total loss of heat in calories per sec. per sq. cm. per 1° C. has been given by Dr. Barratt as 0·000533 at 17° C., and 0·000634 at 100° C. for wire 1 mm. in diameter, and 0·000224 at 18° C. to 0·000265 at 100° C. for a rod 5·85 mm. in diameter.

Dr. Barratt and A. J. Scott later extended their observations to larger areas and greater differences of temperature, and their results are summarized in Table XXX (page 342) in which the radiator is in the form of a tube having the diameter specified. The last line, however, is for a flat surface on edge, and for this they remark that the results are so variable that no definite conclusions can be arrived at except that the total heat loss is greater than any of the cylinders examined.

From these results, therefore, it is concluded that convection varies as $\dfrac{1}{\sqrt{\text{diameter}}}$ within the limits of experimental error.

M'Farlane's experiments, made in 1871 with a sphere 4 cm. in diameter, put the value at 0·000178 at 5° C. to 0·000226 at 60° C. for a copper sphere 4 cm. in diameter with a polished surface, and the ratio of the radiation bright to black was approximately 0·69. Petaval's results, obtained in 1898 with a platinum wire 1 sq. mm. in cross-section, give for normal pressure and low temperature differences the value 0·0006 or 0·0007 at about 100° C. Incidentally Petaval showed that by surrounding the wire with an atmosphere of hydrogen the emissivity increased to about four times its value when in air—a phenomenon which has been taken advantage of for cooling dynamo machinery.

Owing to the importance of the convection component in limiting the shunt temperature, the design of the shunt should be such as will make full use of it. Flat plate shunts should be mounted edgewise, and this is most usually done by mounting them on the back of the switchboard, but it should always be with the line joining the terminals horizontal, as otherwise the upper terminal

TABLE XXX.

Diameter of tube (cm. = d).	Total heat loss: Cals. per sq. cm. per 1° C. excess. Black.	Bright.	Percentage radiation. Black.	Bright.	Heat lost by convection (c = calories).	$\frac{c}{\sqrt{d}}$.
0·62	0·000495	0·000432	16	3·7	0·000416	327
1·275	0·000362	0·000285	26·1	6·1	0·000267·5	302
2·545	0·00028	0·000208	35	7·7	0·000182	290
5·12	0·000222	0·000154	37·1	10·1	0·000139·5	316
Flat surface 70 × 15 × 1·4	0·000317	0·000215	39·5	10·9	0·000192	..

will be heated considerably above the temperature of the lower one and thermo troubles will result.

In some types, instead of arranging the plates horizontally between the terminals they are tilted up at an angle. The air can then circulate much more freely, and the shunt is nearly independent of position.

Evershed increased the cooling surface by clamping the plate between two webbed castings, which are insulated from it by thin mica sheets. The heat generated in the resistance is transmitted through the mica to the clamping plates and dissipated at a much greater surface, thus allowing of a considerable reduction in the size of the shunt (Fig. 6.31).

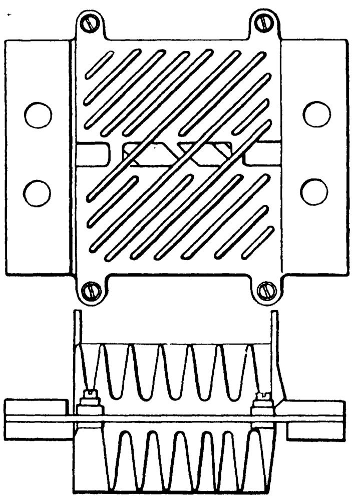


FIG. 6.31.—Evershed radiator type of shunt.

Corrugation, particularly of the black strips, is adopted in some forms of continental shunts with a view to getting a large area in a comparatively small space. Shunts consisting of a series of parallel rods would, were it not for the expense of construction, have several advantages. The cooling area is practically constant for all positions, and good ventilation is easily secured. Fig. 6.32 shows an example of this type of construction, and both

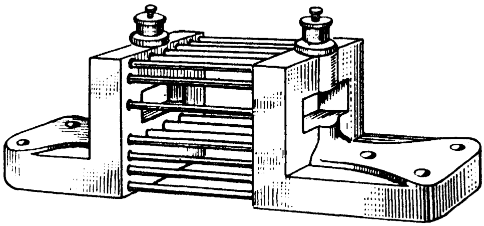


FIG. 6.32.—Parallel rod type shunt.

Evershed and Siemens & Halske have adopted the form in the past for use with portable instruments.

A shunt of the concentric tube type was made by the Cambridge Instrument Co., for use with Unipivot instruments. The tubes have wide slots cut in the sides longitudinally to provide ventilation, since the ends are closed by massive cylindrical plug terminals of the form already described.

In Siemens tubular shunt a short manganin tube is soldered between the current lugs, which are in the form of copper blocks bent to a right angle. The tube is slit along its entire length with a broad slot on the side nearest to the surface on which it is mounted, and the axis of the tube is kept vertical when the shunt is installed, thus securing very efficient ventilation. A standard length of tube 33 mm. clear between the lugs is employed, and for large currents two or more tubes are soldered side by side in the same terminals.

In instruments of 50 amperes range or less the shunt is often included with the instrument itself, and in many cases this is done by bridging the terminals of the instrument where they pass through the case, so that the shunt is supported and protected by the recessed back plate, very much in the same way as the series resistance is arranged in some voltmeters. The base is then mounted on distance pieces away from the surface of the switch-

board, or the edges of the back plate are cut away in order to secure ventilation. An example of such a practice is shown in Fig. 6.33.

In some of the Siemens instruments up to 100 amperes the shunt is located inside the instrument case, but it would seem, on the whole, better not to include this source of heating within an unventilated enclosure, where the ratio of swamping resistance to copper in the instrument cannot be very high.

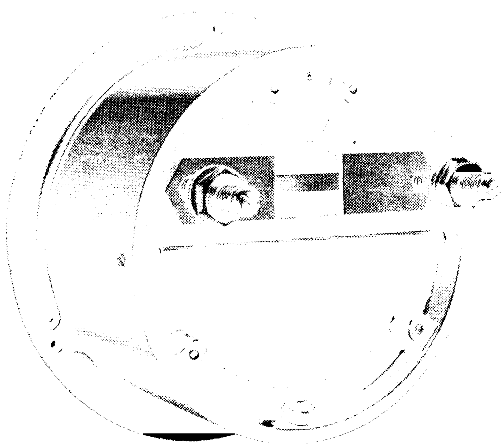


FIG. 6.33.—Shunt mounted in case at back of instrument.

In the case where very large currents are to be measured to a high degree of accuracy, oil-cooled, water-cooled, or combined oil- and water-cooled shunts may be employed. The description of this type of apparatus is given later under the heading of "Standards of Low Resistance."

Design of Ammeter Shunt

As an example of the employment of the above method of calculation let us take the case of a shunt for 1,000 amperes to give a drop of 0·075 volt, and consisting of a series of manganin plates 1 mm. thick in parallel, the specific resistance of the material being 42 micro-ohms, and the full load temperature to be 50° C. above air, provisionally adopting a current density of 2 amperes per sq. mm. Then for 1,000 amperes the total cross-section will be 5 sq. cm.

Now

$$\theta = \frac{0{\cdot}12I^2\rho}{Un^2b^2x} \text{ or } n^2 = \frac{0{\cdot}12I^2\rho}{\theta Ub^2x}$$

If we take a probable value for U of 0·00025, and provisionally make $b = 5$ cm.,

$$n = \sqrt{\frac{0{\cdot}12 \times 42}{0{\cdot}00025 \times 25 \times 0{\cdot}1 \times 50}} = 12{\cdot}65$$

Hence in order to have a uniform set of plates we must increase b to 5·3 and employ 12.

Then the length must be

$$l = \frac{{\cdot}000075 \times 5{\cdot}3 \times 0{\cdot}2 \times 12}{42 \times 10^{-6}} = 11{\cdot}35.$$

Allowance must be made for the resistance at the joints, which will depend on the method of manufacture. In this case it would probably be right to adopt a clear length of between 11·25 and 11·3 cm.

The problem of constructing shunts for very large currents was attacked by M. B. Field.* He pointed out that when currents of the order of 50,000 amperes or more have to be dealt with, the difficulties of making adequate contact, the uncertainty of the current distribution among the separate members of the shunt, and the practical impossibility of testing the shunt under full load conditions until it is installed, render the usual methods of construction inadmissible, and he therefore considered the upper limit for a single unit to be about 6,000 amperes.

For currents larger than this he showed that the method of constructing the resistance in the form of a series of standardized sub-shunts arranged in parallel has very decided advantages. Each of these sub-shunts is provided with the usual potential terminals, from which leads of equal resistance are taken. At their outer ends all the positive leads are joined together and the junction connected to one terminal of a milli-voltmeter, and the negative leads, similarly grouped, are joined to the other terminal of the instrument, as shown in Fig. 6.34. The interesting feature of this arrangement is that the relative contact resistance of each sub-shunt is of little consequence, the summation remaining practi-

* M. B. Field, *Journ. of Inst. of Elect. Engineers*, vol. 58, August, 1920, " On Multiple Unit Shunts for the Measurement of very Heavy Currents."

cally unaffected. Errors no greater than 0·15% occur when the component shunts are simply led across the bus bars without any clamping up at all. Fig. 6.35 shows Field's suggestion for

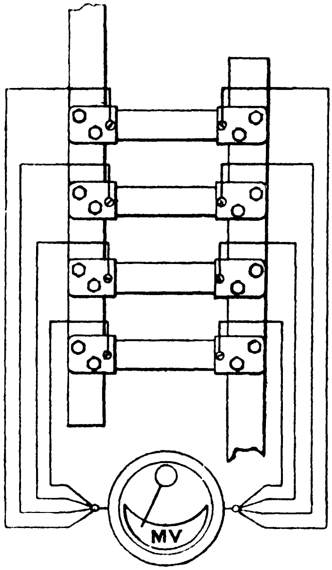


FIG. 6.34.—Field's method of grouping shunts for measurement of large currents.

inserting the shunts in the run of the bus bars, and thus entirely doing away with cast lugs and the necessary machining of them, together with the usual surfacing and jointing difficulties on in-

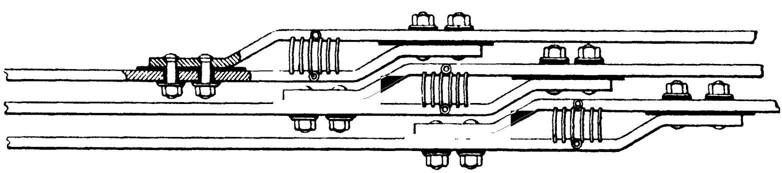


FIG. 6.35.—Field's suggestion for grouped shunts in the run of the bus bars.

stallation. Another interesting feature of the design is that the current distribution in any sub-shunt is practically uniform, since the current path through each strip is made up of the same length of copper and resistance alloy.

MULTIPLIERS

As already mentioned, the moving coil of voltmeters is connected in series with a resistance which reduces the P.D. at its terminals to the required amount, and the range of the instrument is determined by the amount of resistance in series, so that if a series of tappings on the resistance are brought out to suitable terminals, intermediate voltage ranges are available. Such resistances necessarily absorb a certain amount of power which is dissipated in heat, and therefore their position in respect to the instrument will depend upon the expenditure of power in them. If this does not exceed 4 or 5 watts, the resistance may be located within the instrument case, but with instruments for pressures above 250 volts it is usually better to locate the resistance in a separate ventilated compartment or mount it in the recessed back plate behind the instrument, so as to have the advantage of some air circulation for cooling.

The resistance must be wound in an alloy of low temperature coefficient, as constancy of resistance is of great importance, and since thermo-electric troubles do not arise in voltmeters there is no objection to the use of alloys of the constantan class.

In modern instruments bifilarly wound bobbins are now seldom used, as their cooling properties are poor, the insulation between turns for high voltages difficult to maintain unless a large number of bobbins are employed in series, and they are then uneconomical of space.

The flat card type of resistance is now quite general, and bifilar winding is unnecessary, so that such resistances have everything in their favour, since they are neat, rigid, compact, and have excellent heat-dissipating qualities.

The construction of this type of resistance has already been dealt with (see Chapter 2), and certainly in most cases reconstructed mica preparations seem to give the best results, since many of the composite and fibrous insulators, apart from their poor mechanical qualities, have large coefficients of expansion and seriously deteriorate under prolonged heating.

In some cases an independent multiplier is employed, such as those shown in Figs. 6.36 and 6.37, which consists of a resistance contained in a perforated metal case; but such an arrangement is only necessary when the loading is particularly heavy, which

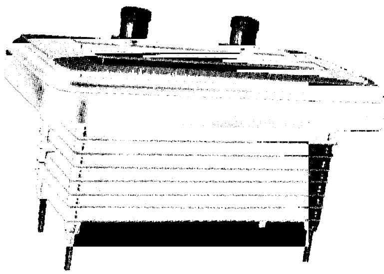


FIG. 6.36.—Type of voltmeter multiplier.

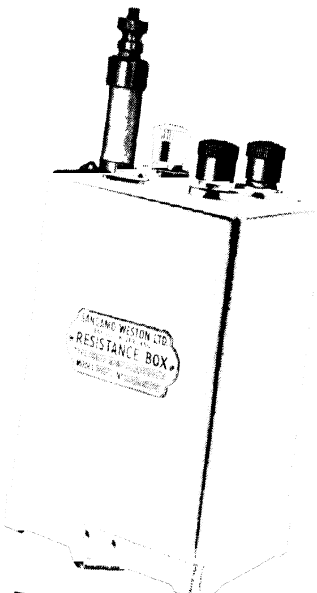


FIG. 6.37.—Type of high voltage voltmeter multiplier. Sangamo-Weston Instrument Co.)

is seldom the case with moving coil instruments. Usually where such multiplier boxes are used the instrument is arranged with a self-contained resistance for the lower range, and the multiplier is then inserted for the upper ranges. Occasionally resistances are constructed by winding the wire in loops over porcelain pillars; the liability to strain is less, and the radiating quality good where there is sufficient space to spread the turns sufficiently, but usually it lacks the neatness of the flat card.

COMBINED INSTRUMENTS AND TEST SETS

Since the moving coil instrument lends itself so readily and accurately to an extension of range by means of shunts and series resistances, the instruments are frequently arranged in compact and portable fashion for direct current measurements.

Double instruments in which two movements are arranged in a single case, one indicating voltage and the other current, are often employed, the usual method of assembly being to put the magnets end to end with a central dial-plate bearing the two scales over which the pointers move. The best position of the magnets is with their yokes together in the centre and the movements as widely separated as possible, so that the interference between the two instruments is reduced to a minimum. An example of this construction is shown in Fig. 6.38. If the pointers are arranged at slightly different levels they may be extended and cross one another, giving a more open scale, as is done in the case, for instance, of the Elliott double instrument.

With this arrangement it is obviously possible to construct a chart between the two scales, consisting of a series of lines so calculated that when the intersection of the pointers is observed, the ratio of the two quantities can be read off without calculation, and if, therefore, one movement indicates pressure and the other current, the chart lines will give resistance. Changes in flux will naturally alter the absolute calibration of either instrument, but if the moving coils are arranged to move in the same magnetic circuit such magnetic changes will affect both instruments equally and their ratio will remain the same, and the resistance scale will therefore be unaffected. This principle was developed by Commandant Ferrié, who also extended it to wattmeters, frequency and wave meters, etc.

Not infrequently, however, two separate instruments are arranged side by side in a portable case, with the necessary multiplier for the voltmeter and set of shunts for the ammeter. The moving coil circuit should be provided with a spring key to protect it from overload or misconnection, and where the shunts are carried in the lid of the case it is a great convenience to make it easily

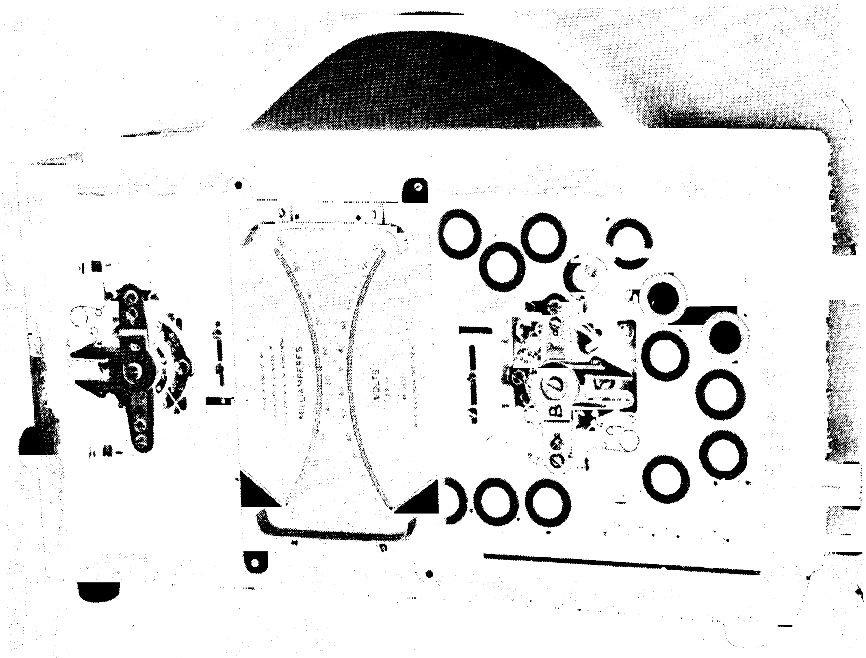


FIG. 6.38.—Form of double instrument. (Evershed and Vignoles.)

detachable, as in the Siemens set, for not infrequently tests have to be made where there is but little room to place both the instruments and heavily-loaded lid out flat, and efforts to prop up the lid in some inconvenient position have often led to disaster with the instruments.

Another system of employing the moving coil instrument for measuring current and voltage is shown in Fig. 6.39. Here a single instrument is employed, and this is connected directly to a well-constructed changeover switch, which in one position connects

a series resistance to the movement converting it into a voltmeter, while in the other position it puts the movement across a shunt, thus converting it into an ammeter. Such a scheme provides a light and portable test set, and is economical in that it requires only one instrument. On the other hand, the switch may be the seat of considerable trouble when in the current-measuring position, unless it is constructed with the greatest care, since any unreliability

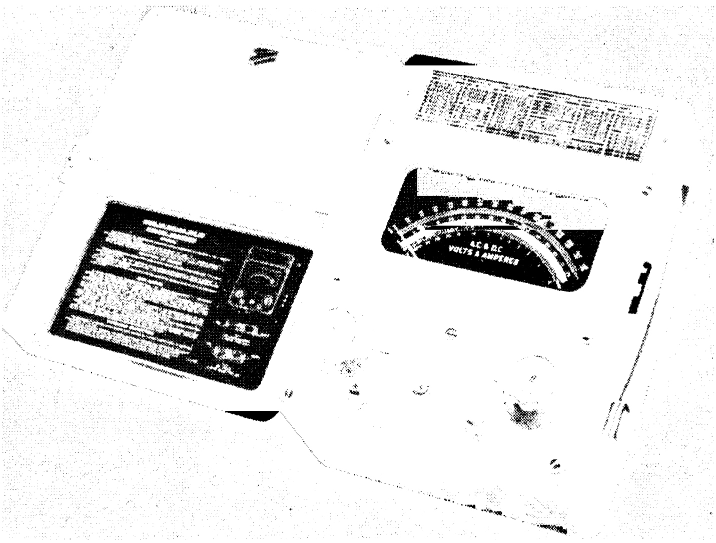


FIG. 6.39.—A modern universal test instrument.
(Sangamo-Weston Instrument Co.)

in the contact will cause very serious error by including resistance in the milli-voltmeter circuit. An obvious objection to the single instrument scheme is that simultaneous readings of volts and amperes cannot be made.

Where independent shunts are employed an alteration of current range involves the breaking of the main circuit for the insertion of the new shunt, thus interfering with the continuity of the test, which is sometimes very inconvenient. To obviate this, two schemes are possible. In the first we may employ a single shunt which is sufficiently large to permit of the full current being carried by

it, and which gives sufficient drop for the milli-voltmeter to give reasonable readings with the smallest currents flowing, the sensitiveness of the instrument being controlled by subdividing its series resistance and bringing the junctions out to a multiple-way switch, as shown in Fig. 6.40*b*. This simple scheme, however, is open to two objections: Firstly, the drop over the shunt has to be sufficient for the smallest currents to give sufficient deflection, which means a bulky and expensive shunt, and secondly, the switch in the millivoltmeter circuit is open to the objection mentioned above.

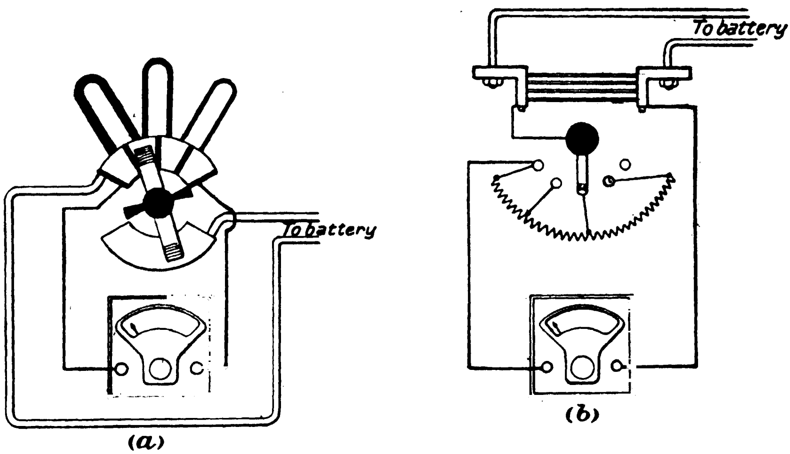


FIG. 6.40.—Two methods of altering current range.

An alternative is shown in Fig. 6.40*a*, in which a series of shunts are arranged like the resistance in a universal shunt-box, a separate shunt being provided for each current range, but the instrument is here permanently connected across them all in series. One terminal of the supply is fixed, and the other is brought to a switch-arm which makes connection to any one of the junctions between the shunts. This switch must be sufficiently massive to carry the maximum current, but it does not produce contact errors. On the other hand, as the shunts are cut out their resistance is included in the milli-voltmeter circuit, and hence the drop must be slightly increased as we pass from low to high currents to compensate for this; but reference to the table on page 335 will show that this is only necessary for the greatest refinement if there is a reasonable amount of series resistance in the milli-voltmeter circuit.

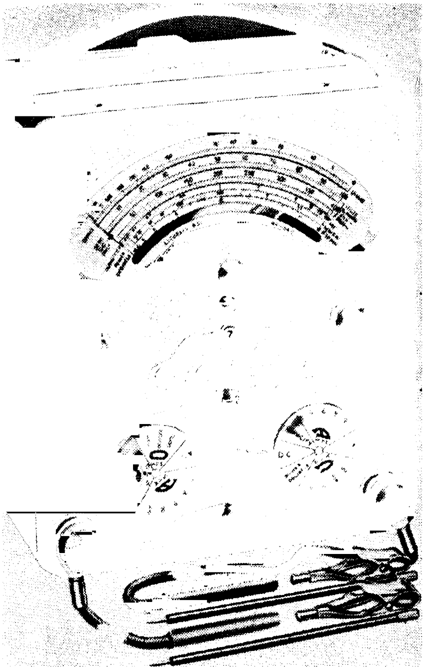


FIG 6.41.—Type of universal test meter. (Automatic Coil Winder Co.)

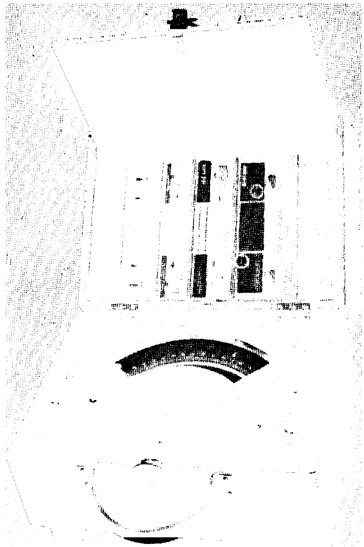


FIG. 6.42.—Type of universal test meter.

There are a number of testing sets now available which have one movement only and are provided with suitable resistances, shunts and range switches, so that a wide range of currents and voltages can be measured. These instruments also contain rectifiers (see page 360) and transformers, so that a similar wide range of alternating current voltages and currents can be measured (Figs. 6.41–42).

ERRORS OF MOVING COIL INSTRUMENTS

The errors that may arise in this type of instrument can be classified as due to the following causes :

(*a*) Friction.
(*b*) Want of permanence in magnet.
(*c*) Heating.
(*d*) Stray field.
(*e*) Thermo-electric errors.

(a) **Friction.**—The friction error with good pivoting and balancing will, in general, be small, want of alignment of the pivots being most usually the chief cause of high friction. As indicated in Table XXXI, the difference in friction for the vertical and horizontal position in portable instruments is very marked, but in no case is it serious. Far more troublesome is the mechanical friction that may occur between coil and core, due to the slight warping of the former on which the coil is wound. It would seem that once started the trouble gradually increases until the instrument is rendered useless. It is in the main due to too small a gap, too light a former, too great a winding tension, or an unsuitable and insufficiently hard-setting cement. In many cases the clearance between coil and core is less than between coil and pole face, and in nearly all such cases the trouble occurs sooner or later. Another type of the same trouble is caused by minute particles of magnetic matter, hair or fluff getting into the gap owing to want of cleanliness in assembly, and thus producing friction by just lightly touching the coil as it passes over them. Magnetic material can be removed from the gap with a fine steel needle, and other matter must be removed with a fine brush or slip of soft wood.

(*b*) **Want of Permanence.**—The permanence of the magnet, as already pointed out, depends upon design and ageing treatment, both of which must be carefully considered, for even with the most perfect design every magnet suffers a gradual weakening as time goes on. In a good magnet this will occur with extreme slowness, but in instruments that are liable to vibration, change of position, and stray magnetic fields, which are particularly the conditions with portable types of instrument, demagnetization may be considerably accelerated, and the error so caused, since it does not give any indication of its existence by mere inspection and in general behaviour, is not discovered unless the instrument is frequently checked against a standard. Errors of 5–10% have been found to develop very rapidly in some forms of portable instrument from this cause, but by proper attention to ageing and reasonable care in use the error can be kept within negligible limits. Usually it is most prevalent in instruments in which the gap density is pushed to extreme limits, so that the sub-permanent magnetism is not entirely eliminated by the ageing process.

It is, however, possible to eliminate almost entirely the effect of magnetic change by the simple device of weakening the mecha-

nical control and attaching to the coil a symmetrical iron or steel needle, which is so arranged that it provides magnetic control, which diminishes as the magnet becomes weaker. This device was first patented by Ayrton and Perry in 1887; a similar system was again patented by Soames in 1896, and it was again independently revived by Weiss in 1902, who claimed that instruments so compensated were unaffected by a change of 20% in the magnet strength.

(c) **Heating.**—Owing to the small current taken by the movement and the consequently large series resistance, voltmeters of the moving coil type show no readable temperature error. It is easily possible to construct instruments with a resistance of 100 ohms per volt, and in some cases higher values are attained, as, for instance, in some of the large precision laboratory standards, which work with three or four times this value, so that the temperature coefficient of the copper coil, which is only a very small part of the total resistance, is entirely swamped out. With ammeters, however, the case is not quite the same, for since it is desirable to keep the voltage drop in the shunt down to a low value, in order that the power wasted with large currents shall be small, the amount of resistance in series with the moving coil is necessarily small, and the swamping, therefore, cannot be anything like as complete.

Campbell has shown that if the moving coil and shunt are connected to auxiliary resistances, as shown in Fig. 6.43 (which is really in effect a Wheatstone bridge in which the battery is replaced by the shunt and the galvanometer is the moving coil), and by making the opposite arms of the bridge of similar material, one pair having a high temperature coefficient and the other pair a negligible one, practically perfect compensation can be attained. For if the relative resistances of the arms as shown in Fig. 6.48A and their temperature coefficients as shown at B (Fig. 6.48), then the temperature coefficient of the combination

$$a_1 = a \frac{(r_1^2 - 1)r - 2r_1 n(1 + a)}{(r + 2r_1 + rr_1)(1 - r_1)}$$

For complete compensation, that is, $a_1 = O$,

$$\frac{r_1^2 - 1}{r_1} = \frac{2n(1 + a)}{r}$$

Then if I_1 is the current which normally deflects the instrument

without compensator, and I is the total current taken by the complete mesh,

$$\frac{I}{I_1} = \frac{1 + 2r + r_1}{1 - r_1}$$

and if V is the voltage across the coil when connected to the compensator, and V_1 the voltage drop over the shunt,

$$V = V_1 \frac{r(1 - r_1)}{r + rr_1 + 2r_1}$$

Another method of compensation, which is not, however, as perfect as the above, consists of connecting the moving coil in

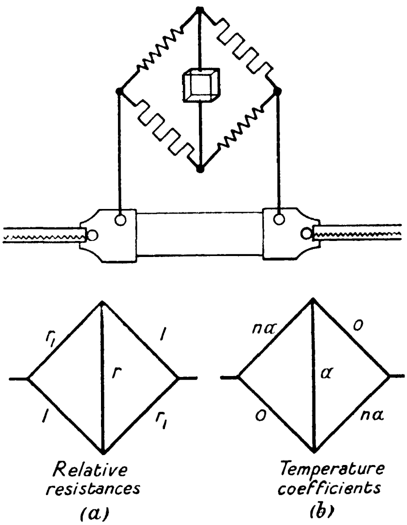


FIG. 6.43.—Campbell's method of temperature compensation.

series with a resistance of negligible temperature coefficient to the terminals in the usual way ; the coil and part of the series resistance is then shunted by a copper coil. It is then possible to calculate the resistance of the various parts of the branched circuit to give fairly good temperature compensation.

In addition to the self-heating error, the control springs and magnet are both subject to change with external temperature. The springs, if made of phosphor bronze, will weaken by about 0·04% per 1° C., and the magnet will also weaken by an amount

which, from tests made on actual instruments, appears to have a value between -0.01 to -0.03% per 1° C., so that the springs and magnet partially compensate one another; and it has been suggested that the residual error can be eliminated by making the combined temperature coefficient of the coil and series resistance positive by the same amount as the residual error is negative.

(*d*) **Stray Field.**—It is popularly supposed that because of the intense field employed in moving coil instruments they are independent of external fields. But some years ago Professor Ayrton pointed out that this was not strictly the case, and that owing to the presence of iron in the working parts of the instrument comparatively weak fields were concentrated and their effects enhanced. In ordinary unshielded moving-coil instruments the greatest error is produced when the direction of the disturbing field coincides with, or opposes, the field in the gap of the instrument. A stray field of only 2 gausses may produce an error of about 1·25% in this position when the instrument is fully deflected, and a field of 10 gausses will increase the error to 7·5%. A field along the rotational axis of the coil has practically no effect, and at right angles to this, across the working field, the figures are 0·1 and 0·65 respectively. As already pointed out, the Cirscale instruments of the Record Electrical Co. are unique in this respect owing to the astatic arrangement of their field. With this instrument, entirely unshielded, the worst direction of the disturbing field appears to be when it coincides with the plane of the C-shaped magnet, but even then the error is less than 0·2% for a field of 2 gausses and 0·7% for 10 gausses. At right angles to this the corresponding errors are 0·14 and 0·57, while a field along the rotational axis of the coil gives errors of 0·07 and 0·36% respectively.

The employment of an iron case for ordinary type instruments is therefore advantageous, and when this form of shield is employed we find that a field of 10 gausses in the most effective direction will produce an error of only 0·75 to 1%, while in the Cirscale movement the error never exceeds 0·2% in the same field.

It will therefore at once be obvious that the employment of unshielded instruments of the ordinary type requires careful consideration, and the close proximity of two instruments may lead to appreciable error in their indications, due to the interference of their fields. In portable instruments this is of considerable importance, and it is therefore perhaps better to sacrifice the advantage

of a light instrument and employ an iron case within the outer wooden case, as the Weston Instrument Co. have done in their Model 45 instruments.

It should not be forgotten, however, that whenever an iron case is employed the calibration must be taken with this in position, as its presence always to some extent modifies the working field, and may cause the instrument to read low by some 4 or 5%.

(e) **Thermo-electric Error.**—This error, of course, pertains principally to ammeters, and is the result of uneven temperature distribution in the shunt, arising from unequal cooling, position of mounting, bad contacts at the current terminals, and the Peltier effect.

As already pointed out, the vertical position of the shunt with one current terminal above the other is the most satisfactory position, and a difference of temperature of several degrees between the upper and lower terminal is sometimes found with shunts arranged in this way.

Heating due to contacts should not exist with well-designed terminals, and with a current density in the contact in the neighbourhood of 30 amperes per sq. cm. of surface, a perfectly satisfactory connection can be made which, by reason of its good thermal conductivity, will assist cooling rather than otherwise.

The Peltier effect produces a change of temperature at each of the junctions between the shunt plates and the copper terminals, in one case heating where the current flows against the junction electromotive force, and in the other cooling where the flow is assisted by the junction E.M.F., and the heat so generated or absorbed is proportional to the Peltier coefficient and the current flowing. Obviously, therefore, this disturbance of temperature will be less the smaller the thermo-E.M.F. of the metals in contact and the lower the shunt temperature is kept, and therefore, where materials with a high thermo-E.M.F. to copper are employed, it is safer to place the positive terminal, which is the one usually heated by the Peltier effect, in such a position that the greatest advantage can be taken of the cooling conditions.

From some experiments made on a constantan shunt, whose maximum temperature was approximately 30° C., the steady difference of temperature of the two junctions produced by the Peltier effect amounted to about 12° C., after thirty minutes from the time of switching on, as shown in the curve (Fig. 6.44).

Undoubtedly the best method of eliminating the thermo-electric error is to employ an alloy of small thermo-electric power to copper, such, for instance, as Manganin, Tarnac, Therlo, or Achenrain, but such materials require careful treatment as regards soldering and protection, and hence materials of the constantan class are more widely employed on the score of cheapness, despite their higher thermo-electric properties, and methods of compensating for the error have to be adopted.

One such method consists of connecting the instrument to the shunt through strips of the same material as that used for the shunt

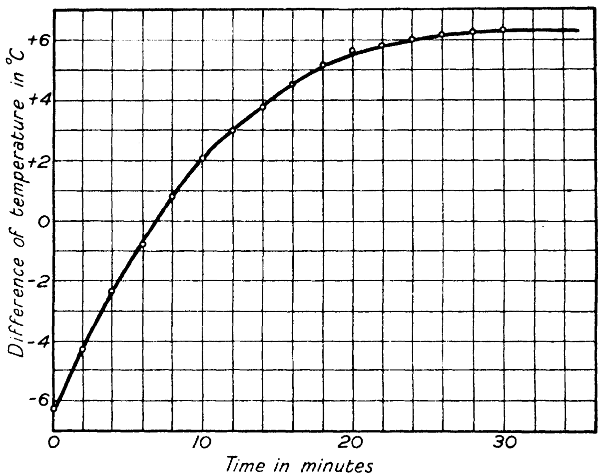


FIG. 6.44.—Curve showing the temperature difference between the two ends of a Constantan shunt due to Peltier effect.

plates, making these tags sufficiently long to ensure that the outer ends, to which the copper leads are soldered, are at the same temperature. Another method consists in modifying the shunt in the manner indicated in Fig. 6.45. Instead of attaching the instrument leads directly to the shunt in the usual way, one only is so attached to one end of the shunt. A strip of the same metal as the shunt plates is then soldered to the other potential point, and brought over to a copper terminal mounted on the first in such a way that, although electrically insulated, it is as far as possible in thermal equilibrium with it, and to this the second lead is attached so that both leads leave the shunt at one end.

The thermo-electric error is detected by switching off the current passing through the shunt after it has attained its maximum temperature and leaving the instrument connected. If the error is present the pointer will not return immediately to zero, but will

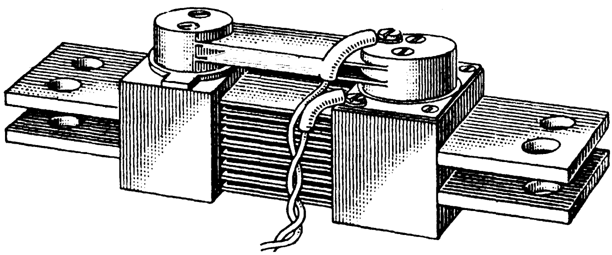


FIG. 6.45.—Shunt arranged to Equalize temperature of potential points.

gradually approach that position as the temperature of the shunt falls. If no other cause of zero shift is present, disconnecting the lead from the shunt should immediately cause the pointer to return to zero.

Table XXXI gives a set of comparative figures for various forms of moving-coil instruments.

RECTIFIER INSTRUMENTS

The moving coil instrument, particularly in its modern form, has the great advantage over all other kinds of indicating instrument in its great sensitivity. It is now an easy matter to construct voltmeters having a resistance of 20,000 ohms per volt and to measure currents as low as 5 microamperes with robust indicating instruments. It is, however, essentially a direct-current instrument, and these high orders of sensitivity are not obtainable with any of the standard forms of alternating current instruments. In view of the extensive use of alternating currents it is only natural therefore that efforts were made to obtain these high sensitivities for alternating current measurements. This has been made possible by the use of the copper oxide rectifier with a moving coil instrument.

The copper oxide rectifier, which was developed in the first place for the rectification of small alternating current powers, is also available for instrument work in a series of small sizes (Fig. 6.46). It is usual to employ a rectifier consisting of four elements arranged

FIG 6.46 —Miniature Rectifiers for Instrument Work. (Westinghouse Brake Co).

in the form of a Wheatstone Bridge (Fig. 6.47). This arrangement gives full wave rectification, and the current through the instrument is a pulsating direct current. The indication will, of course, depend on the average value of the current through the instrument and the latter can be calibrated to read the R.M.S. value of the alternating current, assuming that the supply is sinusoidal and has a form factor of 1·11.

To measure alternating currents of the order of milliamperes the connections of Fig. 6.47 are used, and for voltmeters a suitable series resistance is placed on the alternating current side of the rectifier as shown in Fig. 6.48 Rectifiers have the property that

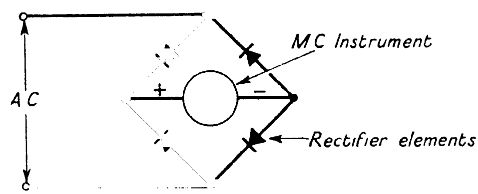


FIG 6 47 —Bridge rectifier circuit

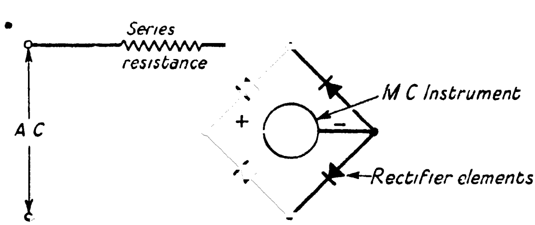


FIG 6 48 —Bridge rectifier circuit for voltmeters.

the voltage drop across them is not proportional to the current through them, and a typical volt-ampere characteristic is given in Fig. 6.49. This means that the resistance is not constant, but varies with the current through it. For this reason it is not possible

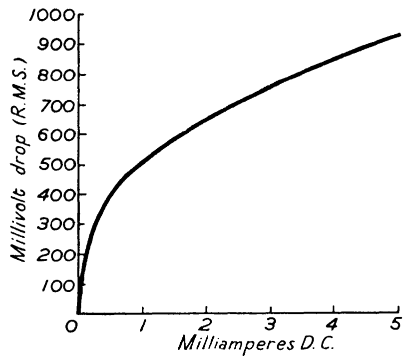


FIG. 6.49.—Rectifier characteristic.

to extend the range of a milliammeter by the use of shunts, and it is necessary to use current transformers. The connections of a typical multi-range milliammeter is given in Fig. 6.50. To obtain the necessary ranges a current transformer with a tapped primary is used. Multi-range voltmeters are obtained by using a tapped

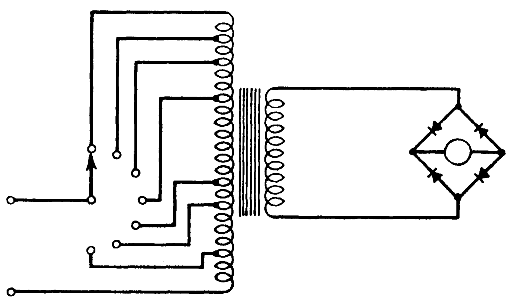


FIG. 6.50.—Connections of multi-range milliammeter.

series resistance, but the variable resistance of the rectifier places a limit on the maximum value of the lowest range obtainable using an instrument with one scale. For a low value of maximum scale the series resistance will be comparatively low, and the rectifier resistance may be an appreciable part of the resistance of the volt-

meter circuit. The rectifier resistance increases considerably as the current through it decreases, and thus with a low series resistance the scale will be cramped at the lower end, and will not be an exact sub-multiple of the scale for a higher maximum voltage. Generally the lowest practical range for a multi-range voltmeter is 0 — 10 volts.

To provide ranges below this it is necessary to resort to step-up potential transformers; thus if it is desired to obtain a range of 0–5 volts, a step-up transformer 5/20 volts would be incorporated as shown in Fig. 6.51, and so obtain a uniform scale. This, however, is at the expense of an increased current consumption or reduced sensitivity.

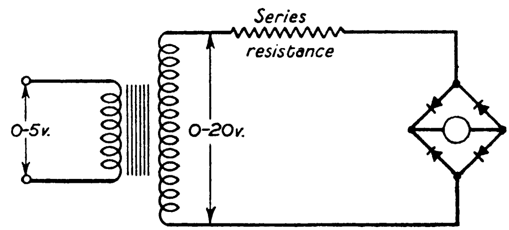


FIG. 6.51.—Low reading rectifier voltmeter with transformer.

The main sources of error in rectifier instruments are variations in temperature and variations in wave form of the current or voltage to be measured. The rectifier elements are not perfect, but have a high resistance in the reverse direction, and an increase in temperature causes a decrease in both the forward and reverse resistances. In a milliammeter, as in Fig. 6.47, variations in forward resistance can produce no error, but variations in reverse resistance are equivalent to a variable shunt across the movement, and there will be a slight reduction in reading with an increase in temperature. This error is not usually very serious. With voltmeters either or both of the series and reverse resistances play a part in determining the temperature error. In a low range voltmeter with a low series resistance, the forward resistance of the rectifier is an appreciable part of the total resistance, and consequently variations of this resistance with temperature cause an error in the instrument reading. An increase in temperature causes a decrease in resistance with a corresponding increase in the instrument indication. At the same time, of course, the reverse resistance will also decrease,

tending to cause a decrease in the instrument reading, but for low ranges, say 0–10 volts, for example, the effect of the forward resistance is by far the greater. On the other hand for high range voltmeters, say 0–1,000 volts, for example, the forward resistance of the rectifier is negligible compared with the series resistance, and consequently variations of this resistance with temperature produce a negligible effect on the instrument indication. The reverse resistance, however, has its maximum effect, and an increase in temperature will cause a decrease in the reading. At some intermediate range in volts the effects due to the forward and reverse resistances tend to cancel one another, resulting in a small and possibly negligible temperature error.

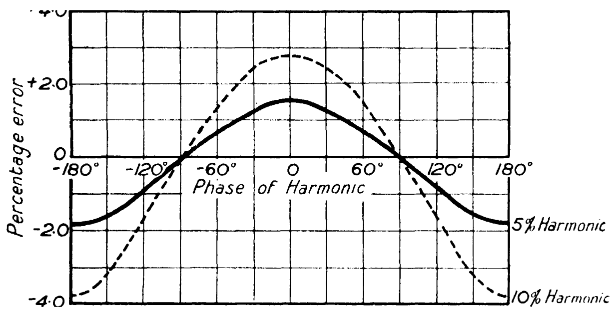


FIG. 6.52.—Rectifier meter errors due to third harmonic.

Efforts have been made to compensate for these temperature errors, but these have not been entirely successful, due to the fact that the law connecting error and temperature is a complicated one. One method used in low-range voltmeters is to wind part or the whole of the series resistance of copper, so that the increase in resistance of the copper with increase of temperature will counteract the decrease in resistance of the rectifier as the temperature increases. This correction can only be partial, since the laws governing the variation of resistance with temperature are different. Another method which has been suggested and used is to replace two of the rectifier arms with resistances, thus reducing to half the number of elements with variable resistances, but at the expense of the current consumption, which is doubled. A number of other methods have been tried, but with only partial success.

As stated above, the moving-coil instrument actually measures the average value, and therefore will only give the true value of an

alternating current if used on a wave-form identical with that on which it was calibrated. Usually a sinusoidal supply is used for calibration, and consequently any variation in wave form which changes the form factor from 1·11 can cause an error. As an indication of the possible magnitude of this error, the curves in Fig. 3.52 show how the error varies with the magnitude and phase relationship of a third harmonic superimposed on a sine wave.

CHAPTER 7

SOFT IRON INSTRUMENTS

THIS type of instrument has been very widely adopted, since it is capable of indicating on alternating as well as on direct current circuits, and permits of a cheap and robust construction. Instruments of this kind depend for their indications on the movement of a piece of soft iron in the field of a coil traversed by the current to be measured. They may be classified into various types according to the manner in which this movement is effected.

Attraction Instruments

(*a*) *Direct pull type.* A small soft iron plunger is attracted axially into a current-carrying solenoid (Atkinson, Hartmann & Braun, Kelvin).

(*b*) *Deflected needle type.* A soft iron needle is pivoted transversely, and is magnetized by a solenoid, so that it tends to set itself along the field of the latter (Miller, Siemens & Halske, Everett Edgcumbe, Evershed). Now obsolete.

(*c*) *Attraction type* (*single iron*). The soft iron rod or disc and the rotational axis are parallel to the axis of the solenoid which surrounds them, but the shaft is displaced from the centre, so that the deflection brings the iron into the stronger field close to the winding (Schuckert).

(*d*) *Attraction type* (*double iron*). The iron is attracted towards the gap between the ends of one or more irons placed so as to complete the field of the coil (Stanley, Evershed).

Repulsion Instruments

The moving iron is mounted parallel to the axis of the solenoid, and is repelled from a similar iron lying parallel to it within the coil (Nalder, Evershed, Weston, B.T.H., Everett, Abrahamson, Hartmann & Braun).

Combined Attraction and Repulsion

The moving iron is repelled from a parallel iron within the coil as above, and attracted towards the gap in another iron as in (*d*) above (Reiniger, Gibbert & Schall, Siemens, Schuckert).

In Fig. 7.1 are given diagrammatically examples of the chief applications of these principles. It will be observed that in every case the deflection of the iron tends to increase the field through the solenoid, and thus its inductance, as explained in Chapter 4. In the case of all the attraction instruments, the iron becomes more strongly magnetized by moving into the stronger field, or setting itself parallel to it ; while in the case of repulsion instruments

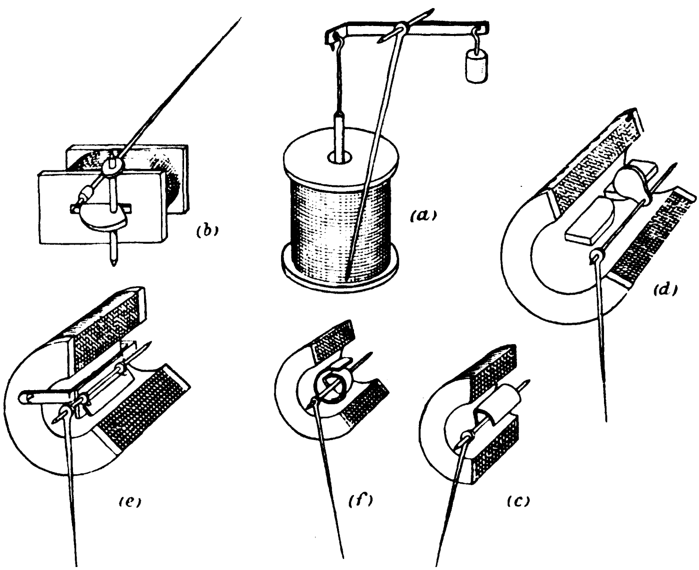


FIG. 7.1.

the magnetic flux in each iron is evidently diminished by the proximity of the other, and it becomes greater the further the irons are separated.

(a) Direct Pull Instruments

The simplest type of this instrument uses a coil and plunger. If a piece of iron be suspended vertically over the mouth of an upright solenoid, it will be drawn in by a force which is a function of the strength of the current circulating in the coil and also of the form and position of the iron. If the suspension is a long helical spring which provides the restoring force, and the iron carries an indicator, recording its position for each value of the current in

the coil on a vertical scale, we have realized one of the earliest and most primitive forms of this type of instrument, due originally to Kohlrausch.

The relation between the current and force for different types of coil and plunger has been investigated by several observers, and in general it is found that with a plunger considerably longer than the coil the maximum pull occurs just as the end of the plunger is emerging from the lower end of the coil. If, however, the iron is shortened, the induced magnetic pole outside the coil begins to have effect, and the maximum pull occurs earlier in the travel; as the iron is still further shortened the point of maximum force shifts nearer and nearer the mouth of the coil, until, if the iron is contracted into a little sphere, the only pull exerted is just as the sphere is entering the mouth.

We may, of course, stop the lower end of the coil by means of a fixed iron core, and under these circumstances the force curve will run up very steeply as the gap between the irons diminishes.

The effect of coning the plunger was investigated by Bruger in 1886, and he found that very considerable modification of the force curve resulted. The pull was, in general, much less, and the maximum occurred when the lower end was well outside the lower face of the solenoid. It is obvious from a study of the force curves such as are shown in Fig. 7.2 that an evenly-divided scale cannot be expected with this simple type of instrument, but by employing an iron of such a small cross-section that it is magnetically saturated at quite small currents a convenient scale can be obtained, and Hartmann & Braun and other Continental instrument makers have constructed such instruments to work as current gauges.

Professor Ayrton also employed the principle in his magnifying spring instruments, in which a very thin, soft tube is suspended and guided within a vertical solenoid. The suspension is a narrow ribbon of phosphor bronze wound in the form of a long helix, the lower end of which was attached to the iron tube through which it passed, and its upper end is then rigidly attached to a pin passing through the glass scale cover. The small downward movement of the plunger produced a large rotatory motion of the end of the spring, and the plunger was thus turned on its axis, and the pointer attached to its upper end moved round a horizontal scale. To make the instrument direct reading, an auxiliary coil of fine wire of such diameter that it would slide over the main bobbin was

connected as a shunt to the main coil and moved up and down until the correct position was found, and it was then permanently fixed.

In the Atkinson modification of this type of instrument the spring is eliminated, and the iron plunger is enclosed in a tube like that of an immersion hydrometer. The solenoid is arranged above, so that the hydrometer is lifted in the liquid (a saturated solution of a

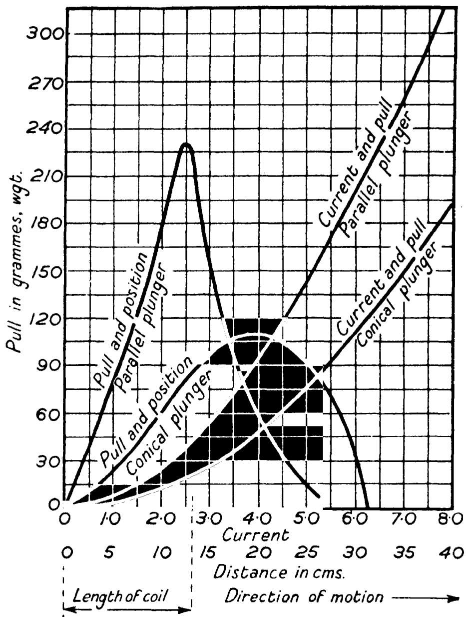


FIG. 7.2.—Curves for coil and plunger.

special salt) in which it is immersed, and its position is indicated by means of a dark band carried by the float against a fixed scale.

The coil and plunger principle has also been adapted to the circular form of instrument by Dolivo, Compagnie des Compteurs, and several other makers, by simply suspending the plunger flexibly from a short arm on the axis of rotation, which is mounted over the vertical coil. The downward motion of the iron core is thus transformed into rotary motion, and moves the pointer over a circular arc, the control being provided by gravity acting on suitably arranged weights. The Kelvin ampere gauges were also constructed

on this principle. In one form the iron plunger is drawn upward by the solenoid above, and is constrained to remain vertical by a fairly heavy weight carried on its lower end well below the axis of the pointer, and terminating in a damper moving in an oil dash-pot. The mass of the iron, weight and damper is partially counter-balanced by a fixed weight hanging from an arm on the opposite side of the axis of rotation. The Kelvin sector instruments, however, embodied the greatest refinements of this type, and in these a good scale was obtained in the following way:

The vertical coil is arranged below the rotational axis, and the thin iron plunger is attached to this axis by a flexible cord working over the edge of a light metallic sector. The pull, which is thus always exerted at a constant radius, is balanced by the leverage of gravity weights, which, since they are on a fixed arm, are subjected to the constant pull of gravity acting at a varying radius with each position of the plunger. Then if F is the magnitude of the pull due to the plunger, and a is the radius of the sector, w the resultant counterbalance exerted at a radius a_1, and considered as concentrated at the centre of gravity of the system, then for a given deflection of the pointer, θ_1, $a_1 \sin \theta$ is the horizontal displacement or radius of the counterbalance w, and the restoring torque is $wa_1 \sin \theta$, balancing the torque produced by the current acting on the plunger in the solenoid, and equal to Fa; so that

$$Fa = wa_1 \sin \theta = T_K \sin \theta,$$

where T_K is the maximum torque wa_1, due to the control weight. Now the purely mechanical torque represented by the right-hand side of the equation depends upon the sine of the angular deflection, while on the other hand F will vary with the magnetizing force of the solenoid, the relative positions of core and coil, and the induction in the iron. By combining the first two of these suitably F is also made to vary according to a sine law, and by ensuring complete saturation of the iron plunger at all currents the third is eliminated, so that an even scale results. Thus in this instrument, if we keep the current in the coil constant and measure the pull on the core in different positions, we obtain the sine curve A in Fig. 7.3; if, on the other hand, we keep the core stationary and vary the current through the instrument, we obtain the curve B in Fig. 7.3, which is very approximately a straight line. Fluid damping is employed, a glass tube containing oil being attached to the

lower flange of the coil into which the plunger moves. The method of suspension is shown in Fig. 2.18, page 51, and is the well-known Kelvin knife-edge type. For heavy currents the coil is sometimes

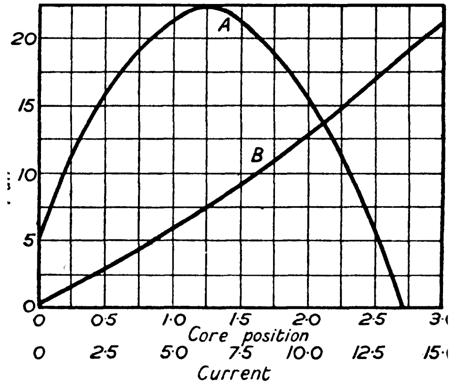


FIG. 7.3.—Curves for Kelvin sector ammeter.

built up by cutting a spiral slit in a series of concentric tubes, the first, third, fifth, etc., tube having a right-handed pitch and the intermediate ones a left-handed one ; these are then joined at the ends in such a way that the current travels up the outer spiral

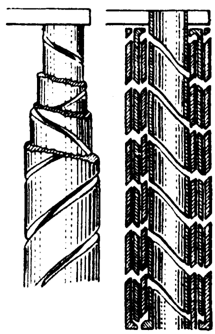


FIG. 7.4.—Coil of Kelvin soft iron instrument for heavy current.

and then down the spiral of the second tube, up the third and so on. Thus a very robust and excellently-insulated compact coil is formed, giving an intense and concentrated field (Fig. 7.4). With instruments for currents exceeding 2,000 amperes the coil is sometimes replaced by a simple bus bar, and the plunger is then

made in the form of a long inverted U of iron wire, which straddles the thin edge of the bar, thus converting the instrument into one of the deflected needle type. The current in the bar magnetizes the plunger in such a way that at the end of one limb of the U a north-seeking pole is formed, and at the other a pole of opposite sign; and both are urged in the field due to the current in the bar in the same direction, and hence the force is double that which would be given by a simple straight plunger, and the advantage of a more completely closed magnetic circuit is also gained. In some instances the movement was located at the bus bars and the scale-box and pointer on the switchboard, the two being mechanically connected by means of a flexible cord.

(b) Deflected Needle Instruments

Among the earliest direct-reading commercial instruments of this type were the Ayrton and Perry ammeters, in which a light, soft iron needle was pivoted between the poles of a large permanent horse-shoe magnet provided with specially-shaped pole pieces. The current to be measured was sent round a solenoid so arranged that it produced a field at right angles to that in the gap of the magnet, and the needle is therefore deflected by this cross field. The control was therefore magnetic, and the large, permanent magnet screened the movement from external fields. Owing to the small moment of inertia of the moving system and the powerful control the time of swing of the needle was very short, and the pointer rapidly reached its position of equilibrium, and therefore no additional damping arrangement was fitted. The final adjustment of the calibration was made by means of soft iron cores screwed into the outer ends of the coil bobbin, which could be advanced or drawn back as desired.

The Whitney Company of America constructed a modified arrangement on somewhat the same principles. In these instruments the permanent magnet was of the ordinary U-shape, but was turned on its side so that one pole was above the other. The coils were arranged between the limbs so as to produce a cross field, as in the Ayrton and Perry instruments, and the vertical spindle on which the needle was mounted was brought through a hole in the upper pole of the magnet and carried the pointer and control spring above. The magnetic axis of the needle was inclined to the axis of rotation, and although it was magnetized by induction from the

poles of the permanent magnet it was not subject to magnetic control. Eddy-current damping was provided by means of an aluminium vane moving between the poles of an auxiliary magnet.

The Thomson inclined-coil instruments also belong to this class. In these, the working coil is a circular bobbin which is supported so that its plane is approximately 45° to the axis of rotation of the needle, which passes vertically through its centre. On this axis the moving element, in the form of a thin iron strip or bundle of laminations, is rigidly and symmetrically mounted, with its plane also inclined at an angle to the vertical. At zero the element is across the plane of the coil, but when current is sent through the coil it turns so that its axis more nearly coincides with that of the coil. By adopting this arrangement a working range of about 100° is obtained, and in the later forms of instrument a laminated magnetic shield is fitted to protect the instrument from external magnetic fields and eddy-current damping is provided.

There are also a number of instruments in which an iron disc mounted eccentrically on the axis of rotation was drawn into the narrow mouth of a rectangular coil, across one flange of which the axis was pivoted.

(c) Attraction Type Instruments (Single iron)

A very simple form of instrument based upon this principle was constructed by Schuckert, and is illustrated in Fig. 7.5. Here a

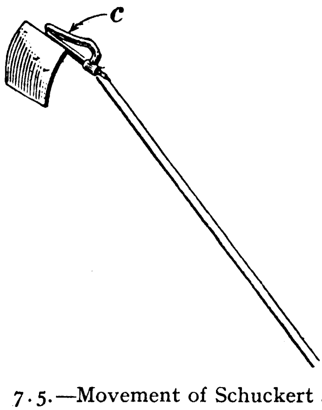


FIG. 7.5.—Movement of Schuckert ammeter.

thin plate of very soft iron is bent to a right angle, and one side of the angle iron so formed is attached to the axis of rotation,

the other side being given a slight curve, so that when fully deflected it will come to rest with its curved face parallel and close to a portion of the inner tube of the coil bobbin, and when there is no current in the coil it falls under the action of gravity to a position nearer the centre of the coil; the action is therefore such that the iron is attracted from the weaker to the stronger portion of the coil field. The moving system is very simple and light, the balance and sensitivity being adjusted by means of a piece of copper wire (*c* in Fig. 7.5), which is attached at one end to the axis of rotation and is capable of being bent in any required direction. No air or other damping arrangements are fitted, the vane moving within the coil being considered sufficient for the purpose. The torque for a given number of ampere turns is, however, small in this type of instrument in comparison with the repulsion type.

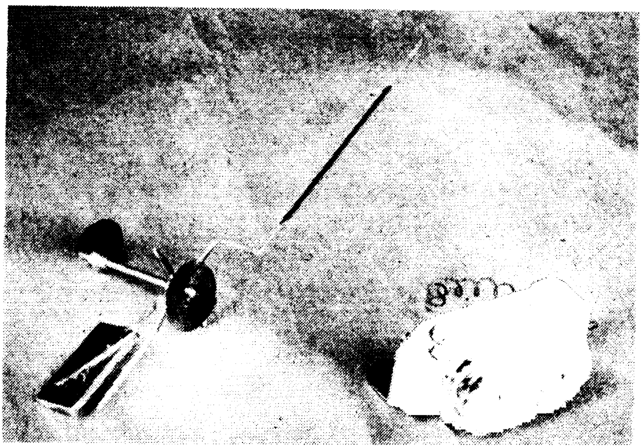


FIG. 7.6.—Typical coil and moving iron (Elliott Bros.).

In the modern forms of attraction instrument the coil is usually wound flat with a narrow slot entry for the moving iron. The moving iron is a shaped disc mounted on a pivoted shaft so that the coin can be drawn into the slot in the coil, and a typical coil and moving iron from an Elliott instrument are shown in Fig. 7.6. The scale obtained from this type of instrument can be seen in Fig. 7.7. Fig. 7.8 shows another form of construction.

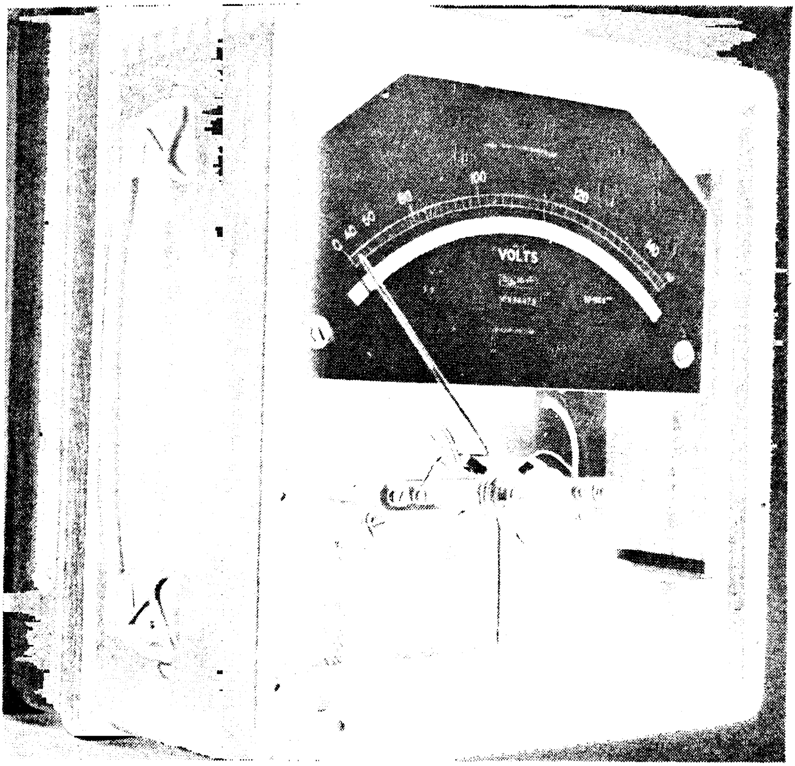


FIG. 7.7.—Complete instrument showing type of scale obtained.

(d) Attraction Type Instruments (Double iron)

One of the earliest forms of this instrument was made by Evershed, and the movement is shown in Fig. 7.9. As will be seen, the bracket supporting the jewel screws carries on its inner flat surface two somewhat large attracting irons (*a*) mounted end to end, with a central gap between them. The moving iron is a little cylinder (*b*) fastened to the shaft by an arm of short radius. The field produced by the current in the instrument coil magnetizes these irons in such a way that the cylinder is attracted down into the gap between the fixed irons, and it is possible to shape the scale by adjusting the gap faces of the attracting irons. Although this scheme permits of a light movement and a short moving iron the attracting masses have considerable bulk, and the introduction

FIG. 7.8.—Another form of moving iron construction.

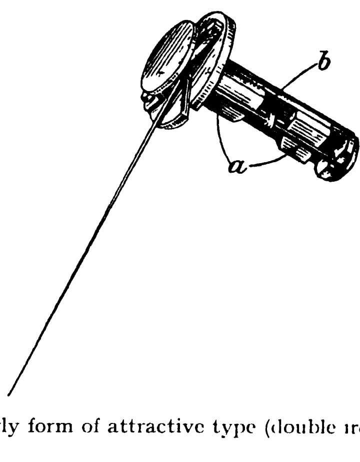


FIG. 7.9.—Early form of attractive type (double iron) (Evershed).

of so large a quantity of iron into the coil considerably increases its inductance. A similar design was adopted in the Stanley instruments, except that these instruments instead of employing a short cylindrical moving iron had an iron sector inserted in a disc of brass, which is mounted concentrically on the axis of rotation and is attracted down into the gap between the fixed irons. The arrangement of these instruments is shown in Fig. 7.10.

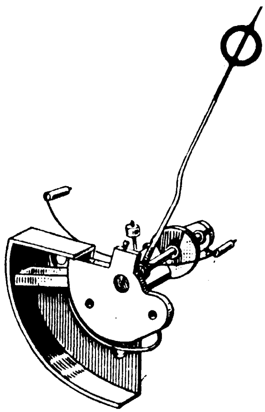


FIG. 7.10.—Movement from Stanley soft iron ammeter.

Repulsion Instruments

This principle has been very widely adopted for soft iron instruments, largely because it has the advantage of beginning to indicate with a smaller current than most other types, and it permits of a certain amount of adjustment of the form of scale by shaping the repulsion iron, and in many cases the upper part of the scale is opened out by fixing a small attracting iron at a suitable part of the framework. In its simplest form an iron is fixed parallel to the axis of the coil along the inner side of the jewel-supporting bracket, and the moving iron is carried at a short radius from the pivot staff and parallel to it. At zero these two irons are close together, but when current circulates in the coil they become similarly magnetized and the moving iron is repelled. In early instruments made by Nalders the repulsion iron was a cylindrical rod and the moving iron a rectangular bar. Most manufacturers, however, used thin soft strips in preference to the heavy solid rod,

thus considerably lightening the movement and at the same time adding something to the damping. The British Thomson-Houston instruments (Fig. 7.11), Abrahamson (Fig. 7.12), and Everett Edgcumbe (Fig. 7.13), are all of this form. In the Abrahamson instrument there is no damper, and the moving iron is made very short axially.

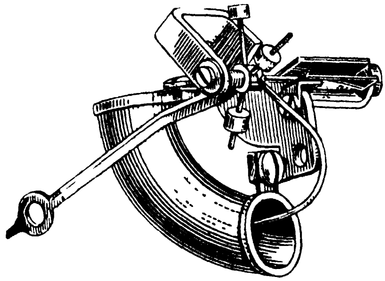


FIG. 7 11.—Movement from B T-H. voltmeter.

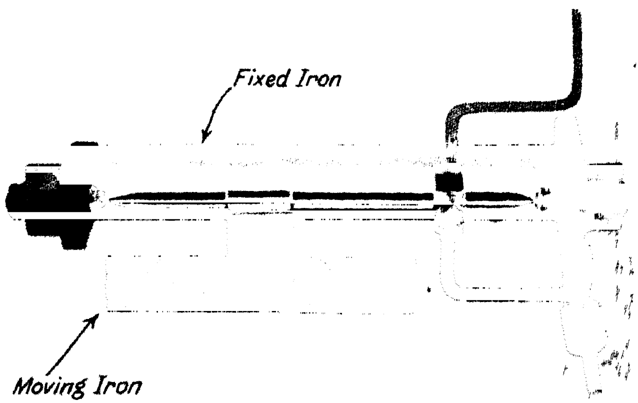


FIG. 7.12.—Moving system from Abrahamson voltmeter.

A modern form of Everett Edgcumbe instrument with the movement partly withdrawn is shown in Fig. 7.14. The damping chamber and movement support consists of a die-casting, and in modern instruments the damping chamber is often in the form of a die-casting or a plastic moulding. The moving system of this instrument is shown in Fig. 7.15, while the fixed iron can be seen partly withdrawn in Fig. 7.14. An example of plastic moulding

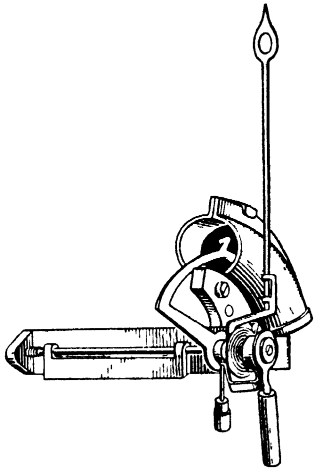


FIG 7.13.—Movement from Everett Edgecumbe repulsion ammeter.

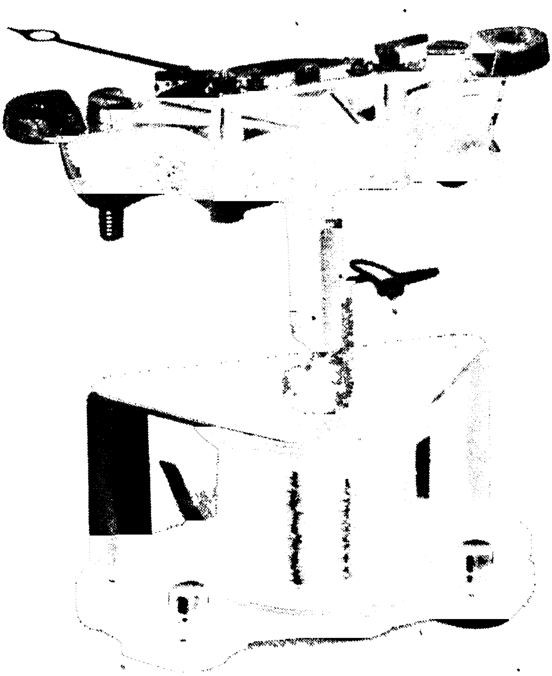


FIG. 7.14.—Modern Everett Edgcumbe meter showing movement withdrawn.

is shown in Fig. 7.16, the moulding including the movement supports and the air damping chamber. The bridge piece carrying the front jewel bearing is mounted on two brass pillars moulded in and the back jewel bracket is secured to the end of the lower centre projection. The fixed iron is also mounted on this bracket. The moulding on this limb below the centre platform is formed in a tubular manner to act as a spigot for centralizing the coil. The

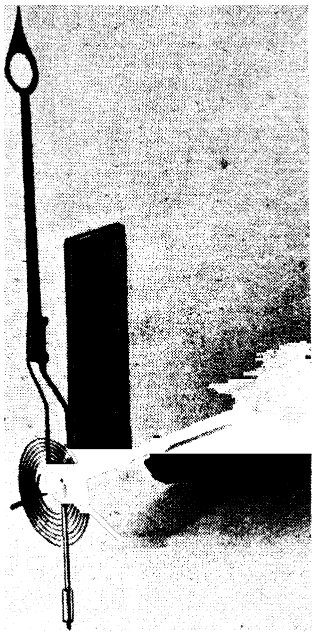


FIG. 7.15.—Moving system of instrument of Fig. 7.14.

lower two side members form supporting bosses for the movement, and have holes for fixing to the base. Fig. 7.17 shows a view of the moving element of this instrument, and it will be seen that a moving iron is welded to a supporting bracket, which is clamped against a collet on the shaft by a distance sleeve which also engages the lugs of the damping vane. The control spring is soldered to a split-spring collet which fits on a circular nut above the pointer fitting. The pointer fitting and balance arm punchings are separate and mechanically interlocked together to prevent movement

between them, whilst the pointer is a light metal punching of curved section secured to the fitting by four tongues bent over. The damping vane is carried close up to the shaft to give both maximum area of vane and strength of support. This instrument is made by the Metropolitan-Vickers Electrical Co., Ltd. The coil is made from copper strip wound on edge with the leads formed to shape,

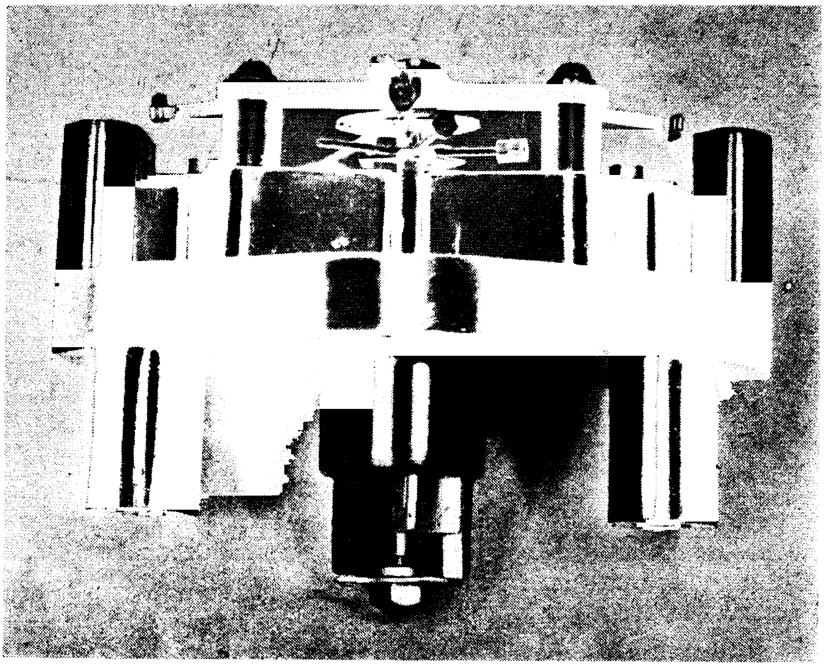


FIG. 7.16.—Plastic moulding carrying movement (Metropolitan Vickers).

and afterwards put into a mould so that the insulation block can be moulded in position. This system avoids any internal joints in the coil system. (See Fig. 5.2).

All the movements above described necessitate a relatively wide opening in the coil, and as usually the amount of iron is fairly considerable, the inductance is relatively high, and this, combined with the fact that the permeability of the iron is not constant, results in the frequency and hysteresis errors being somewhat high.

A modification of the repulsion principle was used in some of

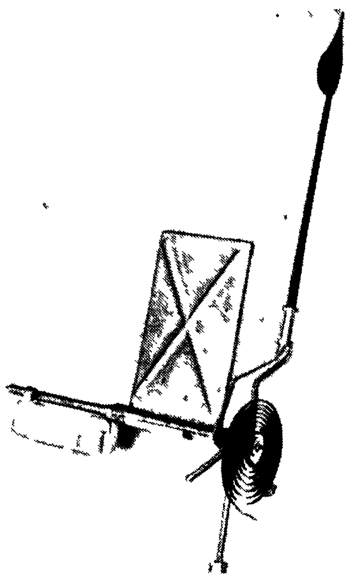


FIG. 7 17 —Movement of meter shown in Fig 7 16.

the early Evershed instruments, in which the moving iron is a segment of an iron cylinder and is mounted concentric with the axis of rotation (Fig. 7.18). The fixed iron is a tapering tongue of thin soft iron sheet, rolled into cylindrical form concentric with the axis, and arranged so that when the movement is on zero the moving iron is parallel to the broadest part of the surrounding tongue, but as the instrument is deflected it rotates towards the narrower part. The form of scale is therefore under control, since it depends upon the shape of the iron tongue.

Many modifications of this have since been designed. Fig. 7.19 is the Hartmann & Braun form. The tapering repulsion iron is seen curving over the tube inside which the moving iron moves. This tube also serves to support the jewels and the damping-box at its front end, in which a light aluminium damper bent into Z-shape moves. This construction allows of an excellent self-contained movement, in which the moving element is protected from dust, damp, and mechanical damage. The Weston instruments are of

similar type (Fig. 7.20), the repulsion iron being inset in a cylinder of non-magnetic material carried on the jewel support, the damping-box being arranged in front. For currents above 100 amperes the coil consists of a few turns of square section copper rod attached to

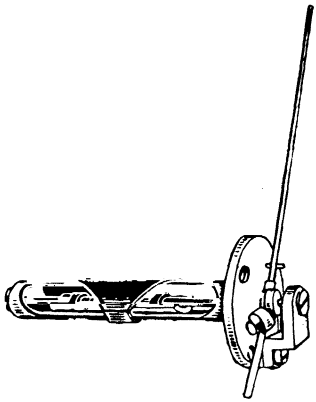


FIG. 7.18.—Moving system of early form of Evershed ammeter.

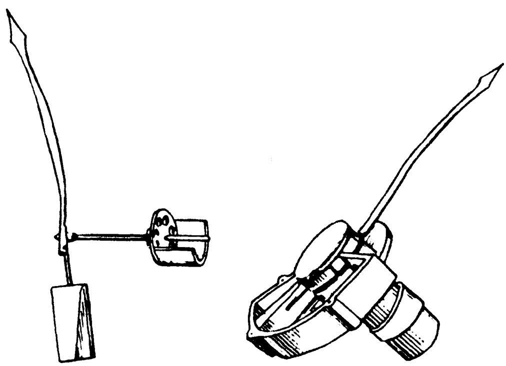


FIG. 7.19.—Details of moving system of Hartmann & Braun ammeter.

the lugs, which makes a neat and well-cooled coil. An exploded view of an Evershed movement is shown in Fig. 7.21.

The Record instruments also employ this form of iron and a movement is shown in Fig. 7.22. The construction is very neat, and is built up principally from brass pressings in such a way that the whole movement is removable from the coil by loosening two

screws. The fixed tapering iron is supported by two metal arms attached to the back jewel support, and in front is a sector-shaped box in which a single vane with upturned edges moves.

The voltmeter bobbins are constructed from brass pressings, and the heavy current ammeters have coils of bare copper strip wound on edge.

In another form of repulsion instrument, instead of using solid rods or thin, flat plates, as in the former examples, the irons are

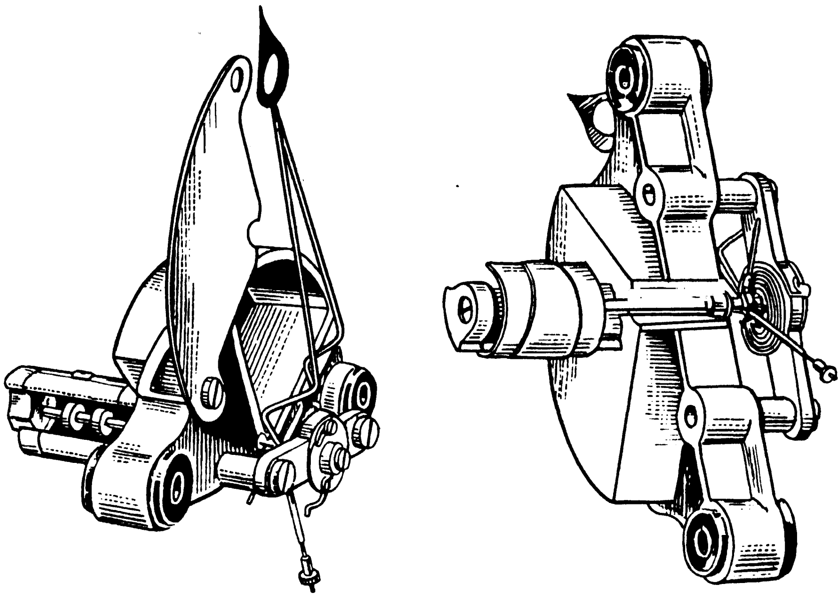


FIG. 7.20.—Moving system of Weston soft iron ammeter.

segments of thin cylinders with parallel edges, one being fixed by one of its edges to the jewel-frame and the other similarly attached to the axis of rotation. At zero the two irons are over one another, the moving iron outside, and when magnetized the moving iron is repelled outwards. The air-damping vane moves in front of the dial plate, and is enclosed by a sector cover which is placed over it and screwed to the dial plate. This form of movement is, however, rather difficult to balance perfectly, and requires a rather large inner diameter to the bobbin.

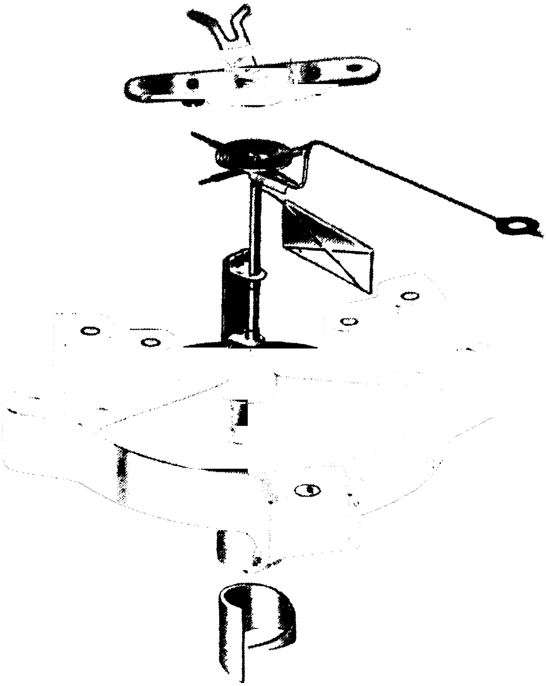


FIG 7 21 —Exploded view of movement of Evershed instrument

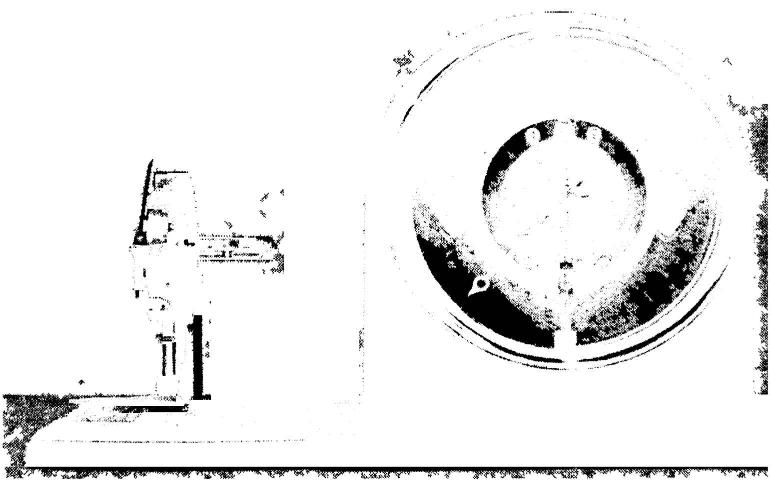


FIG. 7 22 —Double movement repulsion type voltmeter (Record Electrical Instrument Co).

(Photo by courtesy of the Director, Science Museum)

Combined Attraction and Repulsion Instruments

The Harrison instruments were among the earliest employing this principle. In these the movement was mounted inside an enclosing tube like that already described in the Hartmann & Braun instruments, the jewels being inserted in the ends. Within this tube a short repulsion iron and an attracting iron, consisting of two short pieces of iron rod placed end to end with a gap between them, are fixed in suitable positions. The moving iron is also a short rod carried on an arm from the axis of rotation, and at zero this iron is parallel to and close against the repulsion iron. If, now, the surrounding coil is supplied with current, the instrument behaves as a repulsion instrument until the moving iron comes under the influence of the field across the gap of the attracting irons, which thus keeps the upper part of the scale open. Thus, by properly proportioning and placing the irons, a long and fairly uniform scale could be obtained. Holden adapted the principle in a different way. The coil was wound on an iron core which completely filled the interior, and in front a rectangular iron pole piece was attached, and arranged at such an angle that the moving iron (in this case a length of iron wire projecting radially from the pivot axis) lay over and parallel to the face of this pole piece, which was provided with a thin, non-magnetic covering to prevent the moving iron from adhering to it. The back end of the main iron core was also provided with a pole piece in the form of a rectangular iron bar, bent at right angles and laid over the coil so that one end was in magnetic contact with the main core, while the other projected a little beyond the front face of the coil at a position just beyond that corresponding to maximum deflection of the needle. The action of the instrument is therefore obvious: when current circulates in the magnetizing coil, the moving iron is repelled from the pole piece, against which it rests when at zero, and tends to move over towards the other pole piece and thus close the magnetic circuit round the coil. In these instruments the pivot spindle is made quite short, the back jewel being inserted in the front face of the central iron core, the front one being carried by a bracket screwed to the front flange of the bobbin; the pointer and iron are on the same line, and therefore one practically counterbalances the other, and a very light movement is therefore possible. There are, however, important

objections to the iron core and somewhat massive pole pieces which practically preclude the instrument from being used with alternating currents. Fig. 7.23 is another example of the principle by Reiniger, Gibbert & Schall. In this case a split brass tube carries the fixed irons, which are fastened to its outer surface in the manner shown. The repulsion iron is in the form of a thin, tapering tongue (*a*), the broad end of which is bent sharply inwards and passes through a slit in the tube, where it projects radially inwards; the two attracting irons (*b*) are seen on either side near the point of the repulsion iron. This tube is slipped into the interior of the bobbin, and the needle shaft is then inserted and held between

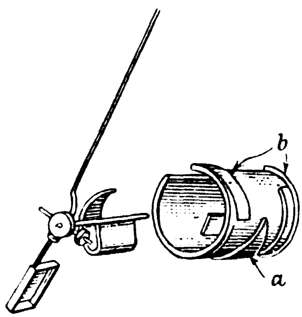


FIG. 7.23.—Fixed and moving iron of Schall ammeter.

jewels carried on bridge pieces attached to the front and back flange of the bobbin. The needle is also in the form of a bent and tapering tongue, and by adjusting the forms of these irons an excellent scale is obtained.

In general the torque in soft iron instruments is small compared with that usually obtainable in moving-coil instruments, and with gravity-controlled types is not easy to measure. The figures in Table XXXII were obtained by employing the little torque-measuring device shown in Fig. 7.24, in which a light aluminium pointer (*a*) is mounted at right angles to a small silver disc (*b*), which is strung by wire or suspension strip between the arms of the brass fork-piece (*c*); the whole movement is carefully balanced so as to be independent of position, and one end of the pointer is arranged to read on a scale of degrees attached to the instrument, while the other end is hooked for calibration and engagement with the pointer of the instrument whose torque is to be measured.

The device is first calibrated by hanging riders of known mass on the pointer hook and turning the whole instrument until the pointer is horizontal, when the deflection is read off and the constant determined at several parts of the scale, or a curve connecting weight and deflection can be plotted. In applying the device to measure the torque of an instrument the latter is first set up with its pointer at zero, and the torque-tester, mounted in a universal holder at the right height, is brought in front of it, and the axis of rotation made to correspond with the line of the pointer of the test instrument is now made to engage in the hook of the torque measurer, and the latter is turned until its pointer is both hori-

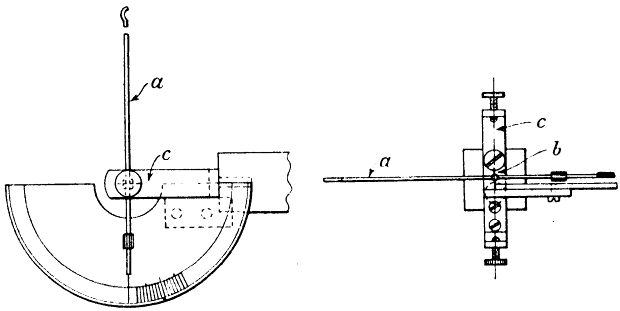


FIG. 7.24.—Torque tester for gravity-controlled instruments.

zontal and at right angles to the pointer of the instrument under test, and therefore pressure on the arm of the measurer is normal; the deflection in degrees is then read off and the radius of the point of application measured. The weight corresponding to the deflection is obtained from the calibration of the tester, and the torque is then simply the product of the pointer radius and the weight.

With spring-controlled instruments the torque is simply obtained by hanging weights on the pointer, as described for moving-coil instruments on page 313, but special care should be taken to see that the movement is sufficiently well balanced, so that on turning it through the required angle to bring the line of the pointer horizontal in the fully-deflected position the instrument is not sensibly deflected from zero when there are no weights on the pointer. Agnew has suggested employing a pendulum whose bob can be attached by means of a light fibre to the pointer of the instrument under test, and this when deflected displaces the pendulum bob through a measured angle; if the connecting fibre is maintained

horizontal, and the constants of the pendulum are known, the horizontal pull can be calculated; but it is also necessary to know accurately the angle between the pointer of the test instrument and the horizontal fibre in the case of gravity controlled instruments, and this is not easily done.

ERRORS IN SOFT IRON INSTRUMENTS

The errors usually occurring in soft iron instruments may be classified under the following headings:

(*a*) Friction.
(*b*) Heating.
(*c*) Stray magnetic fields.
(*d*) Hysteresis and position error.
(*e*) Frequency errors (inductance and eddy currents).
(*f*) Wave-form error (hysteresis and inductance).

The errors (*a* to *d*) affect the instrument both on alternating and direct current circuits; the last two (*e* and *f*) are, of course, absent with direct currents.

(a) **Friction.**—The general consideration of friction errors has already been dealt with in Chapter 2, and the magnitude of the error in soft iron instruments is given in Table IX. Generally the friction error is less in this type of instrument when used on alternating current circuits, probably because of the slight vibration produced by the rapid alternation of flux which occurs in the iron movement. The values given in the table were obtained by setting the pointer to the centre of its scale, and then taking it to each end successively and letting it very gently back to its natural position of rest; the reading corresponding to these two positions is carefully observed, and the difference obviously contains twice the friction error. This is a very drastic test of the pivoting, as shown by the large values obtained for some of the instruments.

(b) **Heating Errors.**—Heating in instruments may result from two causes, viz. that due to internal ohmic loss, and secondly, that due to changes of temperature external to the coil, or what is sometimes known as "chamber error." The resultant heat may affect the instrument in the following ways: (*a*) by causing expansion of the parts; (*b*) by altering the permeability of the iron; (*c*) by altering the resistance of the coil.

As pointed out in Chap. II, the expansion affects the friction

error by altering the jewel setting relatively to the shaft, and the advantage of the brass spindle with inset pivots has been already pointed out; and since the pivots are on a continuous axle, and there is practically no liability to warping of the parts, good jewel adjustment should be always possible. Much could yet be done in many instruments in the way of improving the mechanical design of the supporting bracket to which the jewels and other fixed parts of the movement are attached, and many of the heavy, clumsy, and badly-arranged castings so often employed could well be replaced by light but equally rigid structures of much smaller heat capacity.

The second effect, change of permeability, will generally be too small to seriously affect the indications of the instrument; with some kinds of iron, however, ageing may result from the continuous heating unless the material has been previously aged, and the permeability may thus be reduced and the hysteresis error increased.

The third effect, change of coil resistance, is by far the most serious. If the coil is wound entirely of copper it will increase by about 0·4% per 1° C.; in ammeters such an increase, or indeed any increase, is of no consequence where the entire current is taken through the coil, since it is simply a question of getting the current round the circuit, and an increase in the ammeter resistance merely affects the small P.D. across the ammeter terminals without altering the magnetic effect for a given current. It is, however, desirable to reduce the watts lost in the instrument to a minimum by keeping the resistance to its lowest value, but changes of resistance in a simple series type of current-measuring device cannot produce any error in its indications. With voltmeters, however, this is by no means the case, as they are really ammeters of assumed constant impedance, so that the P.D. is proportional to the current through the coil. If the resistance increases by reason of internal heating, the working current must fall for a given P.D. at the terminals and the instrument will read low. This difficulty may be overcome by winding the coil with an alloy of low temperature coefficient, as is done by some makers, but usually this would result in there being insufficient space for the requisite number of ampere turns, since the gauge of wire employed would have to be considerably increased to get the required resistance and the length of wire necessary. The power consumption also causes excessive heating when confined to the working coil with its limited radiating

surface. These considerations, together with the increased cost of material, have led to a compromise in which the coil is wound with the required number of ampere turns in copper, and it is then put in series with a non-inductive resistance of low temperature coefficient which brings the total instrument resistance up to the required amount. This auxiliary resistance thus forms the major part of the whole resistance, and " swamps " out the temperature changes in the copper coil, or, in other words, the heating changes in the copper portion of the circuit become insignificant. Thus, if R_1 is the resistance of the copper coil and a_1 its temperature coefficient, while R_2 and a_2 are the resistance and temperature coefficient of the series resistance, then the temperature coefficient a of the combination is

$$a = \frac{R_1 a_1 + R_2 a_2}{R_1 + R_2}$$

and by suitably choosing R_2 and a_2, a may be made very small, or the resistance of the instrument will be practically constant at all temperatures.

In general, the maximum error due to internal heating in soft iron voltmeters is about 1%, and as it is usually assumed that the instruments are intended to be continuously connected in circuit, they are calibrated for the upper temperature. If used intermittently, therefore, they will usually be found to be about this amount high, but of course everything depends upon the proportion and character of the swamping resistance and coil.

The effect of external heating will also lower the reading of the instrument, a rise of external temperature of 20° C. producing about 1% change in most cases, independently of the internal heating. Changes of external temperature may, however, occur with comparative suddenness, and their effect on the instrument, in such cases, will depend upon the total heat capacity of the various parts, which will cause a lag in the effect of different amount in each part. Thus, in instruments possessing well-ventilated series resistances, mounted externally at the back of the case, while the working coil is enclosed, it will be found that although the latter is practically unaffected by the temperature change for some considerable time, the former will take up the change quite rapidly. This is usually of no serious consequence, as the series resistance usually has a very low temperature coefficient. In the better class

of instrument, however, it demands consideration, for, if the series resistance is mounted within the case, it is protected from external changes of temperature ; but, on the other hand, the working temperature within the case is increased, and the change in the copper coil resistance is much more serious.

(c) **Stray Field Errors and Magnetic Shielding.**—The errors produced by external fields are of serious importance in all types of instruments employing weak working fields, and this is naturally a characteristic of all moving iron types. The following table gives the values of B under various conditions in a few typical instruments. The average value of B is obtained by dividing the total magnetic flux through the coil and iron by the internal area of the coil, while that of the iron was determined by winding a search coil on it and employing a fluxmeter.

TABLE XXXII.

Maker.	Instrument type and range.	Average B coil and iron.	B in iron full load.
Nalder Bros.	Repulsion voltmeter, 20 v.	276·4	4,330
Everett, Edgcumbe	Deflected needle attraction voltmeter, 130 v.	278	446
Abrahamson	Repulsion voltmeter, 150 v.	332	5,950
B.T.H.	Repulsion voltmeter, 150 v., U.R. type	255	5,070
Hodges & Todd	Repulsion voltmeter, 120 v.	172.5	2,860
Evershed	Attracted iron voltmeter, 150 v.	425	2,104
,,	Deflected needle attraction voltmeter, 250 v.	..	2,400

The effects of external fields are difficult to determine, and the magnitude of the error produced is dependent on the strength of the interfering field, its direction relative to the axis of the instrument coil, and the form and position of the irons in the instrument.

As far as the magnitude of the disturbing field is concerned, such of the few figures that are available usually refer to the field due to 1,000 amperes in a bus bar at one metre from the instrument, that is, to a field of two C.G.S. units, and it is common knowledge that fields of ten times this magnitude are often found at supply station switchboards.

The field will, in general, have its greatest effect when it coincides with the magnetic axis of the instrument bobbin. Thus in the case of an instrument with a horizontal bobbin, a conductor above

or below the instrument will produce the greatest effect, but in some cases cross fields may produce quite serious errors.

In order to investigate the error experimentally, an apparatus was constructed consisting of two large-diameter coils mounted with their planes vertical and parallel in a wooden framework, so that when current was sent through them in series a fairly uniform field was produced in the space between them. The arrangement was then calibrated by reversing definite currents in the coils and measuring the flux through a large search coil (mounted between them in the position to be occupied by the test instrument) connected to a fluxmeter. A curve was so obtained giving the relation between the field and the current through the coils.

The test instrument was then arranged centrally between the coils, and the deviation from the normal reading observed by switching on currents corresponding to various field strengths, both direct and reversed, due care being taken to eliminate permanent magnetic effects by reversals between each set of readings. A large number of observations were made, and the results in some cases show considerable complication, so that it is only possible to give a generalized summary here.

Fig. 7.25 shows the error of an unshielded simple repulsion movement in a field which coincides with that of the coil, the instrument reading about 80% of its maximum (a condition maintained in all the following tests), the second curve in the figure showing the improvement effected by an open-sided iron screen 23 mils thick round the coil and a cast-iron case.

A simple repulsion, a deflected needle attraction instrument, and an attracted iron instrument, all in iron cases, were tested and the results are shown in Fig. 7.26. The disturbing field is in each case along the axis of the bobbin, and when the iron cover is removed the instrument is unscreened except for the iron back plate, which, of course, was not removable. The curves show well the improvement due to the iron case, but they also show that the screening is far from perfect, and not nearly as good as that shown in the previous figure.

With the disturbing field acting across the axis of the coil (corresponding to a bus bar running vertically behind or in front of the instrument) the effect in simple repulsion instruments is much less, seldom reaching 15% in the strongest fields for unshielded instruments. But curious anomalies can occur in a cross field of

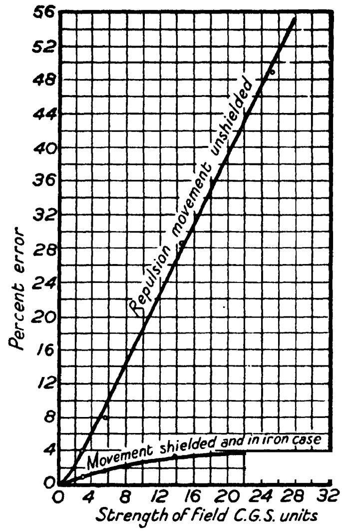


FIG. 7·25.—Curves showing effect of stray field on unshielded and shielded instrument.

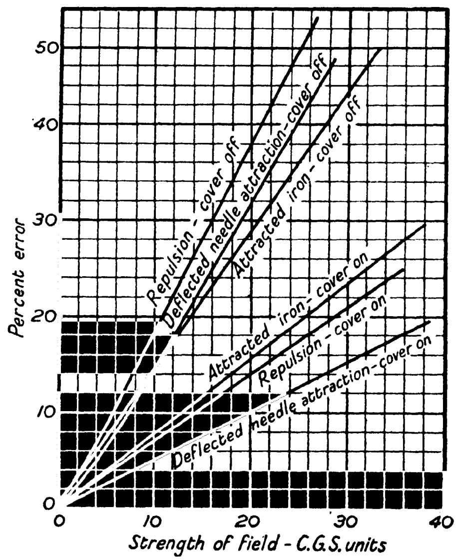


FIG. 7·26.—Stray field error for different types of soft iron instruments.

this kind dependent on the form and position of the irons. In the Keiser & Schmidt instruments, for instance, it was found that the error was greater with a disturbing field in this direction, and the effect is opposite to that produced by a similar coincident field. Thus if the instrument reads high in a given coincident field, and it is then turned through a right angle in the field, it will read low by a larger amount.

With instruments of the deflected needle attraction type, in which a thin plate of iron is drawn into a narrow coil, the possibilities of error are greater, and the effect of a disturbing field in the three directions is shown in Fig. 7.27 for two similar instruments by different manufacturers, both in iron cases.

As we should expect, the least effect is produced by a field passing through the plane of the thin needle—that is, from back to front of the case. But again, we should expect that when the field is along the plane of the needle in either direction (up and down or from side to side) the magnetic axis in the iron will be affected, and the curves show that in these two directions the error becomes quite marked.

With alternating fields the errors are practically the same as for steady fields, providing the disturbing field has the same frequency and is cophasal with the field in the instrument bobbin, but any departure from strict phase relation between the two fields will produce marked changes in the error.

This is shown in Fig. 7.28, which are curves for an instrument in a constant field coinciding with the axis of its coil. The instrument was supplied from the secondary of a phase-shifting transformer, so that the phase of the current in its coil could be shifted from 90° lagging to 90° leading. The curves show that the error rises to a maximum as the fields coincide in phase, and eventually falls to a minimum when they are in antiphase, at full-scale reading.

The employment of iron shields must, however, be carefully considered, for although they may effectively screen the movement from stray fields, they may lead to considerable increase in the hysteresis error if the return path of the lines of force of the instrument coil are practically entirely in the iron shield. In this respect instruments of the deflected needle attraction type have some advantage, since the axis of rotation is at right angles to the axis of the coil, which therefore lies flat upon its base plate, and only a very small part of the return part is in the iron case. Siemens

screen the movement by enclosing the coil in an open-ended box of thin sheet iron, and for such an instrument Edgcumbe and Punga find the stray field error to be 0·5% at 57% load, falling to 0·2% at full load.

Under any circumstances an improvement is always effected by breaking the continuity of the iron case, either by mounting the coil on a non-magnetic back plate, or some similar method which aims at preventing the return flux of the coil from traversing a

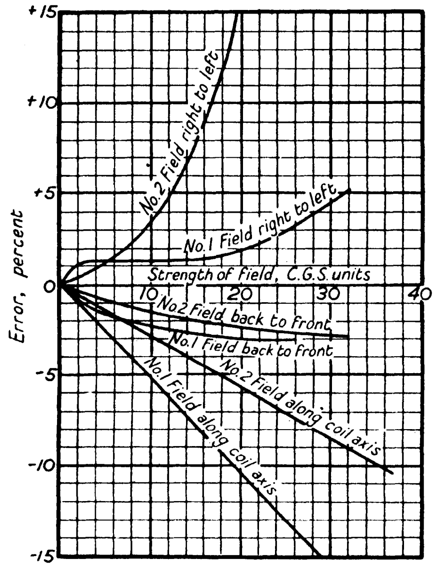


FIG. 7.27.—Effect of direction of stray field on soft iron instruments.

path entirely of iron. When the instruments are being installed on a switch-board, care should be taken that the leads to the instrument and the bus bars of the board lie as close together as is consistent with complete safety, so that their fields may be reduced to a practical minimum, since it would seem better to eliminate stray fields by this means than to protect the instrument from them by devices that may introduce a still more serious error.

(d) *Hysteresis Error.*—The effect of hysteresis in the iron is manifest in two ways : firstly, there is the lagging of the magnetism behind the current, and secondly, the movement of the position of the poles in the iron with the rotation of the system, or what is sometimes known as position error.

The first effect may be detected by calibrating the instrument first with ascending values of the current and then with gradually-

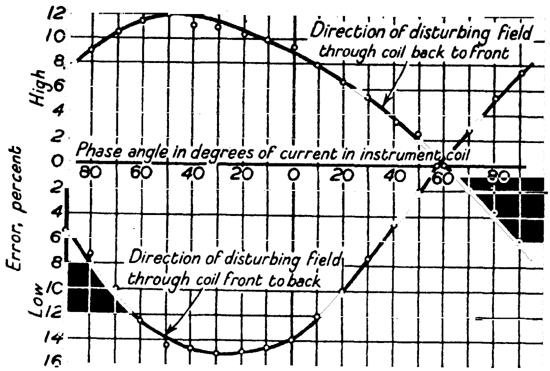


FIG. 7.28.—Variation of A.C. stray field error with phase of disturbing field.

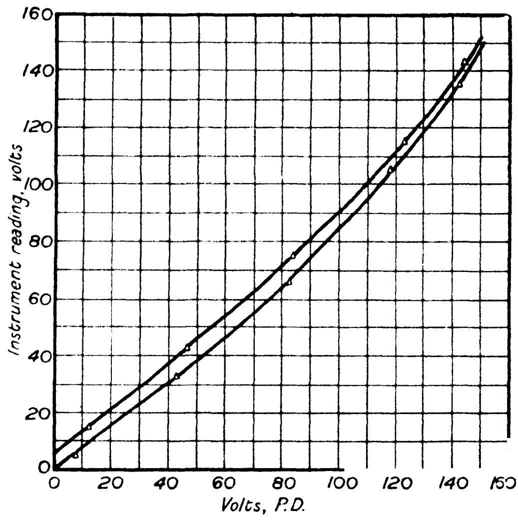


FIG. 7.29.—Hysteresis error of soft iron voltmeter.

descending values; the two resulting curves (see Fig. 7.29) will be displaced from one another by the amount of the hysteresis error, and the descending values will be above the ascending ones, owing to the retentivity of the iron. In commercial calibration

it is, therefore, usual to adjust the current for each reading to a predetermined value and then break and make the circuit before observing the deflection, so as to ensure a reading on the ascending curve. The errors tabulated in Table XXXIII were obtained in the following manner: The current was first brought to a value so that the instrument was indicating approximately half-scale value; the switch was then opened and closed and the reading carefully observed; the current was then raised to its maximum value and then carefully lowered again to its initial value and the new reading observed. The difference of these two readings expressed as a percentage gives the hysteresis error at that point of the scale.

The position error is, in most instruments, very small, except those of the deflected needle attraction type, where it assumes quite serious proportions, and may amount to as much as 3% at half-scale reading, the reason being that in the comparatively broad plate which forms the needle the magnetic axis can be readily shifted.

The position error may be detected in the following manner: The instrument is deflected to its half-scale position and the current is maintained steady at this value; the needle is now mechanically carried to the top and zero of its scale and brought gently back to its position of equilibrium successively, readings being taken both before and after gently tapping the instrument. The difference between the readings after tapping, expressed as a percentage, gives the position error, and the difference before tapping the sum of position and friction errors.

It is possible to reduce the hysteresis error very considerably by taking advantage of the demagnetizing effect of the ends of a very short iron, and by reducing the length of the path of the return flux of the coil in iron to the shortest possible distance. It is also obvious that the saturation in the iron will materially affect the error, either a very low or a very high saturation reducing it. The former usually implies a heavy iron with a consequent increase of friction and inertia, while with the latter errors due to changes of wave form become pronounced.

It is, however, possible to compensate the hysteresis error, and with careful design entirely eliminate it. Professor Perry first suggested the employment of a permanent magnet for the purpose, so arranged that it wiped out the remanent magnetism in the iron.

Another simple method was devised and employed by one of the writers for repulsion instruments of the Nalder type. In this case the spindle to which the moving iron was attached was made of hard steel, which was magnetized like the moving iron under the action of the current in the coil. On the current being reduced, however, the spindle retained its magnetization to a great extent, and as the return path for the flux, due to this magnetized spindle, was to a great extent through the moving iron, it had a demagnetizing effect on the latter, thus eliminating its residual magnetism ; and by carefully proportioning the parts it is possible even slightly to reverse the sign of the hysteresis error. This method is applicable to the majority of soft iron instruments.

Hitherto we have considered the sources of error affecting the instrument, whether working on alternating or direct current circuits. The errors which are essentially due to alternating current will next be considered :

(e) **Frequency Errors.**—These may be broadly grouped into (*a*) inductance errors, which affect voltmeters only, and (*b*) eddy current errors, which equally affect ammeters and voltmeters. The error arising from the self-induction of the coil may, in some cases, be serious. The total inductance of the instrument can be divided into two parts, i.e. that due to the coil and that due to the iron of the movement, and this latter may form a large proportion of the whole, as shown by the figures in Table XXXIII.

Like changes in resistance, any change in inductance cannot produce an error in the indications of an ammeter, since the result is simply a change in the impedance of the instrument demanding a similar change in the P.D. across the coil to get the required current through. With voltmeters, however, any change in impedance involves a variation of current through the instrument for a given P.D., and thus produces a variation in the readings of the instrument.

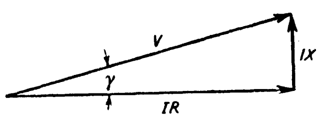


FIG. 7.30.—Vector diagram for electromagnetic voltmeter.

If, therefore, in Fig. 7.30 V is the P.D. at the terminals of the instrument whose resistance is R and inductance X, then for a

given frequency the P.D. will be the resultant of the ohmic drop IR and the inductive drop IX, so that we have

$$I = \frac{V}{\sqrt{R^2 + X^2}}.$$

If X is small compared with R, then we may write $I = \frac{V}{R}\left(1 - \frac{X^2}{2R^2}\right)$, and if θ is the angle between V and IR in the figure, we have

$$\tan \theta = \frac{X}{R} = \frac{L\,\omega}{R} = T\,\omega,$$

where T is the time constant of the circuit, and the error of the voltmeter reading is $\frac{1}{2}\,T^2\,\omega^2$ compared with unity.

Thus we see that the error depends on the time constant which is the ratio of the inductance to resistance, and the reduction of T involves, therefore, small values of L and large values of R.

Now in general both L and R will depend upon the dimensions of the bobbin, for the larger the area enclosed by each turn the greater will be the self-induction, while the length of wire will also be increased, and R will therefore increase, but whereas the resistance only increases in proportion to the length, the inductance increases in proportion to the square of the number of turns ; if, therefore, we can contract down the area enclosed by a turn and reduce the number of turns to a practical minimum, we shall reduce the frequency error. In this respect, as already pointed out, the narrow coil instruments of Siemens & Halske, Land und See Kabel Werke, Evershed and Everett Edgcumbe have the advantage, and the more so since the amount of iron in the coil is small ; the error from this cause, therefore, becomes negligible, since the variation of inductance with load remains practically constant.

Frequency Compensation

The compensation of the frequency error has been effected in several ways, of which the following are typical examples :

Method I: Shunted Condenser

Where the working coil has a relatively large resistance in series with it for swamping, this is a very effective method. In Fig. 7.31

let the coil A represent the working bobbin of the voltmeter, having resistance R_1 and inductance L. Let R_2 be the non-inductive series resistance of the instrument, so that the pressure to be

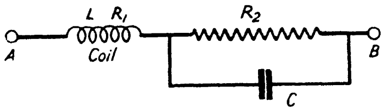


FIG. 7.31.—Frequency compensation by shunted condenser.

measured is applied between the points A, B. Finally, let C be a condenser shunting R_2. The admittance of the shunted condenser

$$Y = \frac{1}{R_2} + jC\,\omega = \frac{1 + jCR_2\,\omega}{R_2},$$

and the impedence

$$Z = \frac{1}{Y} = \frac{R_2}{1 + jCR_2\,\omega} = \frac{R_2(1 - jCR_2\,\omega)}{1 + C^2R_2^{\,2}\,\omega^2}$$

or

$$Z = \frac{R_2}{1 + C^2R_2^{\,2}\,\omega^2} - j\,\frac{CR_2^{\,2}}{1 + C^2R_2^{\,2}\,\omega^2}\,\omega$$

The impedance of the coil is, however, $R_1 + jL\,\omega$, and therefore the total impedance between A and B is

$$R_1 + \frac{R_2}{1 + C^2R_2^{\,2}\,\omega^2} + j\,\omega\left\{L - \frac{CR_2^{\,2}}{1 + C^2R_2^{\,2}\,\omega^2}\right\}$$

In practice $CR^2\omega$ is small compared with unity, and therefore the total impedance may be written

$$R_1 + R_2 + j\,\omega\,\{L - CR_2^{\,2}\}.$$

Hence, if $C = \dfrac{L}{R_2^{\,2}}$ we have compensation for all working frequencies.

As an example, let us consider a soft iron voltmeter taking 0·05 ampere at 120 volts, and having an inductance of 0·5 henry. The total resistance of the circuit will be 2,400 ohms, of which one-tenth, or 240 ohms, may be in the copper solenoid, and the remaining 2,160 ohms in the external resistance R_2.

Then $$C = \frac{L}{R_2^2} \text{ farads} = \frac{10^6 L}{R_2^2} \text{ microfarads}$$

or $$C = \frac{0{\cdot}5 \times 10^6}{2{,}160^2} = 0{\cdot}107 \text{ microfarads}$$

In practice the capacity would be somewhat greater for complete compensation, owing to the demagnetizing effect of eddy currents (see later).

Method II: Condenser on Auxiliary Coil

In cases where there is no resistance in series with the working coil, or when this resistance is insufficient to be effectively shunted by the condenser, a second winding is sometimes put on the coil and connected to the condenser, as in Fig. 7.32.

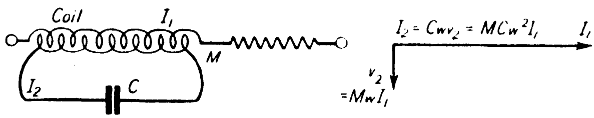


FIG. 7.32.—Frequency compensation by secondary winding and condenser.

The current in the main coil I induces an E.M.F. of $v_2 = M\,\omega I_1$ in the second coil lagging 90°, and this produces a leading current in the condenser $C\,\omega v_2 = MC\,\omega^2 I_1$, which is therefore nearly in phase with the main current, thereby reinforcing its effect as it becomes weakened by increase of impedance. This compensation being in phase is more suited for compensating the effect of eddy currents. It is not difficult to work out an expression for the induced current, but the result is of too complex a character to be suited for convenient numerical calculation, and it is best to determine the correct value of the condenser experimentally.

Method III: Inductive Shunt on Working Coil

The third method of frequency compensation is shown in Fig. 7.33, and consists of shunting the working coil with a choking coil of high inductance, so that when the frequency is low it shunts a small proportion of the working current, while as it rises the shunting effect of the choking coil becomes diminished.

If R_1 and L_1 are the resistance and inductance of the working coil, $X_1 = L_1\,\omega$ its reactance, and I_1 the working current, and

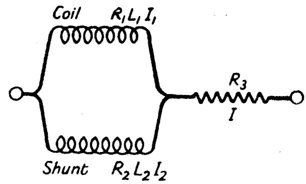


FIG. 7.33.—Frequency compensation by inductive shunt.

R_2, L_2, X_2, and I_2 are the corresponding quantities for the inductive shunt, then we have:
Admittance of working coil

$$Y_1 = \frac{1}{R_1 + jX_1}$$

Admittance of inductive shunt

$$Y_2 = \frac{1}{R_2 + jX_2}$$

Total admittance of coil and shunt in parallel:

$$Y = Y_1 + Y_2 = \frac{R_1 + R_2 + j(X_1 + X_2)}{(R_1 + jX_1)(R_2 + jX_2)}.$$

Hence the total impedance of the whole arrangement:

$$Z = \frac{1}{Y} + R_3 = \frac{(R_1 + jX_1)\,(R_2 + jX_2)}{R_1 + R_2 + j\,(X_1 + X_2)} + R_3$$

Total current for a given P.D. $= V$ is:

$$I = \frac{V}{Z} = \frac{V}{\dfrac{(R_1 + jX_1)\,(R_2 + jX_2)}{R_1 + R_2 + j\,(X_1 + X_2)} + R_3}$$

and the working current:

$$I_1 = \frac{Y_1}{Y} I = \frac{R_2 + jX_2}{R_1 + R_2 + j\,(X_1 + X_2)} I$$

$$= \frac{V}{R_1 + jX_1 + \dfrac{R_3\,\{R_1 + R_2 + j\,(X_1 + X_2)\}}{R_2 + jX_2}}$$

The value of the denominator is

$$R_1 + j\,\omega L_1 + \frac{R_3\{R_1 + R_2 + j\,\omega\,(L_1 + L_2)\}}{R_2 + j\,\omega\,L_2}$$

which is equal to $R_1 + \frac{R_3 (R_1 + R_2)}{R_2}$ at zero frequency. So that for perfect compensation it should have this value at all frequencies. It is fairly obvious from the expression just obtained, however, that this cannot be the case ; but if the resistance R_2 is about ten times R_1 or more, a value of L_2 can be worked out which will give fairly close compensation over a moderate range of frequency. This means, however, that the shunt will generally have to be something like 2,000 ohms resistance for a 120-volt instrument, with an inductance of some henries, and is, therefore, likely to be very costly.

Method IV: By Auxiliary Transformer

The theory of this method is very simple. The coil of the voltmeter has the secondary of a small transformer connected in series with it, the primary of which is connected in series with a high non-inductive resistance to the terminals (Fig. 7.34). Then the

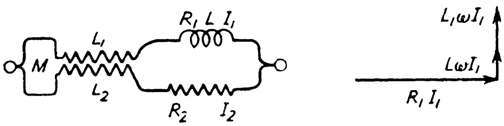


FIG. 7.34.—Frequency compensation by coupled transformer.

P.D. at the terminals is divided into the resistance component $R_1 I_1$ and the reactive component $(L + L_1)\,\omega I_1$ leading in quadrature. But if the resistance R_2 is considerable, compared with the reactance $L_2\,\omega_1$, the current I_2 will be nearly in phase with the P.D., and the induced E.M.F. in the primary will be $M\,\omega I_2$, which will also be approximately in quadrature with the P.D. Hence if $\frac{M\,\omega}{R_2} = \frac{L + L_1}{R_1}$, or $M = \frac{R_2}{R_1}(L + L_1)$, there will be approximate compensation over a large range of frequency.

This method has the disadvantage, however, either of wasting a considerable additional amount of power in the transformer circuit, or of needing a very high resistance in it. On the whole the condenser method of compensation first described appears to be the most correct in principle, most easily applied, and the least costly. It has been used for many years in this country by Messrs. Nalder Bros. & Thompson.

Most makers appear to have devoted their attention chiefly to minimizing the inductive error by reducing the coil inductance as much as possible.

(a) **Eddy-Current Error.**—The effect of massive metal parts in the working field of the coil will, under certain circumstances, lead to an error in the indications of the instrument, since these will be cut by the flux, and eddy currents will result, which will have a demagnetizing effect.

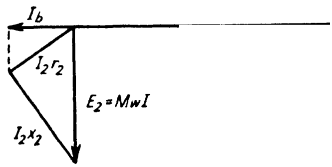


FIG. 7.35.—Vector diagram showing effect of eddy currents in soft iron instruments.

Thus in Fig. 7.35, if I represents the current vector, E_2 will be the induced eddy current E.M.F. whose magnitude is $M\,\omega I$, where M is the mutual induction between the coil and the eddy current circuit, and ω is 2π times the frequency. If R_2 is the resistance of the eddy current circuit and X_2 its reactance $= L_2\,\omega$, E_2 may be resolved into the components I_2R_2 and I_2X_2, where I_2 is the magnitude of the induced eddy current, and we have

$$I_2 = \frac{M\,\omega I}{\sqrt{R_2^2 + X_2^2}}$$

Then if I_b is the demagnetizing component of this induced current, we have

$$I_b = I_2 \sin\phi = \frac{M\,\omega I}{\sqrt{R_2^2 + X_2^2}} \cdot \frac{X_2}{\sqrt{R_2^2 + X_2^2}}$$
$$= \frac{ML_2\,\omega^2}{R_2^2 + L_2^2\,\omega^2} I$$

If we plot this error against the frequency we obtain the curve shown in Fig. 7.36, which rises first according to a parabolic law and then bends over and approximates to a maximum value of $K\dfrac{M}{L_2}$, where K is a factor less than 1, and is smaller the further the metal is from the movement.

It is therefore obvious that metal parts, particularly when these are good conductors, should be placed as far out of the field as possible, or, better still, eliminated entirely.

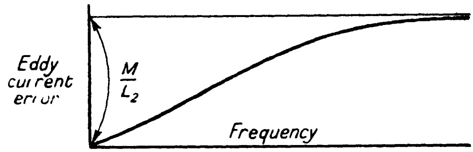


FIG. 7.36.—Curve showing variation of eddy current error with frequency.

With a view to reducing the error as much as possible some manufacturers split the coil bobbin longitudinally; when this is of metal, and where a metal dial plate is employed, this is also split in the neighbourhood of the coil field. But it should be pointed out that where the metal dial is actually screwed to the metal bobbin cheek these two splits should coincide, otherwise the effect of splitting is to a large extent nullified, and we have found several instances in which this is not the case.

On the whole it is better to eliminate the metal bobbin entirely, as is done in the Weston and B.T.H. instruments, where the coil is former wound and held in place by light metal clamps.

(b) **Wave Form Error.**—Moving iron instruments may be seriously affected by wave form, both on account of the change in form of the flux wave and, in the case of voltmeters, the effect of harmonics upon the inductance.

In Fig. 7.37 the curve A is the magnetization curve for the iron of the instrument, then for a sine wave of current such as that

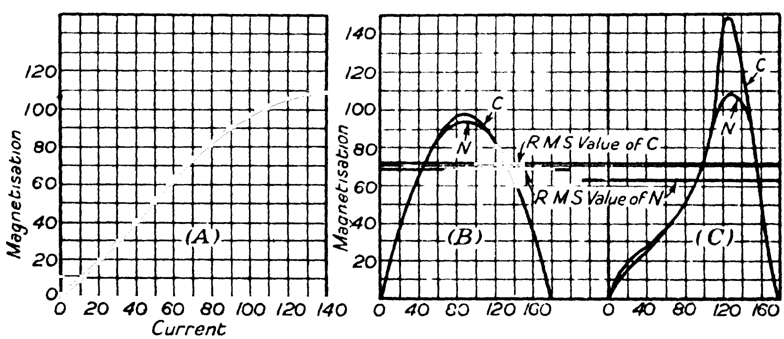


FIG. 7.37.—Curves to illustrate wave form error.

shown at B the flux will be represented by the inner curve (N), whose R.M.S. value nearly coincides with that of the current. The curves in C are for a peaky current wave of the same R.M.S. value as the current wave in B and the corresponding flux wave is also shown, and it will be seen that, owing to the flatness of the top of the magnetization curve, the flux wave shows a considerable departure from the current wave form, and the R.M.S. value of the flux is therefore considerably displaced. Thus an instrument with moderately saturated iron calibrated on a sine wave current, and used on a peaky current wave, will read considerably lower than the true value of the current. The converse is also true, and a flat top wave will cause the instrument to read too high. This shows the importance of keeping the induction in the iron sufficiently low so as not to reach the bend of the magnetization curve under any working condition.

The effect of the harmonics distorting the wave on the inductance may be treated as follows :

Let V_1, V_3, V_5, etc., be the magnitude of the harmonics, then if R is the ohmic resistance of the voltmeter whose inductance is L, and $\omega = 2\pi$ frequency, we have for the harmonic current in the circuit :

$$\frac{V_1}{\sqrt{R^2 + L^2\omega^2}}, \ \frac{V_3}{\sqrt{R^2 + 9L^2\omega^2}}, \ \frac{V_5}{\sqrt{R^2 + 25L^2\omega^2}}, \text{ etc.}$$

and if T is the time constant of the instrument, we may write :

$$\frac{1}{R} V_1(1 - \frac{1}{2} T^2\omega^2), \ V_3(1 - \frac{9}{2} T^2\omega^2), \ V_5(1, \frac{25}{2} T^2\omega^2), \text{ etc.}$$

and the effective value is

$$\frac{1}{R}\sqrt{V_1^2(1 - \frac{1}{2} T^2\omega^2)^2 + V_3^2(1 - \frac{9}{2} T^2\omega^2)^2 + V_5(1 - \frac{25}{2} T^2\omega^2)^2, \text{ etc.}}$$

$$= \frac{1}{R}\sqrt{V_1^2 + V_3^2 + V_5^2 + \text{etc.} - \frac{T^2\omega^2}{2}(V_1^2 + 9V_3^2 + 25V_5^2), \text{ etc.}}$$

or if V_e is the true effective value of the voltage, we may write :

$$V_e\left\{1 - \frac{1}{2} T^2\omega^2 \frac{V_1^2 + 9V_3^2 + 25V_5^2, \text{ etc.}}{V_e^2}\right\}$$

Then if V_e^1 is the voltage indicated by the instrument, we have

$$V_e^1 = V_e \left\{ 1 - \frac{1}{2} T^2 \omega^2 \frac{\Sigma n^2 V_n^2}{V_e^2} \right\}$$

That is, the presence of the higher harmonics on any considerable scale will increase the inductive error of the instrument and cause it to read too low.

Design of Soft Iron Instruments

Soft iron instruments do not, unfortunately, lend themselves to accurate design, chiefly owing to the impossibility of calculating the magnetization of irregular masses of iron, especially in non-uniform fields. But there are some theoretical considerations concerning special types of instruments which may advantageously be discussed, particularly as regards the law of deflection or form of scale, and the calculation of the necessary windings.

Simple Attraction Instrument

A very useful type of soft iron instrument is that in which a thin pivoted iron is attracted into a flat coil, as shown in Figs. 7.6 and 7.10. The chief advantage of the type is its low inductance, which makes it especially suitable for voltmeters.

In Fig. 7.38 is shown a simple oval disc of soft iron in a uniform vertical magnetic field of strength H (the field of the flat coil in

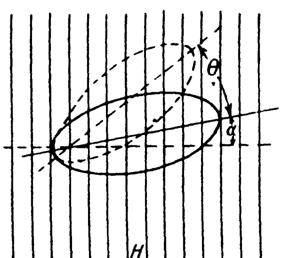


FIG. 7.38.—Design of simple attracted iron instrument.

the practical case is, of course, not quite uniform). Let this disc make an angle a with the horizontal when the pointer is at zero, and the angle $a + \theta$ when the pointer is deflected through an angle θ. Then, if the permeability is regarded as constant (which is not far from the truth in instruments of this class), the magnetization of

the iron will be proportional to $H \sin(\theta + a)$, and the torque T is proportional to $I^2 \sin(\theta + a) \cos(\theta + a)$, or to $I^2 \sin 2(\theta + a)$, which is zero when $(\theta + a) = O$, a maximum when $(\theta + a)$ is 45°, and zero again when $(\theta + a)$ is 90° or the axis of the disc is parallel to the lines of force.

If the control is gravity, the controlling torque can be written $w \sin \theta$, and hence, when the disc is at rest, $w \sin \theta \doteq kI^2 \sin 2(\theta + a)$, where k is a constant from which

$$\frac{\sin \theta}{\sin 2(\theta + a)} = \frac{k}{w} I^2 = k_1 I^2$$

or

$$I \propto \sqrt{\frac{\sin \theta}{\sin 2(\theta + a)}}$$

In Fig. 7.39 are given curves and forms of scales derived from this formula for various values of a, from which several useful points can be observed. In the first place it is obvious that the greatest angular range is obtained when $a = O$, which should give a possible range of 90°, and that every increase of the angle a shortens the scale by an equal amount. On the other hand, when we come to the dividing of the scale, it is obvious that each increase in the value of a renders the scale more uniform. For example, in the case of an ammeter with a maximum reading of about 2 amperes, it will not commence to read at all until the current reaches 0·7 ampere, and nearly the whole of the bottom half of the scale is taken up by the interval between 0·7 and 0·8 ampere. By setting over the iron 5° a great improvement in the lower part of the scale results, which would render it fairly suitable for a switchboard voltmeter; with a increased to 10° the scale becomes much more uniform, so that a readable indication can be obtained at about one-tenth of the maximum current, and the total angular range is about 70°. This would be fairly suitable for an ammeter; but on going beyond this the gain in uniformity is heavily counter-balanced by the reduction in scale length. With spring instead of gravity control the formula would be modified by substituting θ for $\sin \theta$, and would become

$$I \propto \sqrt{\frac{\theta}{\sin 2(\theta + a)}}$$

The effect would be to increase somewhat the openness of the scale at the bottom, and close it more at the top, but the general conclusions above remain unaltered.

When the iron is just outside the coil, as is commonly the case in such instruments, the concentration of the flux causes a force to be exerted on the iron, even when it is exactly transverse to the axis of the coil, so that the scale is continued down to low currents even in this case. But the effect of setting over the iron remains

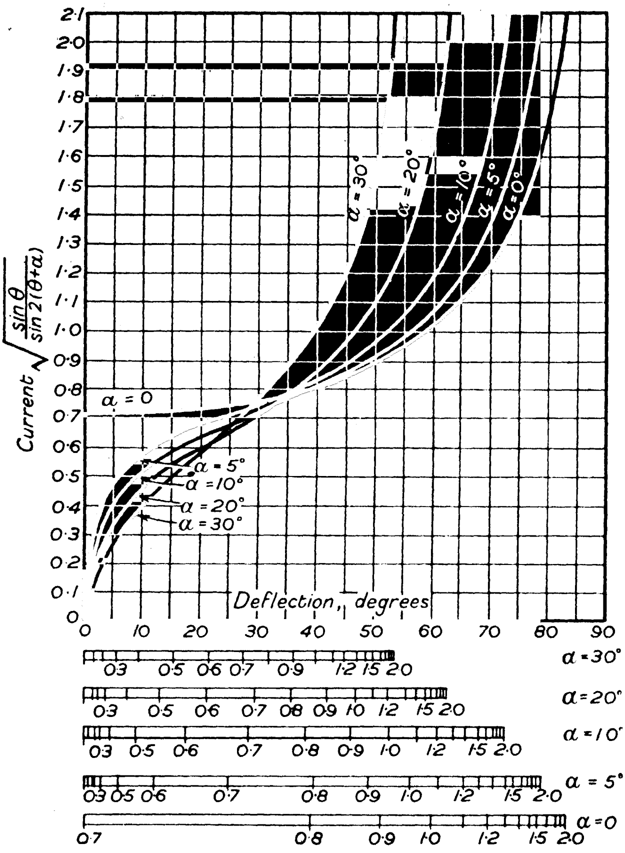


FIG. 7·39.—Theoretical curves and scales for attraction type soft iron instrument.

of the same character as before. It will be found interesting to compare the scales of actual instruments of the type, as in Fig. 7.40, with those deduced from theory.

Simple Repulsion Instrument

In Fig. 7.41 we have diagrammatically the most simple type of repulsion instrument, in which two parallel rods are magnetized by

the coil and repel one another. If we assume that the lateral distance between the iron rods is smaller than their length, this, combined with the obliquity of the action between diagonally opposite poles, makes the effect of the latter negligible, and we may treat the forces as simply existing between the adjacent poles.

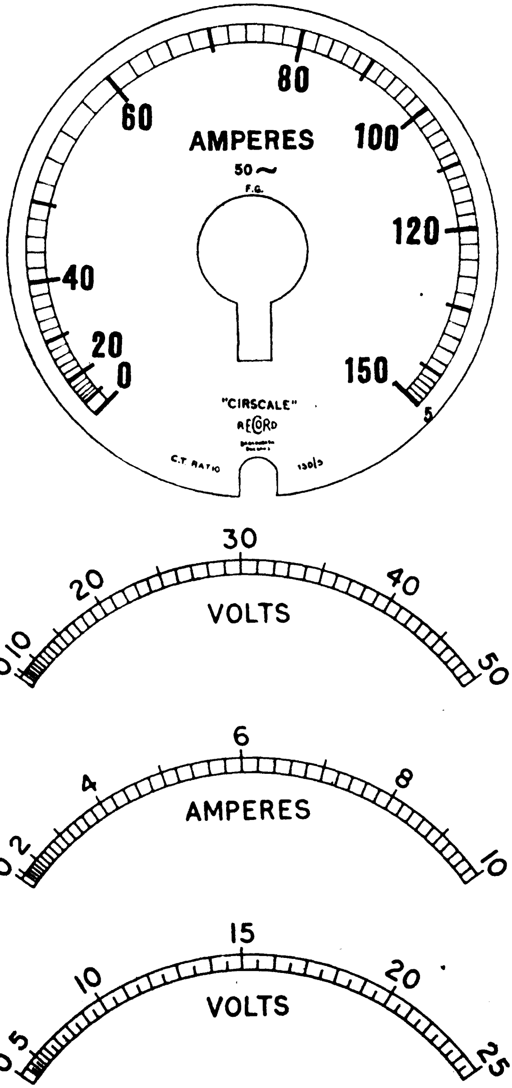


FIG. 7.40.—Typical example of the moving iron instrument scales.

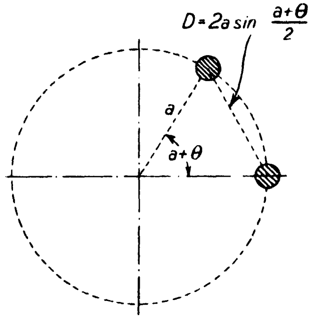


FIG. 7.41.—Diagram of simple type of repulsion instrument.

Hence we have the force $F = \frac{mm_1}{D^2}$, where m and m_1 are the strengths of the poles and D the distance between them, which may be written $2a\ (\sin a + \theta)/2$, and the torque

$$T = Fa \cos(\alpha + \theta)/2 = \frac{mm_1 \cos \frac{a + \theta}{2}}{2a \sin^2 \frac{a + \theta}{2}}$$

which is proportional to

$$I^2 \frac{\cos \frac{a + \theta}{2}}{\sin^2 \frac{a + \theta}{2}}$$

assuming the magnetization proportional to the current.

As before, with gravity control the restoring torque is $w \sin \theta$, from which

$$\sin \theta \propto I^2 \frac{\cos \frac{\theta + a}{2}}{\sin^2 \frac{\theta + a}{2}} \quad \text{or, } I \propto \sin \frac{\theta + a}{2} \sqrt{\frac{\sin \theta}{\cos \frac{\theta + a}{2}}}$$

Fig. 7.42 shows a series of calibration curves plotted from this formula, with different values of a. Of course in this case a cannot be zero, as the moving and repulsion irons cannot be in the same place at the same time with the thickness of iron generally employed. a is usually 20° or 30°.

It will be seen that the character of the scale differs from that of the simple attraction instrument in two important respects. Instead of the utmost theoretical limit of the scale being only $90° - a$, it is now $180° - a$, and it is much more even in its divisions, as is apparent from the much smaller curvature of the calibration curves compared with those in Fig. 7.39.

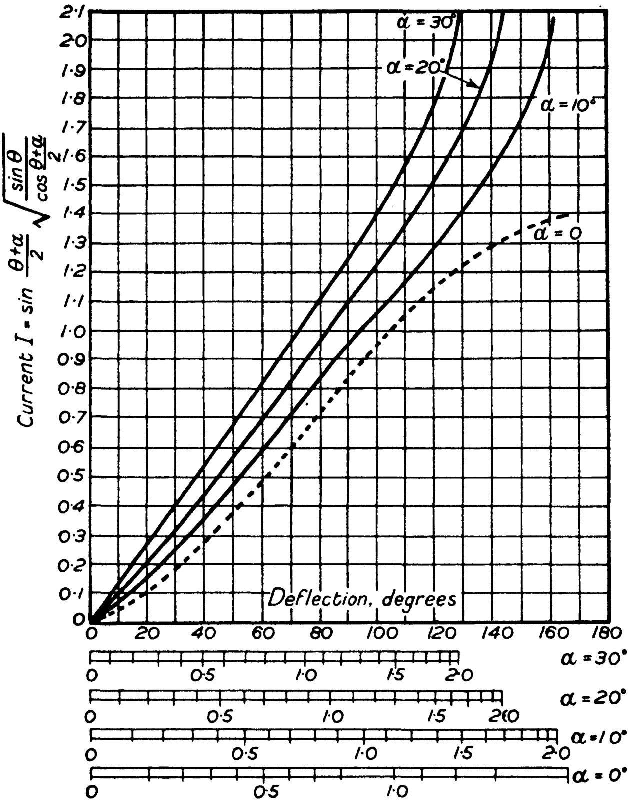


FIG. 7.42.—Theoretical curves and scales for repulsion soft iron instruments.

As a matter of fact, repulsion pattern instruments do not, in practice, have a greater scale length than 90°, but it should be possible to increase this by careful design. Their scales are also not as open at the bottom, as indicated by the formula, owing to the demagnetizing effect of the adjacent poles, but they are decidedly more open than in the attraction type.

Modification of the Scale in Repulsion Instruments

There are two ways of altering the form of the scale in repulsion instruments. One is that which was first introduced in the Evershed instrument (Fig. 7.18), and adopted by Weston (Fig. 7.20), and others, i.e. to make the repulsion iron the form of a tapered scroll so as to graduate the repulsion in any desired manner. A more simple and fairly effective device, however, was introduced by Messrs. Nalder as early as 1888, viz. that of setting the ordinary simple repulsion movement eccentric and out of the axis of the coil, so that the moving iron starts near the centre of the coil and gets nearer to the edges as the pointer moves towards the top of the scale. This combines the repulsion effect with the attraction effect of the Schuckert instrument and opens the scale at the top, making it more suitable for a voltmeter. In the Nalder instruments, therefore, the movements were set centrically for ammeters, and excentrically for voltmeters.

The general conclusion to be drawn from this comparison of the magnetic properties of the instruments is that the simple repulsion principle with cylindrical coil is about the best for ammeters, but that in voltmeters, where an open scale is required near the top of the range, combined with minimum inductance, the simple attraction type with narrow coil and needle, as employed by Siemens & Halske, Everett Edgcumbe, and Evershed, has the advantage.

It should be borne in mind that the great aim to be fulfilled in a voltmeter is to get the largest possible change of inductance with angle combined with as small a total inductance as possible.

Design of Winding.—For each type of instrument there is a certain value of the ampere turns required to give the normal working torque, and this must be approximately the same for the various ranges. In ammeters the actual values of the ampere turns range between 250 and 500, while for voltmeters they are somewhat less, in order to reduce the power loss and time constant.

There is no difficulty in designing satisfactory windings for ammeters of moderate range, as neither temperature coefficient nor inductance are of importance. For example, if we take 100 ampere ammeters with 300 ampere turns, this gives 3 turns, and if the current density in the coil is taken as 310 amps. per sq, cm., a section of 0·322 sq. cm. or $0{\cdot}635 \times 0{\cdot}509$ cm. will suffice. A

mean diameter of coil of 2·5 cm. is amply sufficient for most movements, so that the total length of conductor in the three turns will be about 25·0 cm. and its resistance about 150 micro-ohms, from which the power lost in heating the coil will be $150 \times 10^{-6} \times 100^2 = 1{\cdot}5$ watts, and this can be easily dissipated from the surface of the coil without undue heating.

For voltmeters, however, the problem is much more difficult. Not only is the space factor of the thin wire low, and the heat dissipation less, but as the temperature coefficient has to be kept low, an alloy of low temperature coefficient and high specific resistance must be employed, and therefore the power loss and heating are materially increased. As mentioned above, there are two ways of winding the bobbin, both of which are in general use. One is to wind the working coil with a low temperature coefficient wire, such as German silver, and the other alternative is to wind on the coil sufficient copper wire to give the requisite ampere turns and to swamp its temperature variation by an outside series resistance of negligible temperature coefficient.

As an example, consider the case of a voltmeter with maximum reading of 120 volts and requiring 250 ampere turns. If the maximum current is 0·05 ampere, the total power consumption is 6 watts—a fair average value. The total turns required will be $\frac{250}{0{\cdot}5} = 5{,}000$, and if we provisionally take the mean length of turn as 7·6 cm., the total length of wire will be 380 metres and the total resistance must be 2,400 ohms.

In the first place, wind the coil entirely of German silver wire, the resistance of which is about 17·5 times that of copper, with a temperature coefficient of about 0·035% per 1° C. Hence, 5,000 turns in the space available will necessitate a No. 33 S.W.G. wire 10 mils bare and 12 mils D.S.C., and this gives a coil of 3·8 cm. axial length, with an average external diameter of 3·8 cm. And this will have an exposed surface of 68 sq. cm., and on the basis of a temperature rise of 72° C. for 0·155 watt per sq. cm. we have for the coil 0·0833 watt per sq. cm., or a rise of temperature of 42° C. at full scale reading, and an increase of resistance of $42 \times 0{\cdot}035 = 1{\cdot}47\%$, a value usually found in instruments of this kind.

If now we examine the alternative method of winding the spool with the same gauge of copper wire, the resistance of the coil will be only $2{,}400/17{\cdot}5 = 137$ ohms, and the power absorbed by it will

be $6/17\cdot5 = 0\cdot343$ watts, and, on the same basis as before, the temperature rise is only $2\cdot5°$ C.

The remainder of the resistance will be $2{,}400 - 137 = 2{,}263$ ohms, say, of Constantan, Eureka, or some similar material of negligible temperature coefficient which can be wound and fixed at the back of the case as already described. The temperature coefficient of the whole instrument will then be $\frac{0\cdot4}{17\cdot5} = 0\cdot023\%$ per $1°$ C., and the error caused by current heating only $0\cdot023 \times 2\cdot5 = 0\cdot057\%$ approximately. In this case a higher error will be caused by variations of atmospheric temperature, which may be $10°$ either side of the normal, but this only produces a maximum error of $0\cdot023 \times 10$, or $0\cdot23\%$.

It is evident, therefore, that as good, cheap, alloy wires of low temperature coefficient are available, the advantage lies with the copper coil and external resistance, and this system is now adopted in the majority of modern soft iron voltmeters.

Table XXXIII gives the results of tests made on a number of soft iron voltmeters and ammeters by various makers.

CHAPTER 8

DYNAMOMETER AMMETERS, VOLTMETERS AND WATTMETERS

ALTHOUGH dynamometer instruments chiefly figure in the category of standards, there are a certain number of indicating instruments, notably wattmeters, which are on the dynamometer principle. It is not easy to make a deflectional dynamometer ammeter, as this involves introducing the current into the moving coil through heavy flexible conductors, or by means of mercury cups, both of which devices have obvious disadvantages. This does not, however, apply to voltmeters and wattmeters, in which the current in the moving coil need be only a few milliamperes, while even for ammeters the dynamometer principle may be employed, as in permanent magnet moving coil instruments, by connecting the moving element across a shunt, so that only a small portion of the current passes through the moving coil.

As will be seen, however, this introduces considerable difficulty, as the torque in dynamometer instruments is insufficient without a relatively high number of ampere turns, which make it difficult to get sufficient temperature "swamping" resistance without a high drop across the shunt. An equal or greater difficulty arises from the fact that the inductance of the moving coil produces a decided error in measuring alternating currents, unless the shunt can be made with a time constant equal to that of the coil—a condition which is not very easily fulfilled.

A general defect of the dynamometer type also is that as the torque is proportional to the square of the current the range of the instrument is very low unless special devices are provided for extending it.

Apart from these drawbacks, however, dynamometer instruments are of considerable value, as they are free from all the errors of hysteresis or change of magnetization, and give strictly correct readings with alternating currents, providing eddy currents are avoided by keeping masses of metal away from the neighbourhood of the coils.

The earliest commercial form of dynamometer instrument was the Siemens electro-dynamometer, which, however, was rather in

the nature of a standard, or at least a sub-standard, than an indicating or switchboard instrument, as its indications were read by means of a torsion head.

A diagrammatic illustration of this instrument is given in Fig. 1.11, Chapter 1, and Fig. 8.1 shows a form of Siemens instrument with a 2·5 ampere range. The connection to the swinging coil,

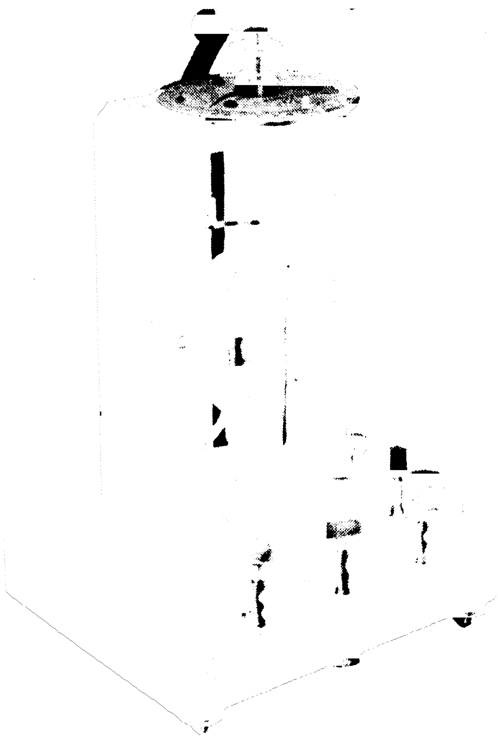


FIG. 8.1.—Siemens' dynamometer ammeter.
(*Photo by courtesy of the Director, Science Museum*).

which is only one turn in all sizes above 20 amperes, was made through mercury cups and in order to increase the range two fixed coils were employed in each instrument. Three terminals are mounted on the base, the centre one being connected to the bottom mercury cup, and the two outer ones to the ends of the two fixed coils respectively, so that when the leads are joined to the centre terminal and to one or other of the outer terminals,

either range of the instrument is employed. As the moving coil is always brought back to exactly the same position by the torsion head, the torque is rigidly proportional to the square of the current, so that if the spring is truly proportional the angle through which the torsion head is turned $D \propto I^2$, from which $I \propto \sqrt{D}$; or $I = K\sqrt{D}$, where K is what is termed the constant of the dynamometer. The useful range of current for each coil is only about 3 to 1, so that the range of the two coils is about 10 to 1.

The chief value of the Siemens dynamometer has been as a sub-standard for calibrating and checking both direct and alternating current ammeters. Since the torque at any instant is exactly proportional to the square of the current, the reading, with alternating currents, gives the R.M.S. or effective value of the current quite correctly, if there are no eddy currents in the instrument. If, therefore, the constant of the dynamometer is determined with continous current, taking care to eliminate the effect of the earth's magnetic field, this constant will give the true effective value when the instrument is employed to measure alternating currents.

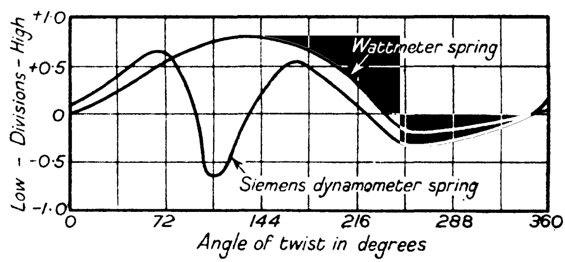


FIG. 8.2.—Curves showing spring errors.

In practice the accuracy of the principle of the instrument is affected in three ways: (*a*) by want of true proportionality in the spring; (*b*) by changes in the elasticity of the spring with temperature; (*c*) by eddy currents or change of distribution of current in the conductors for heavy current instruments. As to the first, investigations have shown that no spring gives a torsion strictly proportional to the angle of twist, and Fig. 8.2 shows typical curves of the relation between the deflection and the square of the current in a Siemens dynamometer, and the relation between the power and deflection for a torsional wattmeter. The form

of these curves and the departure from true proportionality depends greatly, however, upon the exactitude with which the spring is mounted.

As regards the change of elasticity of the spring with temperature, this produces a change of constant of about $-0{\cdot}15\%$ per 1° C., or half that of permanent magnet spring-controlled instruments.

The curve (Fig. 8.3) is for a dynamometer heated in an oven; the torsion head being set to a definite value and kept constant, the current was adjusted to balance at each temperature, and was measured on a separate external instrument; the coefficient from this curve is 0·148% per 1° C.

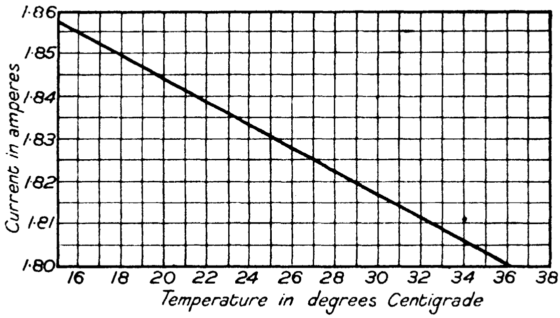


FIG. 8.3.—Temperature curve for dynamometer spring.

Eddy currents produce, of course, a similar effect in dynamometers to that which they do in soft iron instruments but, owing to the absence of iron, it is more easy to work out the law of variation of constant with frequency.

If the circuit of the dynamometer is diagrammatically illustrated by the upper loop in Fig. 8.4, and the path of the eddy currents in adjacent metal by the lower circuit, then we have the eddy currents

$$i_2 = \frac{-jM\omega}{(r_2 + jL_2\omega)}\, i_1 = \frac{-j(r_2 - jL_2\omega)M\omega i_1}{r_2^2 + L_2^2\omega^2}$$

or
$$i_2 = -\left\{\frac{ML_2\omega^2}{r_2^2 + L_2^2\omega^2}\, i_1 + j\,\frac{Mr_2\omega i_1}{r_2^2 + L_2^2\omega^2}\right\}$$

The total magnetic effect of the combination is proportional to $i_1 + Ki_2$, where K is, as a rule, small compared with unity if the metallic mass is outside the coil.

Thus the torque $\propto i_1 \times$ magnetic field :

$$\propto i_1^2\left\{1 - K\frac{ML_2\omega_2^2}{r_2^2 + L_2^2\omega^2} - jK\frac{r_2\omega M}{r_2^2 + L_2^2\omega^2}\right\}$$

The last term is a quadrature or cross-magnetizing current which does not affect the magnitude of the torque if small. The second term is, however, a demagnetizing one which is zero when ω is zero as for continuous currents, and becomes slowly larger, at first according to a square law, as the frequency rises. As $L_2\omega$ becomes

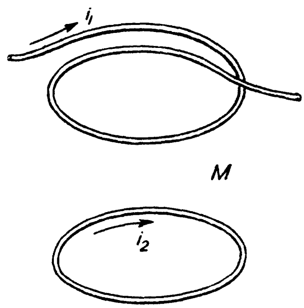


FIG. 8.4.—Effect of eddy currents in dynamometer.

equal to or greater than r, however, the increase of the demagnetizing term gets less rapid, and finally, when ω is very large, it attains the constant value $K\frac{M}{L_2}$.

The variation in the "constant" of the dynamometer is thus represented in character by the curve in Fig. 8.5. An experimental verification of this curve will be given later in connection with the Kelvin Balance.

The Siemens dynamometer was made in two grades, the more simple form having wooden, and the higher grade brass supports and a glass case (Fig. 8.1). The first form is, however, probably preferable owing to its freedom from unnecessary metal. Even so, in heavy current instruments, decided error is introduced with alternating currents of moderate frequency owing to the "skin effect" or tendency of the current to take the longest path round the outside of the large copper plate which forms the moving element.

An indicating switchboard ammeter of the dynamometer type was devised by G. D. A. Parr, and described by him in his book on

Electrical Engineering Measuring Instruments. In this instrument the fixed and moving coils are in the form of flat spirals. The passage of the current through the four coils causes the two moving ones to be repelled from the two fixed ones, only a small movement being possible, and this is communicated to the pointer by means of a lever to which is attached a fine strip passing round a pulley on the pointer shaft. The moving system is controlled by two oppositely-wound spiral springs, and the weight is reduced by making the moving element of aluminium. Current connection is established by terminating the coils with copper ends which dip into mercury cups constructed on the unspillable ink-bottle principle. A vane attached to the movement, and working in a nearly closed chamber, provides the necessary damping. The magnifying effect

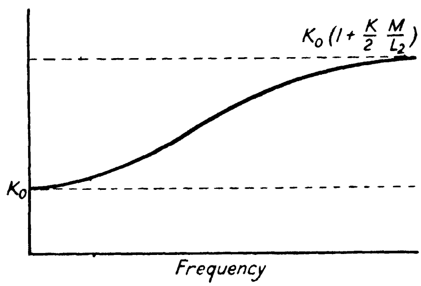


FIG. 8.5.—Curve showing variation of constant with frequency for a dynamometer due to eddy currents.

of the strip and pulley results in a long scale, the pointer moving over an arc of practically 300°, and Parr claimed that the instrument was independent of frequency and wave form. For heavy currents the instrument is shunted with a crimped metal shunt, with the professed object of giving it the same time constant as that of the instrument coils.

This is a not uncommon practice in shunted alternate current instruments, but it should be obvious that such a crimped strip would have, in nearly all cases, a much lower time constant than an inductively wound coil. The error due to the difference in the time constants will be dealt with later.

Most other dynamometer ammeters are constructionally identical with the corresponding voltmeters of the same design, and are restricted to the measurement of comparatively small currents, unless used in conjunction with series transformers or external

shunts, the only modifications being that in some instances the currents in the windings are increased and the resistance reduced by the use of thicker wire, or the fixed coils are wound with heavy gauge wire, and the moving element connected as a shunt across them instead of having all the coils in series, as is the case with the voltmeter. The description of the voltmeters which is given below, therefore, will suffice for both classes of instrument.

Dynamometer Voltmeters.

The dynamometer principle lends itself to the construction of the most accurate form of voltmeter for measuring alternating voltages of moderate magnitude and frequency—50 to 500 volts

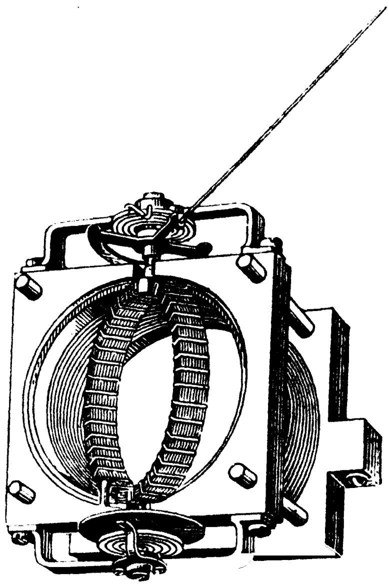


FIG. 8.6.—Early form of Weston dynamometer voltmeter.

at 50 or 60 cycles per sec. No difficulty is experienced in this case as regards leading the current into the moving coil, as its value does not, as a rule, exceed 70 or 80 milliamperes, which can be carried by the ordinary control springs or by fine ligaments.

The first direct-reading voltmeter on this principle appears to have been introduced by Dr. Weston in 1890, and is illustrated in

Fig. 8.6. The fixed and moving coils are of circular form, the former being wound on thin insulated bobbins which are split longitudinally to eliminate eddy currents in them. The coils thus form a solenoid, some 4·6 cm. long, enclosing the moving coil; this is of circular form, and is wound without metallic former to an external diameter of about 3 cm., and is maintained in shape by impregnating with a special hard-setting cement. All three coils, together with the necessary non-inductive external resistance, are placed in series between the terminals of the instrument. The current is conducted in and out of the swinging coil by two spiral springs of phosphor bronze, one above the coil and the other below, these serving also as the control. Balancing is obtained by means of the cross-arm device similar to that employed in permanent magnet instruments. Below the fixed coils is the metal damping-box into which the spindle passes. The two aluminium damping vanes, with upturned edges, closely fit the annular space in this box, which is closed by an external metal cover, the general arrangement of the damping chamber being shown in Fig. 2.29, page 73.

On account of the relatively large proportion of copper to swamping resistance, the temperature coefficient cannot be reduced to an entirely negligible quantity; for the most refined work, therefore, each instrument was fitted with a thermometer which was so bent that its flattened bulb lay over and near the fixed coils; the temperature could thus be ascertained at the time of reading, and the resistance of the instrument adjusted to its normal value by means of a little regulator switch provided.

As regards inductive error, we have $V = V_1\left(1 + \frac{T^2\omega^2}{2}\right)$, as with soft iron instruments (see page 400). In this instrument the inductance of the whole of the coils was about 70 millihenries, and its resistance 2,084 ohms for the 120-volt range, so that the error at 50 cycles was only 0·0056%, while at 300 cycles it only amounted to 0·2%.

In the Weston model 341 instruments the construction is very similar to that of the wattmeter described on page 451. The two metal supports for the fixed coils are erected on the double damping chamber, and the coils are former wound and clamped in position on them by means of a cross-bar, so as to embrace the moving coil, which is connected through two spiral springs to the double bridge-piece above. The whole of the coils are connected in series, and are

provided with the usual series non-inductive resistance, which brings the resistance of the instrument up to about 9·5 ohms per volt.

These instruments are extremely well shielded by placing the movements inside a cylinder of iron laminations, having an internal diameter of 9·4 cm. and a radial thickness of about 4 mm., over the ends of which a series of disc laminations are clamped. The whole is further enclosed in a second drawn steel cylinder which allows of an annular air space between the two shields of about 5 mm. Several other manufacturers have constructed instruments similar in form to the original Weston voltmeter, but in most cases the movement is heavier and much less perfectly damped. The Roller Smith Co. of America incline the plane of the moving coil to the axis of rotation, so that it may be secured to the shaft by two metal lugs projecting from the coil and held between pinching nuts on the spindle.

The General Electric Co. of America adapted the Thomson inclined-coil principle, described on page 373, replacing the iron disc by a moving coil with its axis inclined to the rotational axis, so that when the instrument is deflected the moving coil tends towards a position so as to embrace the grestest flux.

Messrs. Elliott Bros. constructed dynamometer instruments of quite another type, in which the moving coil is wound on a large circular metal former about 5·2 cm. in diameter. This coil entirely embraces the fixed coil, which is wound to an approximately spherical contour, and both the coils, together with a non-inductive resistance, are connected in series (Fig. 8.7). The movement is spring controlled in the usual way, and is damped magnetically by means of an aluminium disc below the moving coil, which rotates between the poles of four little brake magnets similar in shape to those employed for the same purpose in the Thomson Supply Meter. It would appear, however, from general principles that the moving coil should be enclosed by fixed coils if its inductance and inertia is reduced to a minimum, and it is significant that, of recent years, this firm have reverted to the ordinary form of dynamometer construction in which the moving coil is enclosed by two fixed coils for all instruments of the dynamometer type—ammeters, voltmeters, and wattmeters (see Fig. 8.28, page 459).

Despite the disadvantage mentioned above, Siemens & Halske, Hartmann & Braun, and several other instrument makers, both on the Continent and in America, constructed instruments with the

moving coil enclosing the stationary one; probably because such construction permits of the employment of standard forms of pivot and control spring mounting, adjustment is more easily effected, and a slightly greater torque is obtained.

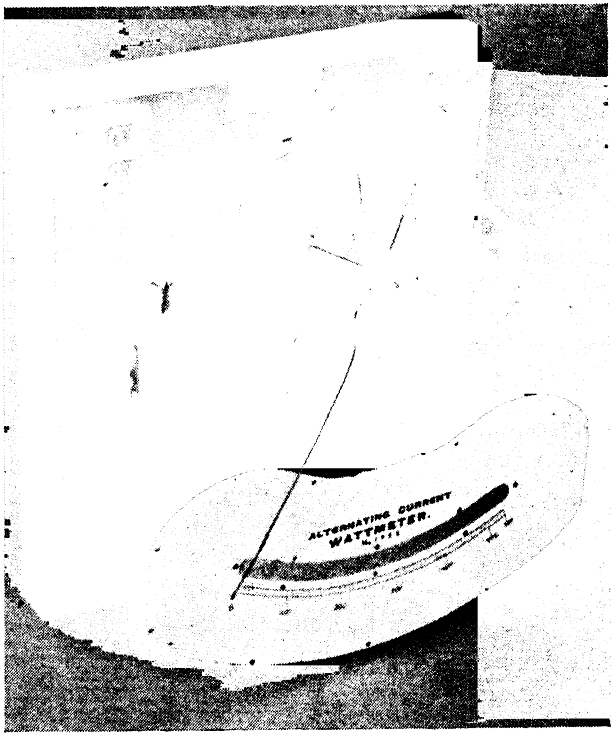


FIG. 8.7.—Dynamometer wattmeter (Elliott Brothers).
(*Photo by courtesy of the Director, Science Museum*).

In these instruments, however, a rectangular moving coil is employed which embraces the fixed coils fairly closely.

Fig. 8.8 shows diagrammatically the construction of the A.E.G. dynamometers. Here the coils are mounted in the centre of a block of iron laminations in such a way that these form a return path of low reluctance for the lines of force due to the coils, but there is no iron within the coils themselves. Enclosed by the fixed coils is the moving coil, mounted on a pivoted spindle between jewels

and spring controlled in the usual way. In this instrument, also, magnetic damping is employed ; the lower end of the axis carries two symmetrically placed, light aluminium vanes which move in the gaps of two permanent magnets mounted on the sides of the iron

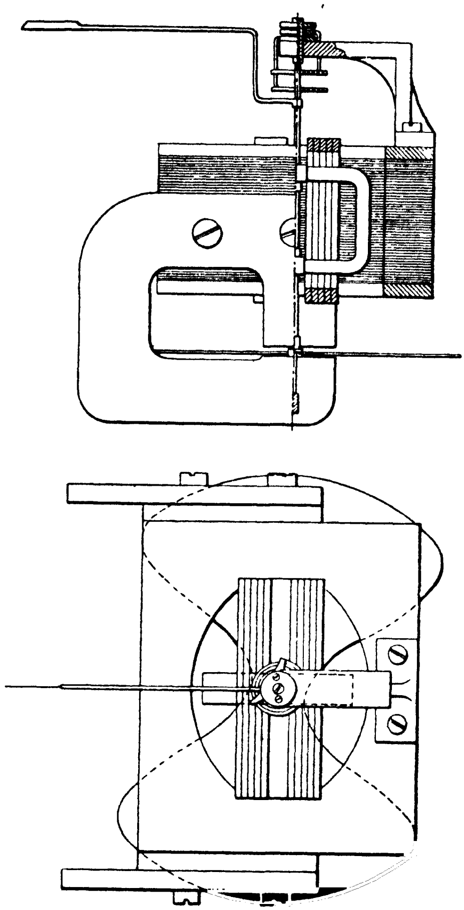


FIG. 8.8.—A.E.G. iron-clad dynamometer voltmeter.

laminations. A series resistance, some 15·4 times the copper resistance, is employed, the current for a 125-volt instrument being 0·56 ampere ; thus the temperature error is kept within fairly narrow limits. It is claimed that the iron laminations screen the instrument from the influence of external magnetic fields, but, on the other

hand, both hysteresis and frequency error are, to some extent, present, although owing to the low induction density in the iron these may not be serious, and the instrument is claimed to be serviceable on both direct and alternating circuits. The use of the iron, however, permits of the efficient application of magnetic damping, as it screens the working fields from disturbance by the permanent magnets, and similarly screens these from the weakening effects of the alternating fields due to the coils, and thus allows of great compactness in construction.

The scheme of employing a heavy laminated shield has also been employed by the General Electric Co. of America, who have constructed a portable dynamometer on similar lines to that of the Weston instrument. The fixed coils are former wound and held in position by being clamped in a light metal framework, which supports them just beneath the insulating cover. The circular moving coil is enclosed by them, but the spindle is extended above, and carries a metal sector moving between the poles of two permanent magnets. Extensions from the supporting frame carry the jewels, so that the whole moving system is self-contained. The circular laminated magnet shield is fastened below the insulating cover, and the movement is inserted from the top.

Hartmann & Braun have constructed a series of dynamometer instruments whose theory was very carefully worked out by T. Bruger.* The case of two flat coils, one of which is fixed while the other is movable about an axis parallel to one of its long sides, so that the combination resembles the covers of a book, has been carefully studied. He showed that if E is the potential energy when the coils are traversed by unit current and the movable coil is displaced by an angle θ, then, in order that the displacement shall be proportional to the current,

$$\frac{dE}{d\theta} = K/\theta, \text{ or } E = K \log \theta + k_1$$

where K and k_1 are constants. If this is plotted as a curve showing the relation between $\frac{dE}{d\theta}$ and θ, and the special cases of, firstly, parallel conductors displaced at right angles to their length, and secondly, inclined conductors rotating in the same plane about their point of intersection, are treated in the same fashion, it is

* E.T.Z. 25, Sept. 15, 1904, pp. 822–825.

found that the curve for $\frac{dE}{d\theta}$ and θ for the first case lies beneath the above curve, and that for the second case, above (Fig. 8.9). It is therefore obvious that by properly proportioning the vertical

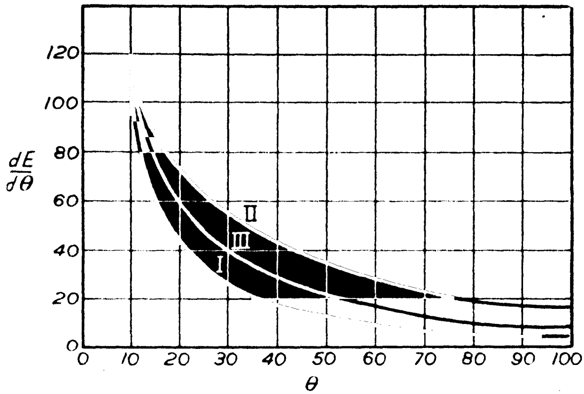


FIG. 8.9.—Curve of relative movement of two flat coils.

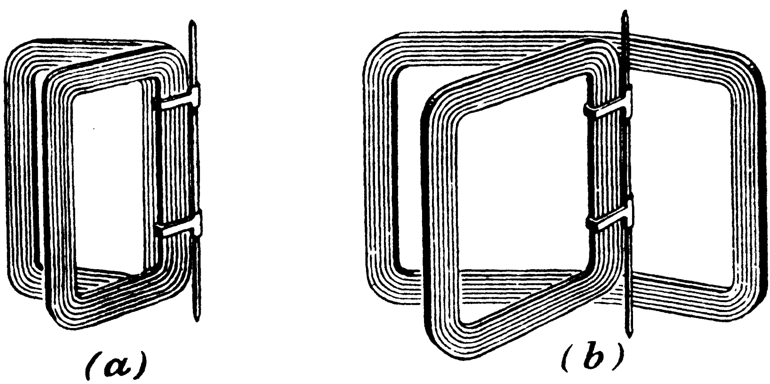


FIG. 8.10.—Arrangement of coils in Hartmann and Braun dynamometers.

and horizontal sides of the coils these two effects can be combined and a practically uniform scale will result.

Two forms of dynamometer based upon these principles have been constructed by Messrs. Hartmann & Braun. In the first, two flat rectangular coils, somewhat narrow in form, are arranged as in Fig. 8.10*a*, the moving coil being pivoted on an axis parallel to one of its long sides.

In the second type the fixed coil is made wider than the moving

coil, and is bent on a line about one-third of its width, so that its plane encloses an angle of about 135°, and the pivot axis of the moving element is about two-thirds that of the fixed coil (Fig. 8.10*b*). To increase the initial sensitiveness, these two coils are mounted within a larger fixed one in the precision types, and the systems are air-damped by means of vanes moving in a closed box below the coil systems. The resulting scale for these instruments is uniform down to 8% of the maximum reading.

Another modification of the same type of instrument consists of two flat circular coils, one of which is fixed with its plane vertical, but inclined at an angle to the axis of a large oval enclosing coil. The movable coil is similar in form to the fixed one, and is carried on a short arm from the centre of the axis of rotation, which is vertical. Thus, in the zero position, the two circular coils are face to face, and on sending current through them they repel one another, the enclosing coils serving to increase the initial sensitiveness, as mentioned above.

In the standard types the voltage drop at full load in ammeters is 0·5 volt, and in precision instruments this is reduced to 0·3 volt. If provided with a suitable series resistance they can be calibrated as voltmeters, the precision type so arranged requiring 0·18 ampere to produce full-scale deflection corresponding to 10 volts.

Another form of astatic dynamometer has been devised by Irwin, the construction of which is described on page 462.

The advantage of the type is that excellent astaticism is obtained with a movement consisting of two **D**-shaped coils so arranged that they move in the radial field of the two embracing field coils. The moment of inertia of the movement is therefore very little greater than an equivalent circular coil, while it is completely astatic.

The device does not, however, lend itself very well to a long-scale deflectional instrument, for to achieve this a wide separation of the fixed coils is necessary, with the result that the torque is very small and the scale crowded at each end.

In order to secure damping, the makers enclose the moving coil in two hemispheres of thin metal which are held between the fixed coil cheeks, and are therefore completely in the working field and become the seat of eddy currents which give rise to an appreciable frequency error. In the high range instruments the coil, together with a non-inductive resistance, are all in series, and the instrument requires a current of 0·03 amperes for full-scale deflection.

The lower range voltmeters are connected internally, like the ammeters described on page 432, that is, the fixed coil is joined in series with a non-inductive resistance, and these two are then shunted by the moving coil in series with a non-inductive and an inductive resistance, the range being finally adjusted by resistance outside the parallel combination. In these instruments the inductance of the moving coil branch is very much lower than that of the fixed coils, and hence, as we should suspect, the instrument has a serious frequency error.

Shunted Dynamometer Ammeters

The great convenience of the shunted moving-coil permanent magnet ammeter for direct currents has led to many attempts at applying the shunting principle to dynamometer instruments for alternate current measurement. As mentioned at the beginning of this chapter, however, this problem is by no means easy of solution, and although some instruments of this type have been made, it can hardly be said that many are entirely satisfactory.

A few words will show the nature of the difficulty. In the permanent magnet instrument the flux density in the gap is of the order of 1,500 gausses, while in the dynamometer instrument, where little, if any, iron is permissible, it is difficult to get a flux density of more than about 50 gausses. With a moving coil of the ordinary type used in a permanent magnet instrument, therefore, the torque would be only about one-thirtieth for the same ampere turns on the coil, and in order to bring it up to a reasonable value the turns on the coil, or the current passing through it, must be greatly increased.

This, combined with the fact that the two coils, the fixed and the movable one, are usually in series instead of having only one coil, as in the direct-current instrument, means that a much greater P.D. must be allowed across the shunt, and that very little swamping resistance can be allowed. For example, in the Weston dynamometer voltmeter, taking 50 milliamperes, about 6 volts are required for the copper coils alone without any series resistance at all.

Most of the instruments of this class are simply modifications of those already described under dynamometer voltmeters. Thus, in the A.E.G. ammeter the fixed and moving coils are arranged in a block of laminations just as they are in the voltmeter, except that the fixed coils are connected in series with the mains, and hence

must be wound with wire of sufficient cross-section to carry the full current. The moving coil is of fine wire, and this is joined across the terminals of a low resistance also in series with the mains (Fig. 8.11). In order that the instrument shall read accurately, this resistance should have the same time constant as the instrument itself, and to attain this approximately, the moving coil is shunted across both the main coils and the resistance, as in Fig. 8.12. The damping is arranged in the same way as in the voltmeter—that is, by means of a sector disc moving in the gaps of two permanent magnets.

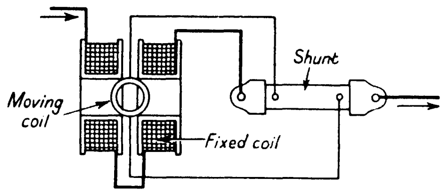


FIG. 8.11.—Arrangement of shunted dynamometer ammeter using non-inductive shunt.

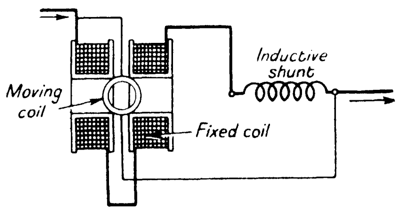


FIG. 8.12.—Arrangement of shunted dynamometer ammeter using inductive shunt.

As regards temperature variation, if the ratio of the " shunt " to the main coil resistance is the same as that of the series resistance to the moving coil, there will be no error when both are equally heated, but there will be if the main coils are made hotter by the passage of the current.

The early Elliott-Heap instrument was also similar to the voltmeter, having a 5·2 cm. moving coil embracing the fixed coil, which is wound with thick wire to a spherical form. The moving coil is also wound with much thicker wire than is the case in the voltmeter, and the metal former is omitted, otherwise the arangements of the two instruments are mechanically identical.

The fixed and moving coils are connected in parallel with one another, and are then joined directly to the terminals of a low-resistance or shunt in one of the mains.

The Irwin astatic instruments were similar in construction to the voltmeter and wattmeter of the same pattern. The moving coil consisted of two strip-wound **D**-shaped coils enclosed between circular mica cheeks, and was enclosed in a spherical enclosure of thin metal so that it serves as its own damper. The coil, in its damping box, is then slipped into position between the fixed coils, and the moving coil is connected as a shunt to one of these. As in the case of the voltmeter, the torque is very low, and, since the movement weighs more than 4 grammes, the friction error is troublesome.

Dynamometer Wattmeters

The most valuable application of the dynamometer principle, however, is to the construction of direct-reading wattmeters. As electrical supply is a supply of energy, it is of the utmost importance to have a means of accurately measuring the rate of flow of energy, or the power given to, or developed by, any electrical machine.

With continuous currents this presents no difficulty, as the power in watts is simply obtained by multiplying the readings of a voltmeter and ammeter. But with alternating currents this is only correct in the case of non-inductive loads where the P.D. and current are in phase with one another ; and in all other cases the mean power is the mean of the product of the number of volts and amperes taken from instant to instant. The wattmeter is therefore in principle a combined ammeter and voltmeter with some device for multiplying their indications continuously, and this is most easily secured by a dynamometer instrument having a fixed coil carrying the main current, and a moving coil in series with a large non-inductive resistance which is connected across the load. At any instant, therefore, the torque between the coils is proportional to the product of the currents in them, and if the current in the moving coil is rigidly proportional to the P.D., the deflection depends upon the average product of the P.D. and current, or upon the mean power. Such an instrument is therefore called a wattmeter, and was first introduced by Profs. Ayrton and Perry in 1881. The first commercial form of instrument was produced by Siemens in 1884, on the lines of his well-known electro-dynamometer, a fine wire-suspended

coil with a series non-inductive resistance being substituted for the thick wire moving coil with mercury contacts.

A large number of wattmeters have since been devised by the various makers, and the chief of them will be described below, but it is desirable first to consider the errors to which such instruments are liable, and the conditions necessary to secure accurate indications. This is the more necessary since for many years the theory of the wattmeter was very imperfectly comprehended, and the first theory given by the inventors was of such a vague character as to alarm both themselves and the electrical world in general, with the result that the dynamometer wattmeter was unduly hampered in its development. It is quite easy, however, to develop the theory of the wattmeter in a perfectly definite and easily comprehensible form, and thus to be able to design an instrument in which the errors are negligible.

Effect of Inductance in the Shunt Circuit

In Fig. 8.13 we have the case of a P. D. represented by the vector V and a current I lagging behind it by an angle ϕ. Then with sinusoidal supply the true power $W = IV \cos \phi$, or is obtained

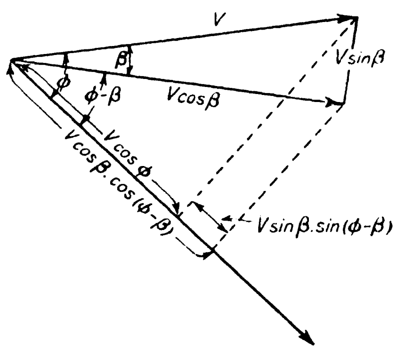


FIG. 8.13.—Vector diagram for dynamometer wattmeter.

by multiplying the current I by the projection $V \cos \phi$ of the P.D. vector upon it.

Now suppose we have a wattmeter which has a certain inductance in its shunt circuit so that the current in the shunt lags behind the P.D. by a small angle β. The effect is obviously the same as regards the instrument reading as if we had a similar instrument with no

inductance in its shunt, but a P.D. of $V \cos \beta$ and an angle of lag of the main current of $\phi - \beta$ instead of ϕ, the reading of the instrument W_1 should therefore be the current I multiplied by the projection of $V \cos \phi$ upon it, or $VI \cos \beta \cos (\phi - \beta)$. We therefore have—

$$\begin{aligned} &\text{True power} & W &= VI \cos \phi. \\ &\text{Wattmeter reading} & W_1 &= VI \cos \beta \cos (\phi - \beta). \end{aligned}$$

In the theory given by Profs. Ayrton and Perry a correction factor k was introduced such that the true power W was obtained by multiplying the wattmeter reading by k.
Hence

$$k = \frac{W}{W_1} = \frac{\cos \phi}{\cos \beta \cos (\phi - \beta)} = \frac{1 + \tan^2 \beta}{1 + \tan \phi \tan \beta}$$

If T is the time constant of the voltage circuit and $\omega = 2\pi \times$ frequency, $\tan \beta = T\omega$, and

$$W = kW_1 = \frac{1 + \tan^2 \beta}{1 + \tan \phi \tan \beta} W_1 = \frac{1 + T\omega^2}{1 + T\omega \tan \phi} W_1$$

From this it appears that $W = W_1$ or the instrument reads correctly only when β is zero or when $\tan \phi = \tan \beta$, i. e. the lag of the main current equals that of the shunt current. It is also evident that if $T\omega$ is small compared with unity the error will be negligible for any small value of ϕ, or the instrument will read fairly accurately at high-power factors. With low-power factors, however, for which ϕ is nearly 90°, the formula breaks down and becomes ambiguous, as tan 90° is infinite and rises extremely rapidly as this angle is approached. If the load is leading instead of lagging, as in testing condensers or unloaded cables, $\tan \phi$ is nearly $-\infty$, so that the denominator may actually be zero or negative and the correction factor may be infinite. On account of this ambiguity it was for a long time thought that the dynamometer wattmeter was unsuitable for tests at low-power factors, and Profs. Ayrton and Perry actually developed the electrostatic method in preference to it.

A slightly different treatment of the error, however, entirely removes this ambiguity. It is obvious from Fig. 8.19 that on inductive or lagging loads the wattmeter reads too high, as the projection $V \cos \beta \cos (\phi - \beta)$ is greater that $V \cos \phi$. Instead of a correction factor, therefore, let us write

$$W = W_1 - k_1$$

where k_1 is a correcting term which has simply to be subtracted from the wattmeter reading to get the true power.
Then

$$k_1 = W_1 - W = VI \cos \beta \cos (\phi - \beta) - VI \cos \phi,$$

which easily works out to

$$k_1 = VI \sin \beta \sin (\phi - \beta).$$

This can be instantly seen from Fig. 8.13, since the difference between the two projections $V \cos \beta \cos (\phi - \beta)$ and $V \cos \phi$ is evidently the projection of $V \sin \beta$ upon I, and since $V \sin \beta$ is perpendicular to $V \cos \beta$, its projection on I is $V \sin \beta \sin (\phi - \beta)$. We then have

$$W = W_1 - k_1 = W_1 - VI \sin \beta \sin (\phi - \beta).$$

Now in practice β is always small, so that $\sin \beta = \tan \beta = T\omega$ as above, and since the whole term k_1 is small, we may write $\sin \phi$ for $\sin (\phi - \beta)$, and we have

$$W = W_1 - T\omega VI \sin \phi = W_1 - T\omega \sqrt{(VI)^2 - W_1^2} \text{ approx.}$$

There is nothing in the least indeterminate about this at any power factor. In fact, the correction is actually most simple when the current is nearly in quadrature, as then $\sin \phi$ is nearly + or − unity, and $W = W_1 \pm T\omega VI$, the minus sign being for lagging and the plus for leading currents. This is again obvious from the diagram, as, if the current vector I is swung round to lag or lead by nearly 90°, it becomes nearly parallel to the vector $V \sin \beta$, so that the projection of it on the current is equal to $\pm V \sin \beta$, and the instrument reads high or low by the amount $VI \sin \beta = T\omega VI$. It reads, in fact, as if its zero were shifted forward by the amount $T\omega VI \sin \phi$. This simple formula not only allows us to get perfectly accurate results with a wattmeter of which the time constant is known, but it enables us to lay down the conditions necessary for its error to be so small as to be non-readable, i.e. that the amount $T\omega VI$ shall not be readable on the instrument scale. If, therefore, we have an instrument having, say, a scale of 100 divisions which we can estimate to 0·1 division, then VI corresponds to the maximum reading when the P.D. and current are in phase. Hence, $T\omega$ must not be more than $\frac{0 \cdot 1}{100} = 0{\cdot}001$, and if the frequency is 50 cycles per

second $\omega = 314$ and $T = \frac{L}{R} = 0{\cdot}001 \div 314$, from which $R = 314{,}000\, L$ or 314 ohms per millihenry of shunt inductance. If this resistance is attained or exceeded, as it can easily be in practice, there will be no readable error on any power factor with moderately sinusoidal supply.

Effect of Capacitance in the Shunt Circuit

In order to reduce the current in the shunt circuit, and also its time constant and temperature error, it is customary to connect a fairly high non-inductive low temperature coefficient resistance in series with the moving coil. Unless care is taken in the winding of this resistance an error of the opposite kind to that of inductance is introduced by the electrostatic capacitance existing between the adjacent wires of the resistance. If two insulated wires are laid side by side there is, of course, capacitance between them, and if they are joined together at one end to form a non-inductive loop this capacitance still exists, but is reduced to half its effective value, as when a P.D. is applied between the free ends the average P.D. between the wires along their whole length is half this value.

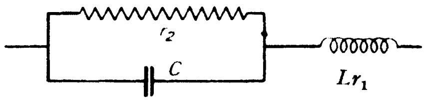


FIG. 8.14.—Equivalent circuit of moving coil.

For ordinary frequencies the arrangement of non-inductive resistance and moving coil is equivalent to Fig. 8.14, in which the resistance r_2 is shunted by the half capacitance C.

Then the admittance of the shunted condenser

$$= \frac{1}{r_2} + jC\omega = \frac{1 + jCr_2\omega}{r_2}$$

end the impedance of the whole combination

$$Z = r_1 + jL\omega + \frac{r_2}{1 + jCr_2\omega}$$

$$= r_1 + \frac{r_2}{1 + C^2 r_2^2 \omega^2} + j \left\{ L - \frac{Cr_2^2}{1 + C^2{}_2 r^2 \omega^2} \right\} \omega$$

and if $Cr_2 \omega$ is small compared with unity as it is in practice,

$$Z = r_1 + r_2 + j(L - Cr_2^2)\,\omega,$$

from which

$$\tan \beta = \frac{(L - Cr_2^2)\,\omega}{r_1 + r_2} = O, \text{ if } Cr_2^2 = L$$

in which case

$Z = r_1 + r_2$ simply.

The effect of capacitance in the resistance is therefore to compensate for, or even reverse, the effect of inductance, and when the whole of the non-inductive resistance is in the form of one coil the capacitance effect usually predominates.

The expression for tan β can be simplified in practice when r_2 is large in comparison with r_1, as we have

$$\tan \beta = \frac{L}{r_1 + r_2} - C\frac{r_2^2}{r_1 + r_2} = T_1 - T_2 \text{ approximately,}$$

where T_1 is the inductive time constant $\dfrac{L}{r_1 + r_2}$ and T_2 the capacity time constant Cr_2, and the wattmeter correction formula becomes

$$W = W_1 - T\,\omega VI \sin \phi = W_1 - (T_1 - T_2)\;\omega VI \sin \phi.$$

Anti-capacity Resistance

In order to avoid this source of error the added resistance should be so wound as to be of negligible capacity. The most simple method of doing this is to subdivide it into a number of bobbins. For example, if C is the capacity of the resistance when wound on one bobbin, dividing it into two bobbins reduces the equivalent capacity to $\frac{1}{4}\,C$, since the capacity of each bobbin is halved, and the P.D. across it is also halved. Hence, if the resistance is divided into n sections the capacity time constant is reduced to C/n^2, and is therefore easily rendered negligible.

Other useful anti-capacitance devices are also employed, such, for instance, as to wind the wire on mica strips (diagrammatically illustrated in Fig. 8.15), or again by weaving the resistance wire between asbestos or silk threads so as to form a kind of cloth; a ribbon of this material 190 mm. wide has an inductance of about

0·015 millihenry per thousand ohms. This method of producing resistances was first devised by Prof. Ayrton, and was later revived by Messrs. Duddell & Mather for use with their wattmeter. It is the most perfectly non-inductive and anti-capacity winding

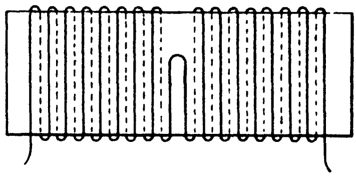


FIG. 8.15.—Anti-capacitance series resistance element for wattmeter pressure circuit.

possible, but the mica-supported coil, described above, is preferable mechanically, and is more than sufficiently accurate for any practical cases. It has been adopted by the Weston Company, who also employ a very satisfactory flexible varnish for coating the wound mica strips.

Eddy-current Errors

Another very important source of error in wattmeters is that due to eddy currents induced in any metal parts. We have already discussed the effect of these in connection with the Siemens dynamometers, but in that case we were only concerned with the demagnetizing action of the eddy currents, and not with their cross-magnetizing or out-of-phase component. In the wattmeter, unfortunately, this latter component is of considerable importance.

In Fig. 8.16, if I is the current in the main coils an E.M.F. V_2 lagging 90° behind it is induced in any adjacent metal, and this

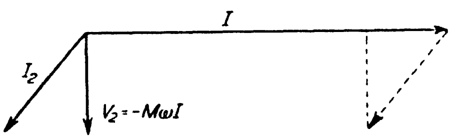


FIG. 8.16.—Vector diagram showing effect of eddy currents in dynamometer wattmeter.

produces a current I_2, which will lag somewhat behind V_2. The effect of this current is to reduce the field due to the current I, and to make it lag somewhat. A very slight eddy-current effect in

metal of high resistance will actually compensate for shunt inductance, as it can make the field, due to the main current, lag by an equal amount to that in the shunt.* But any amount beyond this produces an error which is similar to that of shunt inductance. Great care should therefore be taken to avoid metal cases or supports for the coils unless they are of specially chosen metal of high specific resistance or are laminated or otherwise arranged so that no appreciable eddy currents can be induced.

General Effect of Wattmeter Errors

Besides the above sources of error in dynamometers, there are others, such as those due to mutual inductance between the coils, and to irregular wave forms. But although these need consideration for work of the highest precision, it should be understood that none of them are at all serious in a well-designed instrument free from extraneous metal.

The errors of a wattmeter can best be determined by comparing it with a carefully-designed standard instrument at unity and at zero power factor and at two different frequencies. In the majority of cases the errors, though appreciable, are not large, and they amount to only 2- to 7-tenths of a division on the scale.

When the error is due to inductance only the reading is on the scale or positive at zero power factor, and the presence of appreciable eddy currents in the pressure coil circuit is indicated by a negative or behind-zero reading at zero power factor.

TYPES OF WATTMETERS

For purposes of description we may classify dynamometer wattmeters into two groups:

(*a*) Torsion instruments.

(*b*) Direct indicating or deflectional instruments.

Torsion Wattmeters

In the first group (*a*) the instruments are usually of the suspended-coil type, the moving coil being brought back to its initial position relatively to the fixed coil by means of a torsion head attached to

* This compensation if correct for one frequency will also be correct over a considerable range, as the induced eddy currents in high resistance metal are proportional to the frequency, as also is the lag due to shunt inductance. A small eddy current error can therefore be compensated by adding inductance to the shunt circuit, or by shunting the moving coil by a condenser.

the metallic suspension, or to a spiral spring like that of a Siemens dynamometer. The torsion head is provided with a scale of even divisions, and the angle of twist is read off when balance is obtained, and this, multiplied by the " constant " of the instrument, gives the power.

The earliest types of wattmeter were all constructed on this plan, and the absence of friction error, together with the device of bringing the moving system always to zero, render this type of instrument, when carefully constructed in other respects, an excellent secondary standard.

All designs are essentially similar, and differ only in details of construction. In fact, as before mentioned, the earliest torsion wattmeter was simply a modified Siemens dynamometer in which the swinging coil consisted of a light wooden or ebonite former on which was wound a large number of turns of fine wire ; only a portion of this winding was inductively wound ; the rest, being made non-inductive by double winding, current was conducted through it by terminating it at two copper pins which dipped into mercury cups in the usual way. The main current circuit through the fixed coil of the instrument was identical with that of the current dynamometer already described.

Swinburne modified the dynamometer wattmeter in such a way as to make it more portable and sensitive. In this instance the fixed main coil was wound in two sections, like the coils of a Kelvin galvanometer, and these were held in place by slipping them over brass pins projecting from thin upright metal supports, so that when they were secured in position connection between them was automatically made. The pressure circuit consisted of a fine wire coil, wound on a short mica tube and suspended by a fine metallic suspension, kept taut above and below the coil. The fixed coils entirely embraced this pressure coil, and the suspension served the purpose of leading the current through it, thus entirely eliminating mercury cups ; the movements of the system were observed by means of a long pointer moving over a short scale before a window in the lower part of the instrument case, and the torque was balanced by torsion applied to the top suspension, As at first constructed, this instrument showed very considerable eddy-current error, due to the presence of the metal supports of the main coils, and it has now been entirely superseded by the more modern forms described below.

Drysdale Wattmeter

This instrument was designed as a result of a study of the theory of the errors of the wattmeter as given above.* The conclusions arrived at from this study were: (*a*) that the shunt resistance should be at least 3,000 ohms per millihenry of inductance; (*b*) that its capacity should be negligible; (*c*) that eddy currents should

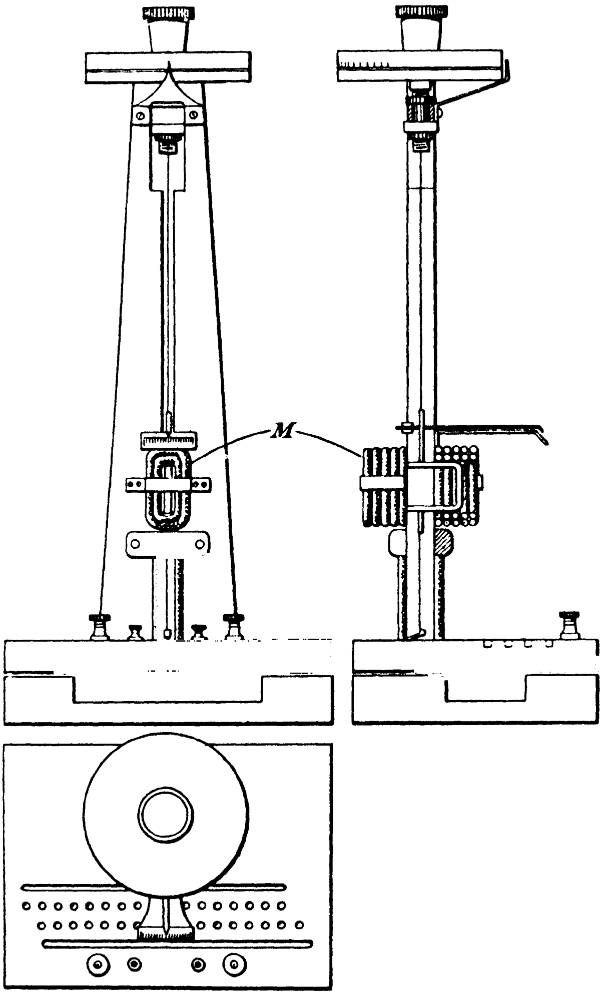


FIG. 8.17.—Early form of Drysdale wattmeter.

* See also *Electrician*, March 15, 1901.

be avoided by keeping all metal out of the field; and (*d*) that the main coils should be subdivided to avoid eddies and to allow of grouping the sections in series and parallel for extending the current range.

A wattmeter embodying these principles was also described, and is illustrated in Fig. 8.17. The main coils M were two in number, each of ten turns of eight-strand cable, the strands being very carefully twisted to get the most uniform magnetic effect. The 32 ends were brought down to two rows each of 16 mercury cups on the base of the instrument, arranged as shown, and the turns could therefore be grouped in five ways, viz. (1) all in series, (2) eight series, two parallel, (3) four series, four parallel, (4) two series, eight parallel, and (5) all parallel. The top range, therefore, varied from 5 to 80 amperes, and good readings could be obtained with 0·5 ampere.

The pressure coil consisted of 300 turns of 2·4-mil. wire, and had an inductance of 7 millihenries; it was suspended by a long, thin, circular wire from a large torsion head at the top of the instrument. The normal shunt current was only one milliampere, which meant a resistance of 100,000 ohms for 100 volts, and the time constant $T = \frac{L}{R}$ was thus only 7×10^{-8} sec.; the maximum error in the power factor was therefore $T\omega = 628 \times 7 \times 10^{-8} = 4{\cdot}5 \times 10^{-5}$, which is absolutely unreadable under any condition of load. As the whole of the supports were constructed of wood, and all other metal was excluded from the fields, eddy currents were entirely avoided.

This instrument was found of great service for dielectric testing and general tests of generators, motors, and transformers, and was subsequently re-designed in the double form for polyphase working.

The instrument as constructed by Messrs. H. Tinsley & Co. was further modified and improved by making the system astatic, as shown in Fig. 8.18, and by replacing the torsion wire by a cylindrical spring, which is more easy of adjustment, while the mercury contacts have been replaced by cylindrical commutators which effect the required changes of range by turning the cylinder. The construction of this commutator is shown in Part II, where a development of its contact surfaces is given, the dotted circles showing the positions of copper pins which connect diametrically opposite contacts through the ebonite cylinder. On each side of the cylinder are nine phosphor-bronze brushes, and when these make

contact along the lines 1/10 all the coils are in parallel, when at the lines 2/5 the coils are arranged in five groups of two in series, and so on, while the main terminals are connected to the two heavy end bearings.

Flexible ligaments connect the swinging coil, which is astatically wound on a mica support, to the shunt terminals. This coil has an inductance of 1·7 millihenries, and a resistance of about 50 ohms,

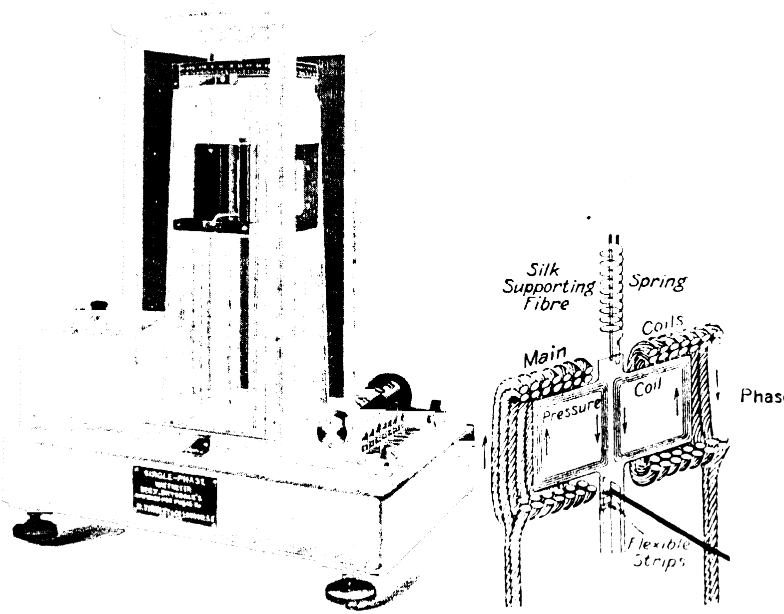


FIG. 8.18.—Drysdale astatic single phase wattmeter.

which is made up to 100 ohms by a coil under the base. A sub-divided resistance box with a total resistance of 100,000 ohms, made with coils wound on mica strips, serves for measurements up to 2,000 volts, the normal shunt current being 0·02 ampere.

Details of tests on the double form of this instrument have been published,* and show that the series parallel commutator is a very reliable method of changing the range. The general appearance is shown in Fig. 8.18. The stranded main coils are of narrow, oval form, and each is enclosed in an Ivoroid box and secured to the teak

* "The Double Dynamometer Wattmeter," *Electrician*, Jan. 14, 21 and 28, 1916.

uprights rising from the base of the instrument. The upper ends of these supports carry the framework on which the torsion head is mounted; this consists of a horizontal teak platform with a central hole; in the centre of this a little plate with an upright metal tube is carried on a thin vertical metal support, which passes down inside the spring to an Ivoroid bridge-piece on the main supports. The suspending fibre passes up through the tube and is secured at the top. This tube acts as a centering pin for the rotating head, which has a flat metal ring whose inner diameter is a little greater than the outer diameter of the spring, so that when it is in place the upper end of the spring can be adjusted concentrically with it. A cross-bar spans the ring and has a central hole through which the suspension tube passes, thus centering the head. A second similar cross-bar is then slipped over the tube, and registers with the one attached to the ring by engaging two screwed pins, which permit of their being clamped together after the upper end of the spring has been slipped between them.

This arrangement allows of an easy adjustment of the constant to an exact figure, for, after clamping the spring at approximately the right position, the constant is determined electrically. Now, since the standard length of spring has 50 turns, it is not difficult to calculate by what fraction of a turn the spring should be shortened or lengthened in order to make the constant correct. The spring is then unclamped, the head turned by the required amount, the spring drawn through and clamped again; usually a single setting is all that is required to bring the constant to an exact figure, to a high degree of accuracy. The lower end of the spring is soldered to an upright carried on the mica coil support at the correct radius from the axis, and by adopting these methods of mounting there is practically no deformation of its natural curvature, and hence the constant remains almost invariable at all positions of the torsion head.

The spring is constructed of round German silver wire, and is wound on a standard mandrel and annealed before removal.

The pressure coil is of rectangular form and is stitched to a reconstructed mica vane, half the turns being on each face, the coil itself being divided into two elements in which the current circulates in the opposite sense for the purpose of making the system astatic. The fine phosphor bronze leading-in ligaments are soldered to terminal tongues carried on the mica support and the main frame-

work of the instrument, and are so arranged that they exert practically no control.

The movements of the system are indicated by means of a long pointer attached to the vane, which moves over a short scale engraved on an Ivoroid bar on the front of the main supports. All metal is excluded from the sphere of action of the working fields, non-metallic screws being employed wherever possible.

The instrument is enclosed in a cover with four glass sides, through which the position of the pointer and divided head may be observed, while the torsion head is operated by means of a large teak disc mounted on the top of the cover. The mica vane on which the pressure coils are mounted acts as a fairly efficient air damper.

Duddell-Mather Wattmeter

In many respects this instrument is very similar to the Drysdale instrument. The main coils (Fig. 8.19) are stranded, the strands being grouped to form ten systems which can be connected in series

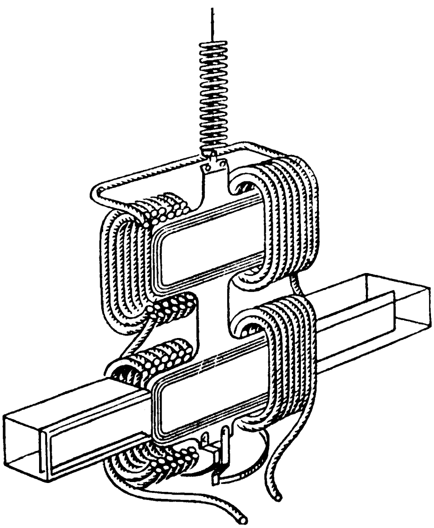


FIG. 8.19.—Principle of Duddell-Mather wattmeter.

and parallel by means of a separate plug commutator. The main leads are brought to the terminals on the bus bars of the box, and each element of the winding to the ten pairs of plug blocks by means of a stranded flexible cable. The moving coil is joined to the two

terminals on auxiliary blocks at the remote end of the bus bars, which permits of their connection to either one or other lead, and for convenient reversal.

The fixed coils are mounted on ivory pillars attached to the ebonite framework, which is supported from a marble base-plate on levelling screws. The whole of the working parts of the instrument are enclosed in a glass-sided case, above which the torsion head is mounted, and this latter is provided with a stop which prevents a rotation of more than 370°. The torsion spring is attached to this head and is screened from heat, arising from the main coils, by being enclosed in a separate little chamber, open to the atmosphere.

The moving system consists of two coils of fine wire mounted on the face of a mica vane, which has two prolongations at the level of the lower coil, and these project into two glass boxes attached to the fixed framework. These prolongations serve both as an index and as dampers to the moving system, for they fit the glass chambers fairly closely, and on the front of each box a zero line is engraved. The whole system is suspended by a silk thread which passes up axially through the spring and is attached to the torsion piece.

The pressure circuit is designed to carry a maximum current of 0·125 ampere, but for ordinary working on circuits of high power factor, the current is limited by means of external resistances of the woven type to 0·01 ampere, and is increased to 0·1 ampere for low-power factor working. The leading-in ligaments are of gold strip, attached to two little metal arms fastened below the vane, and so shaped that the points of attachment are on the axis of rotation.

The design excludes all metal from the instrument itself, and there are no terminals in the instrument case, all connections being effected on the plug commutator, and all leads entering the instrument are therefore properly stranded together so as to produce no external field. Unfortunately, however, the desire to exclude eddy currents has been carried to such a pitch as to make the instrument somewhat liable to distortion by heating after long use.

In a slightly modified form the instrument is constructed for direct measurements on circuits up to 75,000 volts. At these high pressures, both the instrument and its commutator are enclosed in the glass case, which is provided with a central dividing partition, allowing of the operation of the plugs without opening the portion

of the case reserved for the instrument; both parts of the instrument are mounted on corrugated porcelain insulators. The torsion head is operated by means of a long ebonite handle passing, at one end, through the side of the case, and at the other engaging with a pinion gearing into a wheel on the head of the instrument. For the highest pressures the instrument is further surrounded by a metal shield, provided with windows, through which the readings may be observed. This screen secures the instrument against electrostatic disturbances and obviates brush discharge.

The woven-wire pressure circuit resistances are mounted between porcelain insulators, supported beneath the cover of a ventilated containing-box for ordinary pressures, and for pressures above 15,000 volts an oil tank is substituted. The resistance and metal oil tank themselves form a capacity which, at 15,000 volts, 50 cycles, gives a capacity current of 0·0023 ampere, while the capacity current for a 30,000-volt resistance is 0·0035 ampere. For circuits above 250 volts the resistances are arranged so that a portion shunts the moving coil with the rest in series, and on low-power factor circuits a simple change of connections increases the current through the moving coil tenfold.

Northrup Wattmeter

In this instrument the two large diameter main coils are mounted in the case of the instrument according to the Helmholtz plan—that is, the distance between the mean planes of the coils is made nearly equal to their radius, this having the object of securing as strong and uniform a field as possible in the space where the moving coil swings. The coils are wound with stranded silk-covered wires in order to eliminate eddy currents in them, and the winding has a centre tapping so that 10 or 20 turns may be put in circuit.

The moving system consists of two light, circular coils, which together constitute an astatic system, and they are suspended by means of a long spiral spring, which not only provides the torsional control but also sustains the weight of the movement.

The pressure circuit current is 0·04 ampere, and the movement is damped by means of a mica vane dipping into an ebonite box containing thin oil.

The torsion head is mounted above the case, and consists of an ebonite disc whose position is read by means of a vernier. A light pointer moves over a short scale mounted on a pillar inside the case,

and its position is observed by means of a tubular eye-piece inserted through the top of the case. It is, however, doubtful if the use of large-diameter main coils is satisfactory, since, apart from the disadvantage of a non-concentric field, the drop in the main coil circuit becomes unduly great, whilst the employment of the spring to support the weight of the moving system is also contrary to general practice.

The Whitney Company of America have constructed a direct-reading torsion wattmeter in which the moving coil is pivoted and controlled by two spiral springs like those employed in ordinary indicating instruments. The pointer attached to the axis of the moving coil is arranged to move over a short scale, which is so divided as to read by deflection the residual power, whilst the torsion head, which passes through the glass cover of the instrument, has an indicator reading on a circular scale of watts. Thus it is claimed that it is possible to make a setting with the torsion head, and add or subtract the residual value indicated by the position of the pointer on the short scale.

Deflectional or Indicating Wattmeters

In this type of instrument we have, as before, a fixed main coil whose field interacts with that produced by the moving coil, which carries a current proportional to the P.D. at its terminals, but, unlike the torsional instruments, the relative position of fixed and moving coil is different for every load, so that, by attaching a pointer to the moving coil, its position, and therefore the power, is indicated on a suitably divided scale.

As, in switchboard instruments, it is often convenient to be able to estimate the reading of the instrument from a considerable distance, and moreover, the range is usually selected so that the upper part of the scale is most generally useful, it is usually considered desirable to have the greatest accuracy of reading at this part, and manufacturers have therefore aimed at providing a scale of uniform divisions or opening out at the upper end; but for the same percentage of accuracy at all points of the scale the divisions should be logarithmic and not uniform, or, in other words, the scale should be open at the lower readings, and close up at the upper values.

As in the torsion instruments, in most cases, the main coils are stationary and entirely embrace the moving coil, and when the

current does not exceed 200 amperes the whole current may be carried by this coil.

Hartmann & Braun also made a switchboard wattmeter in which the moving pressure coil is outside the fixed main coils, the general arrangements of the instrument being shown in Fig. 8.20.

In all cases where heavy currents are to be dealt with it is desirable to strand or laminate the conductors of the main coils in order to reduce the eddy-current error, but although the elimination of metal from the working fields of the instrument is as highly

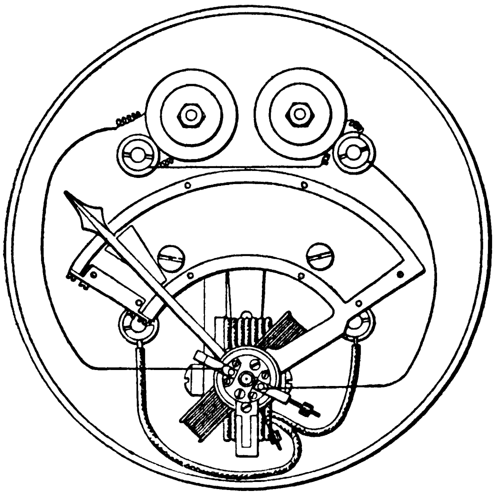


FIG. 8.20 —Hartmann and Braun indicating wattmeter.

desirable in this type of instrument as in the torsion instruments, it should not be carried too far in the case of purely commercial instruments, since the too free use of non-metallic and insulating substances for supporting heavy coils may prove very unsatisfactory, in view of the fact that all such materials have poor mechanical properties and tend to warp and twist under the repeated heat cycle, and thus throw the instrument out of calibration.

No better instance of the advantage of careful design can be found than that of the Weston Instrument Co.'s wattmeters of this type. The general construction of the switchboard instrument is shown in Fig. 8.21. Here the current coils are of rectangular copper, braided and former wound, the ends being brought out and shaped for attachment to the main terminals in such a way as to

produce practically no disturbing field. The supports are punched from a thin sheet of a special alloy, which has a high specific resistance and good mechanical qualities.

These supports are held in position on the base by means of projecting tenons which are first faced with strips of softer metal in order that they may be machined and fitted to the taper slots in the base intended for their reception ; by this means the

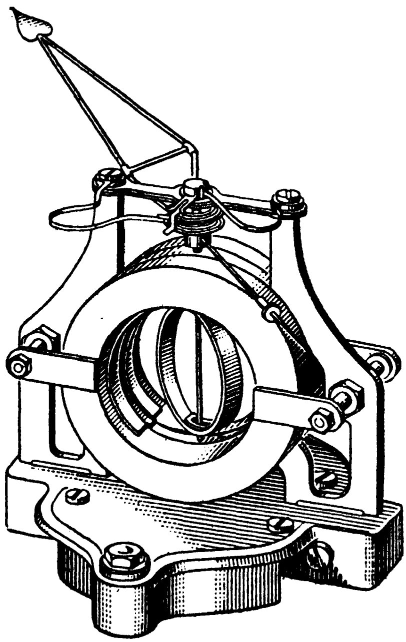


FIG. 8.21.—General arrangement of Weston indicating wattmeter

fitting does not in any way impair the mechanical strength of the support, which is finally held in position by a bolt and nut.

The coils are accurately centred by two arc-shaped flanges, which are recessed into the support only to a depth sufficient to prevent the face of the field coil from coming into actual contact with the support ; thus the outer faces of these flanges engage with the inner surfaces of the coils and keep them in true alignment.

The moving coil is wound to a circular form, the turns being spread at two diametrically opposite points to allow of the passage of the pivot spindle. After winding, the coil is impregnated

with a special cement and baked, which makes it astonishingly rigid, and entirely eliminates warping and consequent change of balance. The pivot axis is centred by means of two little carved aluminium plates cemented to the inner surface of the coil, which is then secured in position by two pins passing through holes in the spindle and engaging with the upstanding ears on the curved plates.

The lower end of the axis carries a light cross-arm with two symmetrical aluminium vanes, whose edges are turned up so that they form shallow trays; these move in two sector-shaped damping-boxes cast in the base, which are closed by two closely-fitting metal covers. Balancing is obtained by means of the cross-arm device, similar to that employed in the permanent-magnet moving-coil instruments.

Electrical connection to the pressure coil is made through the spiral control springs, which are carried by suitable fork terminals at the top of the spindle. The upper jewel bridge consists of two thin metal parts spanning the upper ends of the main coil supports; each of these parts is insulated, one from the other, and also from the supports. The jewel is threaded into a sleeve which is a part of the upper component of the bridge, and projects clear down through a similar sleeve in the lower plate. Each spring terminal tongue is centred on these sleeves against a flange turned in them, and is held tightly in position by means of a spring washer. The whole bridge is accurately centred by passing over two insulating flanged sleeves, which are in turn slipped over the upright pins of the main coil supports. The connecting leads are brought to two washers which also slip over these sleeves, but which are suitably insulated by means of washers, and connection between these leads and the spring tongues is made by means of S-shaped metallic strips, so that all adjustment of the movement can be made after completing the connection, and hence the electrical resistance of the circuit remains definite and unchanged.

The original standard wattmeters by the same makers were somewhat different in construction. The main coil, wound in two sections, was directly supported from the under side of the insulating cover. The moving coil is similar in construction to that described above, but the upper jewel bridge is fitted in a little recess in the cover, and spans an opening which is covered on the outside by an ebonite plate, so that when this latter is removed, the upper spring tongue terminal is accessible for zero adjustment. To the lower

end of the spindle are fixed the damping vanes, which are supported from an aluminium disc attached to the spindle, both disc and vanes being formed out of a single sheet of metal. The dampers move in annular chambers in a brass damping-box, which is supported from two brass pillars below the coils, and in the centre portion of the box is the lower jewel bridge and control spring, the box being closed by a metal cover. The series resistances for the pressure circuits of both types are of the mica plate, being wound with fine crimped strip on the mica sheets, which are then assembled in the form of a pack on four corner pins, each unit being separated from its neighbour by fibre washers. This resistance is fixed on the space below the dial plate of the instrument.

By reference to Fig. 8.22 it will be seen that there are two methods of connecting a wattmeter in circuit. In the scheme shown at (*a*)

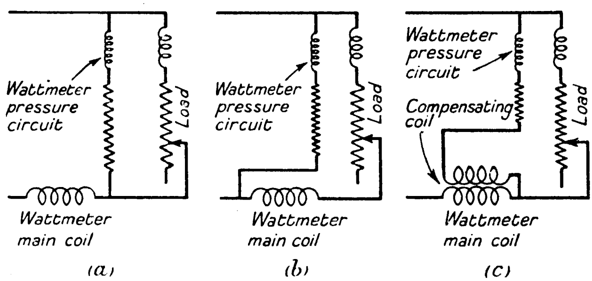


FIG. 8.22.—Alternative methods of connecting wattmeter to load.

the main coil has to carry not only the current of the load, but also that of the shunt coil circuit. We may, however, alternatively connect as in (*b*), but here, again, the indications include the power loss of the main coil. For instruments of high current range the error due to these causes is comparatively small and may be neglected, but with small-current instruments the error may be very appreciable. To compensate for this the Weston Company employ the device shown at (*c*) in all instruments below 3,000-watt range. In series with the pressure coil is a compensating coil consisting of a fine wire winding which is wound on with the main coil, and this produces a demagnetizing effect, in exact proportion to the shunt loss, on the main coil field, and thus automatically subtracts the power lost in the pressure circuit.

Some of the instruments of this type have also been fitted with a commutator for changing the current range, similar in principle

to that employed in the Drysdale Torsion Wattmeter already described. The main coil is then divided into four sections and brought to the brushes of the commutator, which is mounted on the side of the case and enclosed by a transparent cover. The barrel of the commutator is rotated by means of an external milled head, and the current range for each position is marked on this.

In the Weston portable standard wattmeters the movement is entirely enclosed in a double shield of special soft iron, which so perfectly screens it from external fields that when the instrument is brought into a field of 30 gausses in its most effective direction it does not produce an error of more than 2%, although this field, if applied to an unshielded instrument, would render it unreadable. The presence of this shield does not interfere with the performance of the instrument on low-power factors, for the reading of such an instrument, tested at full load zero power factor is less than one-tenth of a division from zero.

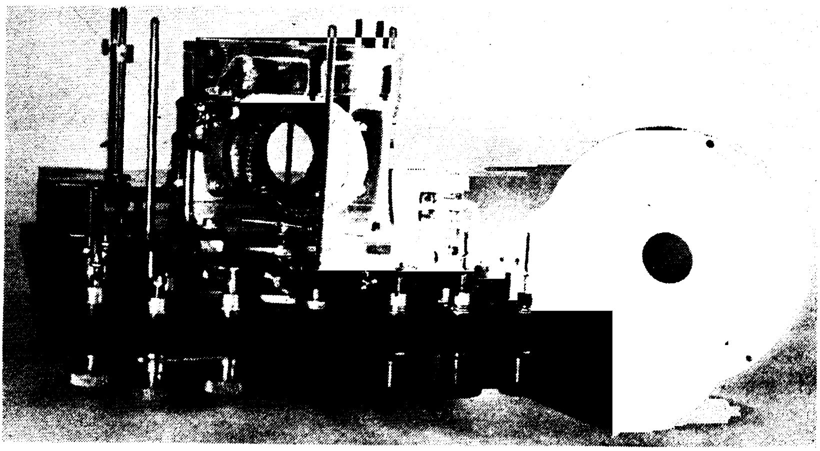


FIG. 8.23.—Form of construction of movement of a laboratory standard wattmeter (Elliott Brothers).

A form of construction employed by Elliot Bros. is illustrated in Fig. 8.23, which shows the movement of a laboratory standard wattmeter made by this firm. The damping in this instrument is by means of an arm vane in a damping chamber. This instrument, which has a scale 12 in. long, has the accuracy required by the

B.S.S., Clause 24. Fig. 8.24 shows a similar type of construction by Crompton Parkinson, Ltd., the damping chamber being just visible below the coils. The movement is screened by a rectangular case, open at the top.

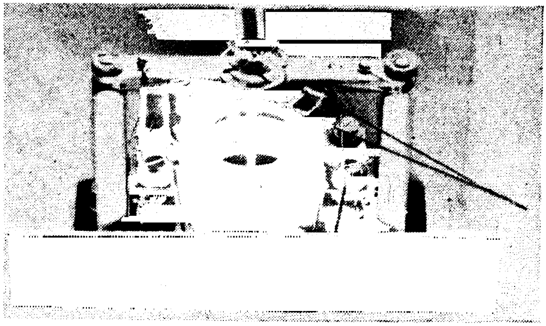


FIG. 8.24.—Construction of single phase wattmeter (Crompton).

Fig. 8.25 shows the details of a Siemens & Halske laboratory-pattern precision wattmeter; here the main coils are rectangular in form and the two sections are supported one on each side of a fibre partition, which is held vertical by slipping into two grooves in the thicker side brackets, which are also of fibre. Within these coils is the moving coil, which is of rectangular form, and encloses a comparatively much smaller area than the main coils; thus, when the planes of both fixed and moving coils are parallel, there is practically one centimetre clearance between them all round. The moving coil is very lightly constructed, and has no supporting former, and the pivots are attached to the inner horizontal sides, and bear in jewels inserted in a curved horizontal arm of fibre, which is arranged on a metal support in such a way that it centres with the main coils. The spring tongues and their attachments are mounted concentric with the jewels on the top and bottom faces of this arm, and are clamped tight by means of a notched washer. The zero set is obtained by means of a curved lever which moves the lower spring tongue when actuated by a screw from outside the case. The arm of the piston damper and the pointer attachment are supported from the upper pivot shaft, the cylinder of the damper being carried on the back of the jewel support.

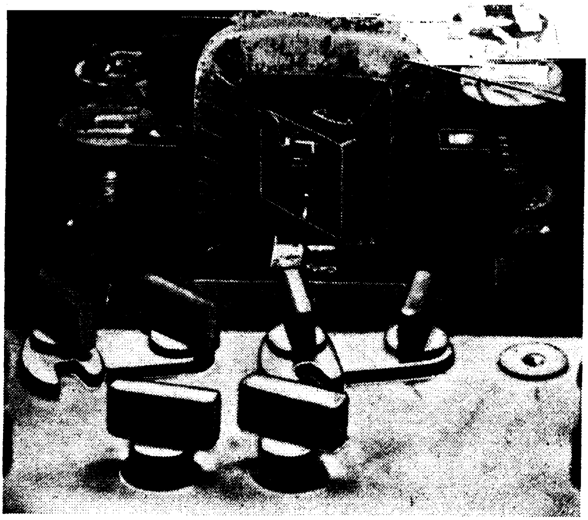


FIG. 8.25.—Laboratory pattern precision wattmeter (Siemens & Halske).

The pressure circuit of the Siemens instruments is adjusted to a fundamental resistance of 1,000 ohms and a current of 30 milliamperes by means of a shunt and series resistance, and the instruments are provided with a 1,000-ohm terminal which allows of their being used with external series resistances as well as the normal pressure ranges, connection to any of these points being made by a multiple-contact switch, as shown in Fig. 8.26. The use of a

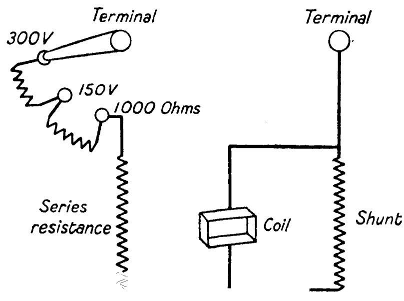


FIG. 8.26.—Pressure circuits of Siemens & Halske wattmeter.

shunt resistance adjustment in the pressure circuit is, however, open to serious criticism.

The switchboard precision instruments differ from the above in several respects. In these the main coil consists of several strips of copper connected in parallel (in a 200-ampere instrument eight are employed), and these are bent into a rectangular coil; at the top and bottom the strips are expanded, as shown in Fig. 8.27, so that the current has to divide into two semicircular paths which

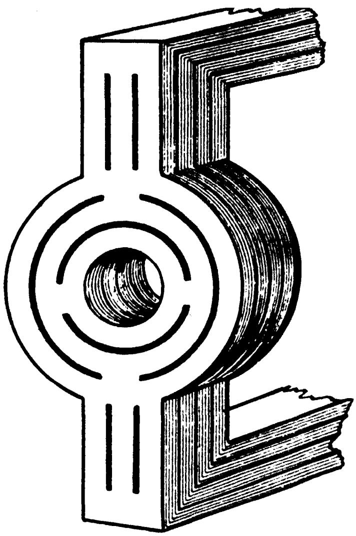


FIG. 8.27.—End of current coil of Siemens & Halske switchboard wattmeter.

have a radius approximately equal to that of the moving coil; at these parts the strips are provided with a series of narrow slits, as indicated in the figure, in order to restrict the path of the current in it, and so produce a radial field which links with the ends of the moving coil, and so contribute a torque which results in a uniformly divided scale. The pressure coil is rectangular, and is carried on a steel pivot shaft which passes independently through the two coil systems, and bears in jewels in a separate metal bracket piece, so that, on assembly, the shaft is slipped through and the moving coil mechanically attached by means of pinching screws.

From the centre of the spindle the curved arm of the piston damper is also attached by an independent collet and screw. The

control springs are mounted on the upper arm of the jewel bracket, above the main coil, and connection between them and the pressure coil is made by two small wires which rise stiffly from the coil, and are bent sharply at right angles inwards to the spring attachments. The series resistance in the pressure circuit is wound on a series of cylindrical bobbins, which surround the pillars which support the dial plate of the instrument.

In a portable wattmeter by Messrs. Reiniger, Gibbert & Schall a single-section rectangular main coil is employed. This consists of 10 turns of bare copper strip, wound with a pressboard separator and then silk-taped. The coil is of greater width than height, and is supported by the insertion of one of its long sides into a recess in the fibre base plate.

The moving coil is small and of rectangular form, and is pivoted to a fibre arm, like the Siemens laboratory instruments, this arm being built up to centre by fibre pieces attached to the base plate. The jewels are inserted in holes in this arm, and are held in position by transverse grub screws. Zero adjustment is provided by means of a forked arm, which operates the lower spring attachment when moved by an eccentric pin, which can be turned from outside the case.

The damping system is identical with that employed by the same makers in their moving-iron instruments, described on page 387, a little tray-form damping vane being carried on a prolongation of the pointer, and moving in a sector-shaped metal box mounted on the fibre coil support.

The A.E.G. switchboard dynamometer wattmeter has a simple and neat construction. The two sections of the main coil are wound in circular form and then taped and varnished ; they are then rigid enough to be self-supporting from the metal terminal lugs to which they are clamped.

The pressure coil is also circular and has no supporting former, and the pivot spindle passes diametrically through it. The ends of the coil are brought out at the front of the movement to the two control springs, whose attachments are concentric with the outer jewel, and are carried between insulating washers on the upper bracket. The outer spring attachment is provided with an upstanding ear, which is drilled and threaded to engage with a tangent screw operated from outside the instrument case. A spiral spring surrounds the tangent screw, one end of which presses against the

ear on the spring abutment and the other against the bearing in which the screw turns, thus keeping the whole arrangement tight and free from backlash, while the travel of the ear on the screw is limited to a small arc by means of a little milled nut. This form of zero setting is common to most of the A.E.G. spring-controlled instruments.

In an early form of Everett Edgcumbe switchboard wattmeter the main coils are arranged with their planes horizontal, the two coils being clamped to the upper and lower faces of an ebonite plate which is held in position by a bracket frame of German silver. The

FIG. 8.28.—Modern form of 4 in. indicating wattmeter (Everett Edgcumbe).

moving coil is wound in two grooves in an ebonite former, through which the pivot axis passes, and a single spiral control spring is employed, and this serves as one connection to the coil, the other end being attached to a flexible metallic ligament which exerts practically no control.

A curved piston damper is fixed on the front arm of the German silver framework, which also carries the jewel and spring attachment.

A modern form of Everett Edgcumbe wattmeter is illustrated in Fig. 8.28, which shows the partially-dismantled view of a 4-in. indicating wattmeter. The field coil is shown removed, and shows

the damping chamber and coil supports, which are in the form of a plastic moulding.

Switchboard indicating wattmeters are now quite common on switchboards and control gear, and many of these are of the dynamometer pattern. An example of this type of instrument is given in Fig. 8.29. This shows a front, horizontal view of the complete movement of an instrument made by the Metropolitan-Vickers Electrical Co., Ltd. A moulded damping chamber is used, very

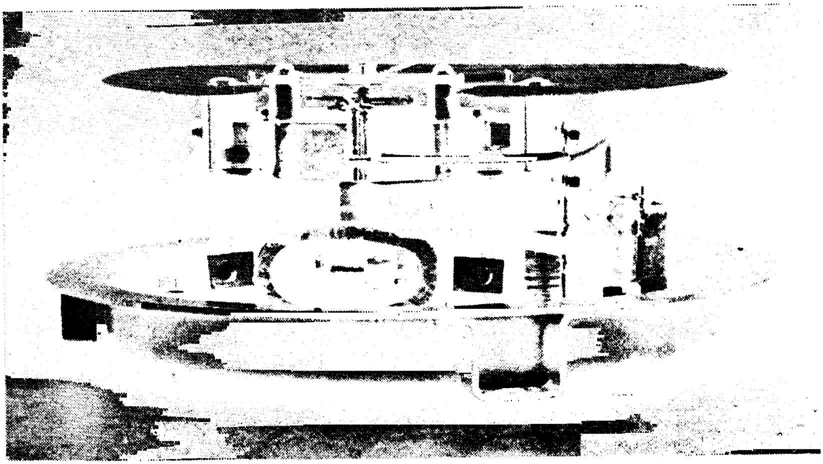


FIG. 8.29.—Switchboard dynamometer wattmeter (Metropolitan-Vickers).

similar to that employed in this firm's moving-iron instruments, except that the centre portion is removed to provide room for the coils. By using this form of support the amount of metal in close proximity to the coils is kept to a minimum. The coils are elongated in shape to reduce the overall height of the movement so as to fit in a standard switchboard case. The two field coils are fixed to the supports by small metal brackets which are taped in with the coil insulation. The complete movement from this instrument is shown in Fig. 8.30. Here again the damping vane, pointer fittings, etc., are of the same design as this firm's moving-iron instrument (Fig. 7.16), and are fixed in a similar manner. The moving coil is varnish impregnated and fixed to the shaft with metal clamps, one of which is fitted with a collet and set screws.

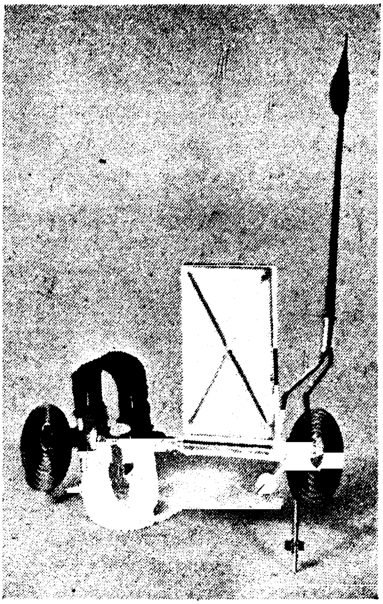


FIG. 8.30.—Movement from switchboard wattmeter (Metropolitan-Vickers).

Astatic Wattmeters

Owing to the importance of the stray field error in unshielded dynamometers, several attempts have been made to overcome this difficulty by adopting an astatic arrangement of the coils. It is, of course, possible to make any of the instruments described above astatic by dividing the coil systems into two parts and mounting them so that the two moving coils are on a common axis, and reversing the currents in one set of coils. Such an arrangement has, however, some disadvantages, for it usually necessitates a much heavier movement and a deeper instrument case, the coils are more difficult to support, and the whole instrument becomes more bulky and expensive.

The Kelvin Engine-room Wattmeter was an example of this type of construction which is shown in Fig. 8.31. The two main coils are wound with copper strip and are mounted with their axes vertical, one in front of the other, on a horizontal bracket support projecting from the base plate. Each encloses a little circularly-wound and taped volt coil, the central opening of which is plugged with a disc of

hard wood ; the two coils are then inserted in an aluminium frame, and fastened in position by screws passing through the sides of the frame into the wooden discs, so that the moving system somewhat resembled a pair of spectacles. At each end is the usual Kelvin knife-edge arrangement on which the coils turn, the hook supports being fixed to the upper part of the main-coil framework. Two spiral control springs of palladium alloy are employed, one at each end, and these serve also to lead current in and out of the coils; at the front end of the axis the pointer is prolonged below, and terminates in a little flat vane which moves in a narrow metal

FIG 8 31 —Coils from Kelvin engine room astatic wattmeter

box filled with oil which is attached to the lower main-coil bracket, and this provides the necessary damping.

The front spring attachments are on a vertical metal plate which is capable of movement over a short arc, and can be clamped in position by a screw working in a curved slot in it, and by this means zero adjustment can be made.

An outstanding feature of this instrument was the large amount of metal employed in its construction, which not only involved an unduly heavy moving system, but also produced a considerable eddy current, as shown by the figures in Table XXXVI.

It is, however, possible to obtain fairly perfect astaticism without making the coil systems double : thus, in the Irwin dynamometers, made by the Cambridge Instrument Co., the two main coils are fixed face to face 2.5 cm. apart, as in the voltmeter and ammeter

already described, and are connected so that when traversed by current the fields oppose and produce a strong radial field. In the space between, the movable coil is pivoted. This coil consists of two **D**-shaped coils held between mica cheeks, and so connected that the current in them flows in opposite directions in the outer curved sides, which are, in the radial field, due to the main coils, as shown diagrammatically in Fig. 8.32. The arrangement does not, however, seem to lend itself well to the production of a satisfactory

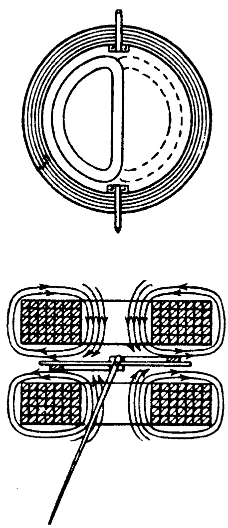


FIG. 8.32.—Diagrammatic arrangement of Irwin astatic wattmeter.

long-scale indicating instrument. In all the forms the torque is small and the torque efficiency very low, and in consequence the friction error is serious and troublesome while the scales close fairly rapidly at each end.

The inductance of the moving system is only 3·6 millihenries, and the total resistance of the pressure circuit is 5,000 ohms for a 150-volt instrument, so that the inductive error is small; but offset against this it will be noticed that there is a considerable amount of metal in the working fields and, as a result of this, the instruments show an appreciable negative reading on zero power factor.

Another form of astatic dynamometer has been devised by Messrs. Hartmann & Braun, in which the moving coil consists of a single oval coil bent through two right angles, which give it a ‾|_ shape. This is suspended in the field of an enclosing and fixed main coil connected in the main circuit, and the torque produced is independent of the position of the moving coil over a wide range, and produces an evenly divided scale throughout. The movement is air-damped by means of vanes moving in an enclosing damping-box below the coils.

Ironclad Wattmeters

One of the principal defects of the ordinary dynamometer wattmeter is the small torque obtainable when the shunt reactance and heating are kept within the necessary limits.

There has been a very strong prejudice against the employment of iron to increase the magnetic fields, the impression being that if sufficient is used to materially improve the torque, it would necessarily introduce errors of a quite unallowable magnitude.

It has, however, been shown* that this is by no means the case, and that by suitably proportioning the iron circuit and air gap a wattmeter can be constructed which has all the required accuracy.

In order to preserve the strict dynamometer principle it is necessary that the pressure-coil circuit should be as free from inductance as possible—that is, it should enclose as small an area as possible and be free from iron. On the other hand, iron may be used to increase the field of the main coils, since inductance in this part of the instrument has no deleterious effect if it does not produce an undue inductive drop, and by reinforcing the field in this manner we secure the additional advantage of making it less sensitive to the effects of external fields and minimize the effects of eddy currents.

The presence of iron in the instrument may affect the readings in three ways, viz. :

(1) By increasing the inductance of the shunt circuit.

(2) By introducing eddy currents.

(3) By causing the magnetism to lag behind the magnetizing current by the angle of hysteretic lag, and distorting the wave of flux variation with respect to that of the current.

* "The Use of Iron in Dynamometer Wattmeters," *Electrician*, Dec. 10, 1909.

The effects of shunt inductance and eddy currents have already been dealt with, and to reduce the latter the iron core must obviously be well laminated.

The effect of hysteresis in causing phase displacement of the main field is shown in the accompanying Fig. 8.33. The main current I is resolved into two components, a wattless or magnetizing current I_M, and an energy or core loss current Ic. The main flux

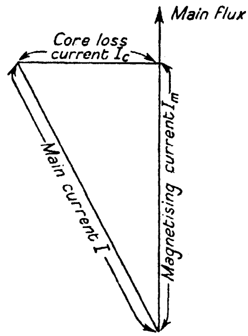


FIG. 8.33.—Vector diagram showing effect of hysteresis.

being in phase with I_M lags by the angle β, if there are no eddies, such that $\tan\beta = \dfrac{Ic}{I_M}$; with closed magnetic circuits it is not infrequent for these components to be equal, the lag therefore being 45°; this is more than a hundred times the permissible angle in a good wattmeter. The obvious method of reducing this is to use softer iron and to introduce an air gap into the magnetic circuit.

In order that we may calculate the phase displacement, let h be the hysteresis loss in ergs per cubic centimetre per cycle for the iron used, f the frequency, and v the volume of the iron.

Then the total hysteresis loss in watts is

$$W_h = hfv/10^7$$

And if A is the area of the core, and $\hat{B}$ the maximum induction density in it, the effective core E.M.F. is :

$$\bar{V} = \frac{2\pi}{\sqrt{2}} \frac{10^8}{A\hat{B}f}$$

Hence the effective core loss ampere turns :

$$I_c = \frac{\text{watts}}{\text{E.M.F.}} = \frac{hfv}{10^7} \cdot \frac{\sqrt{2}}{2\pi} \cdot \frac{10^8}{A\hat{B}f} = \frac{10}{\pi\sqrt{2}} \frac{hl}{\hat{B}},$$

where $l = \frac{v}{A}$ = the length of path in the core.

For the magnetizing ampere turns $\hat{I}_M$ we have :

$$\frac{4\pi}{10} \hat{I}_M = \Sigma\hat{B} \frac{l}{\mu}, \text{ or } I_M = \frac{10}{4\pi\sqrt{2}} \hat{B} \left(\frac{l}{\mu} + l'\right)$$

where l' is the air-gap length.

Now

$$\tan\beta = \frac{\bar{I}_c}{\bar{I}_M} = \frac{4hl}{\hat{B}^2\left(\frac{l}{\mu} + l'\right)} = \frac{4\mu h}{B^2\left(1 + \mu\frac{l'}{l}\right)}$$

If the air gap is large l/μ is negligible compared with l', and we have

$$\bar{I}_M = \frac{10}{4\pi\sqrt{2}} \hat{B}l'$$

and

$$\tan\beta = \frac{4h}{\hat{B}^2} \cdot \frac{l}{l'}$$

Now if h follows the law $h = \eta\hat{B}^a$,

$$\tan\beta = \frac{4\eta}{\hat{B}^{(2-a)}} \cdot \frac{l}{l'}$$

and this is equal to β for small angles.

Hence we may write

$$\frac{l'}{l} = \frac{4\eta}{\beta B^{(2-a)}}$$

where η is a constant for the particular kind of iron employed. Since β represents the maximum error in the power factor which can be caused by the phase displacement in the wattmeter, it follows that for this to be limited to any given amount there is a minimum value of $\frac{l'}{l}$ which must be employed.

Both Mr. Wild and Dr. Sumpner have shown that at very low induction densities the exponent a in the expressions above is more nearly 2 than the value 1·6 originally given by Steinmetz, and by analysing the values for Stalloy and Lohys iron obtained by Wilson, Winson, and Odell, we find for the range $\hat{B} = 50$ to $\hat{B} = 500$ the law,

$$h = 0{\cdot}00011\ \hat{B}^2 \text{ for Lohys iron}$$

and

$$h = 0{\cdot}000057\ \hat{B}^2 \text{ for Stalloy}$$

Therefore assuming the law as $h = K\,\hat{B}^2$ we have

$$\tan\beta = \beta = \frac{4h}{\hat{B}^0}\frac{l}{l'} = 4K\frac{l}{l'}$$

which is independent of B.

Then for Lohys iron we have

$$\beta = 0{\cdot}00044\frac{l}{l'}, \text{ or } \frac{l}{l'} = 2{,}270\beta$$

and for Stalloy

$$\beta = 0{\cdot}000228\frac{l}{l'}, \text{ or } \frac{l}{l'} = 4{,}380\beta$$

By plotting curves like those in Fig. 8.34 for the iron to be employed we can read off the ratio $\frac{l}{l'}$ for the corresponding values of β.

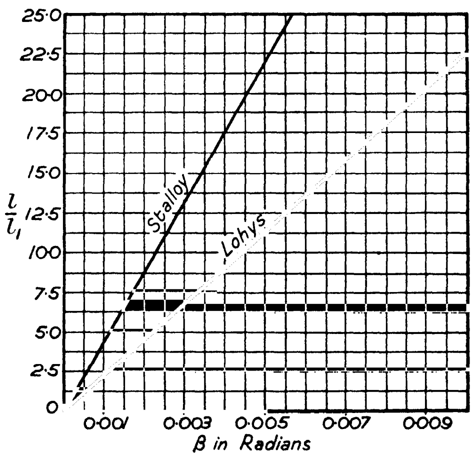


FIG. 8.34.—Curve showing relation of l/l' to β for iron-clad wattmeter.

Examples of Ironclad Wattmeters

There have been very few practical attempts to realize an ironclad instrument; probably the first was made by the A.E.G., which is identical in appearance with the wattmeter and voltmeter already described in the first portion of this chapter (Fig. 8.8, page 427).

The main coils are fixed inside a block of iron laminations, and the moving coil turns within them in the usual way. Apart from the increase in torque the instrument has the advantage of being magnetically damped.

A 100-volt instrument of this type has been stated to take 0·04 ampere in its pressure circuit, thus having a resistance of 2,500 ohms and an inductance of 7 millihenries, or a phase displacement of 0·00088 radian at 50 cycles per second. The torque is given as 0·8 gm. centimetre, and the weight of the movement 4 gm.; the loss in the main coils is 2·4 watts.

An ironclad wattmeter has also been designed by L. Murphy in which the moving coil is of the usual wide rectangular form; this is pivoted in the usual way, and moves in the gap between a central cylindrical laminated core and a concentric annular laminated yoke. The central core is provided with two deep slots running the whole length on each side, and in these the main coil is wound.

Instruments of this type were also made by Siemens & Halske and the A.E.G. in Germany. The latter manufacturers divide the turns of the main coil into two groups where they emerge from the slots at the top of the laminations and bend them into semi-circular arcs, so that the core and movement can be withdrawn from the main coil without disturbing it.

Dr. Sumpner also designed a wattmeter with an iron circuit, but the special features of this instrument will be dealt with later.

Two types of instruments have also been designed by one of the authors. The first of these involved a new principle, and is shown diagrammatically in Fig. 8.35. The main coils are wound on the limbs of a laminated iron circuit, and are so arranged that the flux crosses two narrow gaps between the poles so formed. Two thin rectangular pressure coils are suspended in the gaps, each consisting of a single layer of wire between thin mica cheeks, the finished coil being only about 0·02 of an inch in thickness. These are connected in series, but the current in one coil circulates in the opposite sense to that in the other. Their horizontal sides are

between the two sets of poles, and since they carry currents in opposite directions, one tends to move upwards while the other moves downwards.

As originally constructed, both coils were free to move, and to the upper edge of each a fine wire was attached which passed over a little pulley on the pointer shaft, giving it a rotation proportional to the movement, each coil being controlled by horizontal springs. In this arrangement not only is the moving system astatic, but the inductance is reduced to a practically negligible quantity.

In the second form of instrument an entirely different principle is adopted. The main coils are wound upon the inwardly-projecting

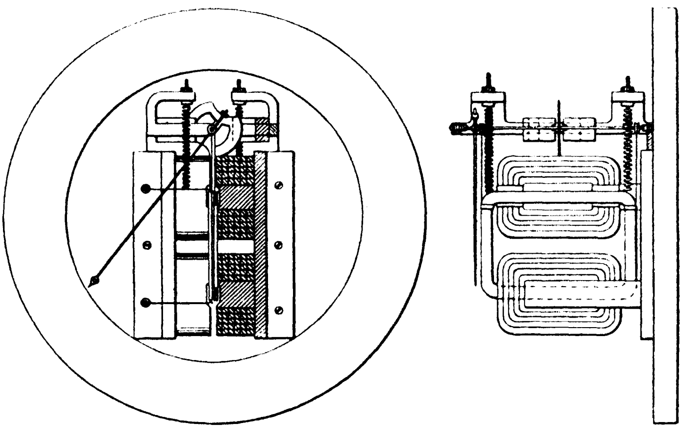


FIG. 8.35.—Early form of Drysdale iron-clad wattmeter.

poles of an enclosed type of magnet. These produce a flux across the gap whose return path is round the two outside yokes, the whole magnetic circuit being built up of thin Stalloy punchings.

In the gap a narrow moving coil, wound between mica cheeks, is pivoted between jewels and spring-controlled in the usual way. The movement is magnetically damped by having at the front end of the spindle an aluminium sector whose outer edge is turned up at right angles to its plane, and this edge moves between the poles of a circular permanent magnet attached to the front jewel bridge.

By proportioning the gap and magnetic circuit according to methods indicated in the first part of this section, and by employing the narrow Ayrton-Mather type of moving coil, the inductance of the shunt circuits is brought down so as to produce no appreci-

able error, while at the same time a torque of over one gramme centimetre is obtainable with a scale of practically uniform divisions over the greater part of the range.

Of recent years a number of successful commercial instruments have been produced both in this country and in America. In particular attention has been paid to the provision of an instrument with a long scale 270° angular motion, and also one which will withstand the onerous requirements of the fighting forces. As

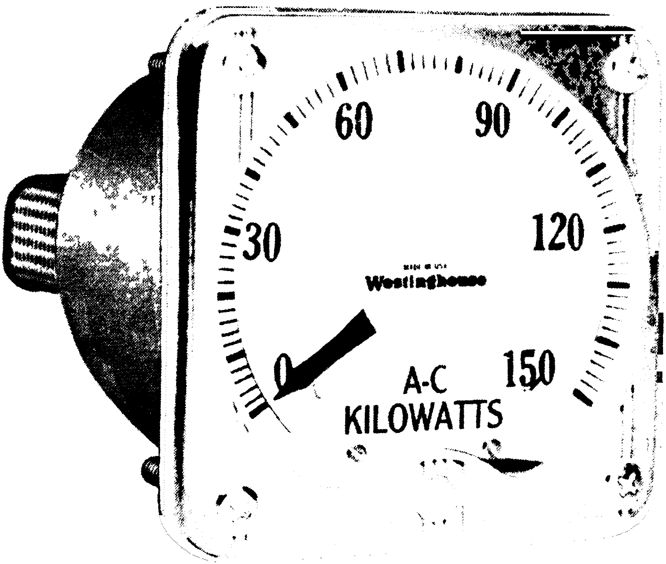


FIG 8 36 —Westinghouse iron-clad wattmeter.

an example of these instruments may be quoted the single and double element wattmeters made by the Westinghouse Electric Corporation of America. The external appearance of this instrument is shown in Fig. 8.36, and this is one of a complete range of instruments, similar in appearance, all of approximately 250° angular deflection, which range includes moving coil and moving iron in addition to the iron clad dynamometers.

The construction of the double element type of wattmeter is shown diagrammatically in Fig. 8.37, and the single element is similar to this, but without the back element. In designing this

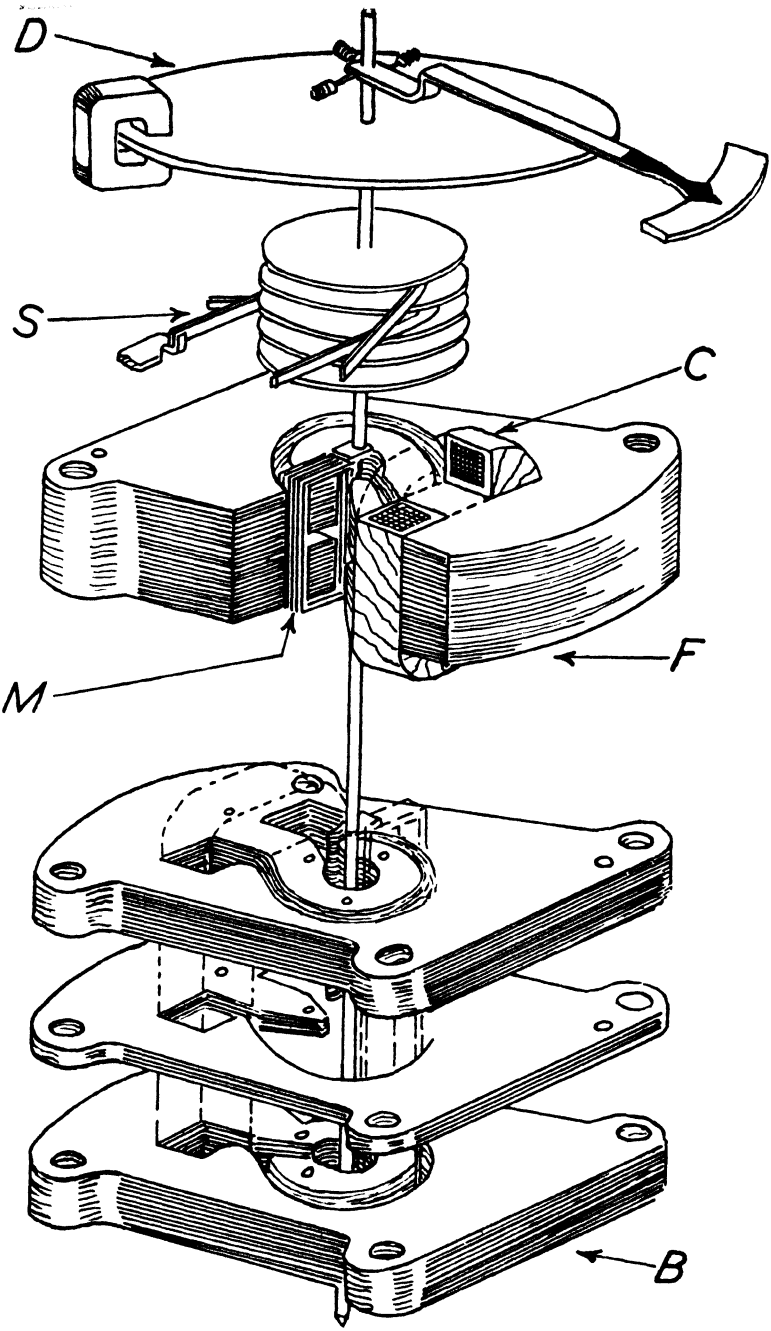


FIG. 8.37.—Diagram of double-element wattmeter. D = damping disc. S = conducting strips. C = stationary coil. M = moving coil. F = front element electro-magnet. B = back element electro-magnet (exploded).

instrument, first consideration was given to ease of manufacture and of servicing, and this has been attained by providing a number of sub-assemblies which can be built up completely before the main assembly, and can later be dismantled if required without disturbing any of the adjustments. The moving element, for example, is built up complete and balanced as a unit. The stator assembly, including the current coil, the laminations, the die-cast frame and the steel bolts used to fasten the stator to the moulded base of the instrument is also a complete unit, which is not disturbed in the final assembly with the moving element. The lamination iron is stamped in one piece to ensure permanence of calibration and to minimize calibration adjustments. The circular core of the lamination has been made hook-shaped by a radial slot for inserting the moving coil when mounting the moving element in the stator. If the laminations were so assembled that all the slots were in line, then the wattmeter would deflect when energized by voltage alone, due to the solenoid action of the moving coil. By arranging the stator laminations so that half of them have the slot on the opposite side of the long axis of the lamination this effect has been practically eliminated. The two oppositely-arranged groups of laminations are separated by laminations having the hook-shaped core omitted to allow the sides of the moving coil to pass when the moving element is rotated during the assembly.

Damping in this instrument is provided by permanent magnets and an aluminium disc mounted on the shaft of the moving element.

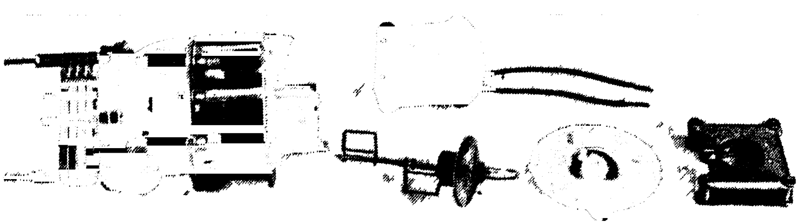


FIG. 8.38.—Components of double element wattmeter.

Fig. 8.38 shows the components of this wattmeter, and the damping magnet assembly can be seen at the extreme right, while the moving element with its damping vane can be seen in the centre

front. In this figure can also be seen one current element and the top plate assembly.

Double Wattmeters

For many purposes, notably the measurement of power in polyphase circuits, either two-phase or three-phase, a double wattmeter consisting of two similar dynamometer wattmeters, with their pressure coils mechanically coupled together, is of great value. In the case of two-phase supply with two independent circuits it is obvious that each wattmeter will measure the power

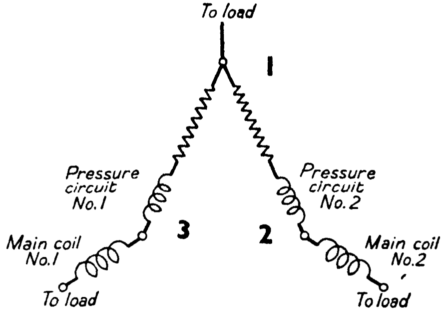


FIG. 8.39.—Connections of double wattmeter for measuring three-phase power.

in one phase, so that the reading when the two moving coils are coupled gives the total power supplied by the two phases.

With three-wire or three-phase supply with three conductors, however, the double wattmeter will measure the total power if connected as shown in Fig. 8.39, where 1, 2 and 3 are the three mains. One wattmeter has its main coil connected in lead 1 and its shunt coil between 1 and 3, while the other wattmeter has its main coil in lead 2 and its shunt circuit between 2 and 3.

The total power supply by the system may be considered as in three parts: one by leads 1 and 2 alone, one by leads 2 and 3 alone, and one by leads 3 and 1 alone. It is obvious that if lead 2 is broken the first wattmeter will measure the power supplied between 1 and 3, and similarly the second will measure the power supplied by 2 and 3 when lead 1 is broken. But if lead 3 is broken, leaving the two shunt coils connected together, we have the two wattmeters in series across 1 and 2, each of which measures half the power between these leads, as the shunt current is now half its normal value; but since the moving coils are mechanically

connected, the reading of the instrument in this case will be the power supplied between leads 1 and 2, and it does not matter in the least what the potential at the point 3 is, as the only result of altering it would be to increase the reading of one wattmeter and diminish that of the second by equal amounts.

When all three leads are supplying current, therefore, either continuous, single phase, or polyphase, the sum of the readings of the two wattmeters gives the total power supplied, and the mechanical coupling of the coils means that the wattmeter reads the total power.

The use of two wattmeters for three-phase measurements was suggested in the early days of polyphase power supply, but the first double wattmeter for directly indicating power on such circuits was designed and constructed by one of the original authors in 1902.*

It was on exactly the same lines as the single-phase instrument already described, but with two similar systems mounted one above the other, with their magnetic axes at right angles, so that one system did not interfere magnetically with the other. The currents were introduced into the moving coils by the top and bottom suspensions, and by two additional ligaments attached to terminal tongues on the main supports. Fig. 8.40 shows the construction of this form of instrument, with astatic systems and series-parallel commutator device; the details are otherwise identical with the single wattmeter described on page 442.

The following test was made to demonstrate how completely the magnetic interference between the two systems is reduced; with the full current in the two moving coils in series (0·02 ampere) and the main coils carrying their full current the following readings were obtained:

	Divs.
Lower system only	500
Upper system only	501
Top main coils and lower moving coil	—1
Lower main coils and upper moving coil	—1
Both systems assisting (0·01 amp. in moving coils)	499·5
Both systems opposing (0·01 amp. in moving coils)	—0·25

The small interference shown by this test is due to the astatic construction adopted, which causes the field of one system to

* *Electrician* Jan. 14, 21, and 28, 1916.

affect the other even when they are accurately perpendicular; but approximate compensation can be secured by manipulating the rising leads to the main coils, or by adding a turn or two in series with each moving coil when in the field of the other main coil. This method has been adopted with excellent results.

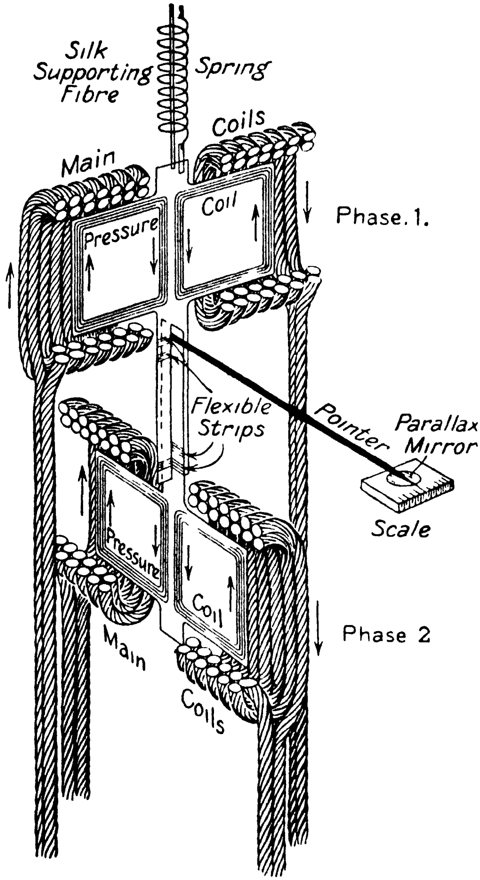


FIG. 8.40.—Drysdale double astatic wattmeter.

In some of the commercial forms of double wattmeter the two systems are simply placed one above the other without turning one through a right angle; the interference can then be compensated by employing an auxiliary resistance inserted between the junction of the two pressure circuits and the third lead.

This method has been attributed both to Dr. Weston and also to Dr. Franke, and its theory may be stated as follows :

Instead of taking the wattmeter reading $W = V_1 I_1 + V_2 I_2$, let it be $W_1 = V_1 (I_1 + KI_2) + V_2(I_2 + KI_1)$, where K is a small fraction representing the amount of interference between the systems (assumed symmetrical). Let r be the resistance of each pressure circuit (including the external resistance), and r_1 the small insertion resistance in the third lead (Fig. 8.41), R being taken as $r + r_1$ and $K_1 = \frac{r_1}{R}$.

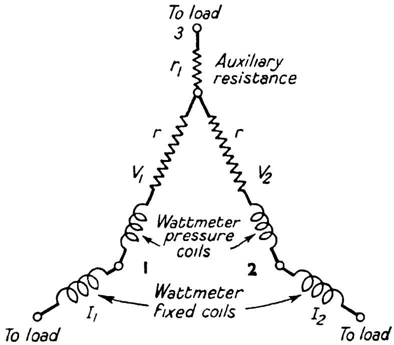


FIG. 8.41.—Connections of three-phase wattmeter with compensating resistance in third lead.

Then it is readily seen that instead of the current in the shunt being

$$i_1 = \frac{V_1}{R}, \text{ it is } i_1 = \frac{V_1 - V_2 r_1/R}{R\{1 - (r_1/R)^2\}}$$

or the equivalent voltage

$$V_1^1 = \frac{V_1 - K_1 V_2}{1 - K_1^2}.$$

Hence, the wattmeter reading becomes

$$W_1 = \frac{1}{1 - K_1^2}\{(V_1 - K_1)\,(I_1 + KI_2) + (V_2 - K_1 V_1)\,(I_2 + KI_1)\}$$

$$= \frac{1}{1 - K_1^2}\{(1 - KK_1)\,(V_1 I_1 + V_2 I_2) + (K - K_1)\,(V_1 I_2 + V_2 I_1)\},$$

which reduces to $V_1 I_1 + V_2 I_2$ if $K_1 = K$ or if $\frac{r_1}{R}$ equals the ratio of the interference reading to the normal reading.

This correction, although small, cannot be perfect at all parts of the scale in the case of a deflectional instrument, as K must vary somewhat with the position of the moving coils, but it is absolutely correct for the case of a zero reading instrument if the interference of each system on the other is the same. It is not easily applicable, however, when multiplying resistances are employed for varying the P.D. range, and in this case correct magnetic compensation is desirable.

The double wattmeter was first employed only for differential tests. Potier used an instrument of the Siemens type for this purpose, with, however, only a single moving coil which enclosed two fixed main coils, but probably the first attempt to employ two complete wattmeter systems was made by Dr. Kennelly in 1892. For his purpose he used two unifilar dynamometer systems of the Kohlrausch type set at right angles one above the other, the swinging coils being rigidly connected by an 8 cm. aluminium shaft and then suspended by a copper wire 32·6 cm. long and 0·0115 cm. in diameter, the torsional moment of the suspended system being 18·6 dyne-cm. per radian. The residual mutual induction between the two systems was compensated by means of auxiliary coils, set by trial, outside the dynamometer, and with this arrangement tests on single-phase transformers were made.

The indicating double wattmeter has now become quite general for polyphase testing, and most instrument makers are manufacturing instruments of this type.

The switchboard pattern of the Weston Instrument Company consists of two single-phase systems identical with that described on page 450 set one over the other, the interference of the fields being corrected by an insertion resistance as described above; this is wound on circular bobbins mounted behind the ordinary series resistances, which are of the flat mica and strip type already described. Three control and leading-in springs are employed, two under the outer jewel bridge, as in the single-phase instrument, and the third mounted between the two moving coils, to which connection is made by an insulated tongue carried on the main coil support.

In a similar instrument of the Weston Company, the interference between the two systems is corrected by interposing between them a magnetic screen consisting of a series of oval-shaped laminations.

The following test was made to examine the effectiveness of this device :

Volts on upper pressure coil, 150. Amperes on lower main coil, 5. Deflection, 0·003 kW.

Volts on lower pressure coil, 150. Amperes on upper main coil, 5. Deflection, 0·003 kW.

Upper system only, volts, 150. Amperes, 5. Deflection 0·75 kW.

Lower system only, volts, 150. Amperes, 5. Deflection 0·75 kW.

Both systems assisting reading, 1·495 kW.

Both systems opposing reading, —0·005 kW.

The Siemens and Halske precision wattmeter is arranged in similar fashion. The rectangular main coils, which in the heavy current sizes are punched from thin copper sheet, are mounted one above the other in the same plane, the adjacent horizontal sides of the two sets of coils being only 0·5 cm. apart.

The two pressure coils are rigidly connected by a light brass tube, and the whole moving system is supported on inwardly-projecting pivots fastened to the upper and lower horizontal sides of the coil pair ; as the coils have no formers, it is claimed that their own elasticity renders the instrument very portable.

In this case four control springs are employed which also serve to conduct the current to the coils.

Here, again, compensation for interference between the coils is made by a resistance in the third lead, and the main coil range is made double by employing a double plug or link commutator device similar to that described with the single-phase instrument.

The Nalder Double Wattmeter is shown in Fig. 8.42, and in this instrument the two systems are set with their planes at right angles to one another. The main coils are wound with thin strip on ebonite bobbins which are supported in pairs from two brass pillars.

The two pressure coils are on a single shaft and are wound on light metal formers. The shaft is divided in the centre by an ivory piece, which serves to support the central spring abutments and the curved arm and piston of the damping arrangement.

The series resistance of each pressure circuit is divided between two porcelain bobbins fastened to the non-conducting back of the instrument, and the whole is screened by a cast-iron case.

The narrow coil ironclad instrument can also be very conveniently adapted to the double form, and a very compact instrument results.

Two iron circuits are employed, each half the thickness required for the equivalent single-phase instrument, and one of these is turned through a right angle relatively to the other. The two-volt

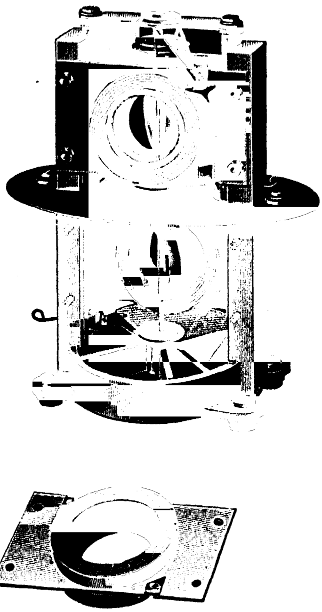


FIG. 8.42.—Nalder double dynamometer wattmeter.

coils are shortened correspondingly and attached to a common spindle with their planes mutually at right angles. With this arrangement the interference between the two systems is *nil* with the moving coils in the central position, and is very small for all other positions. A typical recent double wattmeter of the ironclad type is described on page 470.

Heavy Current Wattmeters

The construction of wattmeters for currents exceeding 500 amperes presents considerable difficulties, notably on account of the large sectional area of the conductors, their want of flexibility, the risk of eddy currents, and non-uniform current distribution or " skin effect " in them, the intensity of the magnetic field due

to the leads, and the size and difficulty of manipulating a series parallel device.

Starting from the essential principle of a dynamometer instrument, that the torque produced by the fixed on the moving element is proportional to the rate of change of mutual induction between them, it occurred to one of the authors* that the most simple, compact, and easily constructed arrangement was one in which the moving element was in the form of a disc carrying two oppositely-wound **D**-shaped coils, and revolving in its own plane between four stranded main coils, also of **D**-shape, as shown in Fig. 8.43,

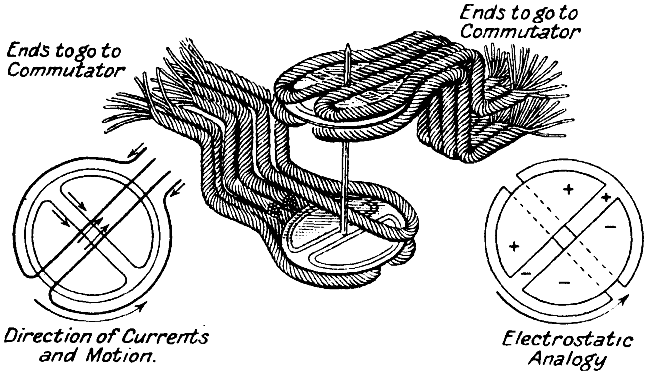


FIG. 8.43.—Drysdale heavy current wattmeter.

the arrangement being the electro-dynamic analogue to the quadrant electrometer, or more strictly its predecessor, the divided ring electrometer.

With this construction the main coils are very easily formed and are mechanically independent of the moving coil, so that the latter is not in any way disturbed by the removal of the main coils.

The commutator for connecting the coils in series parallel has, on account of the heavy currents to be dealt with, to be specially designed, and operates flexible butt contacts by means of a cam (see Part II).

A very pretty solution of the problem was found by Dr. P. G. Agnew, of the Bureau of Standards at Washington.† It is to make the main conductor in the form of a double concentric tube

* C. V. Drysdale.
† P. G. Agnew, *Bull. Bureau of Standards*, vol. viii, July, 1912.

united at one end. The field in the annular space between is then quite independent of the distribution of current in the section of the tubes, since the line integral of the magnetic force in the space round the inner conductor must be 4π times the current in it, whether it be near to or far from the central axis. There is, of course, no external field whatever, as the arrangement is concentric.

Fig. 8.44 is a diagram of the arrangement and is self-explanatory, except that the two astatic moving coils are not really as shown, but are perpendicular to the plane of the paper in order to lie parallel to the annular field. With an inner tube 2·54 cm. outer

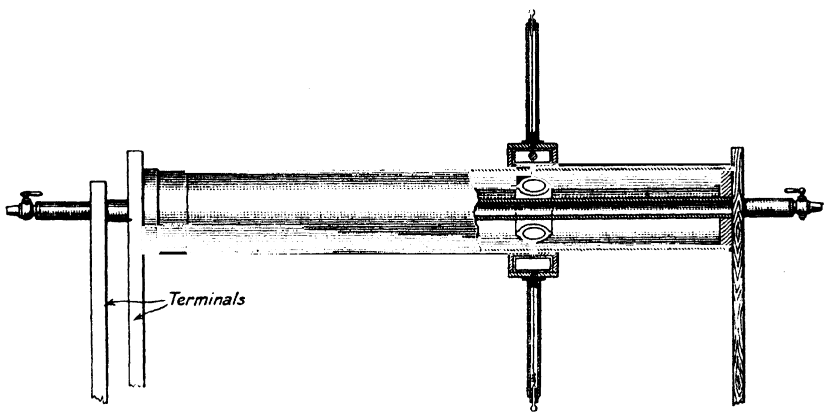


FIG. 8.44.—Diagrammatic arrangement of Agnew's heavy current wattmeter.

and 1 cm. inner diameter water-cooled, and an outer tube 14·14 cm. outer and 12·82 cm. inner diameter, the instrument gave accurate readings with currents up to 5,000 amperes and frequencies up to 900 cycles per second. In order to equalize the current distribution in the outer tube a groove 5 cm. wide was turned in it excentrically, so that it was 2 mm. deep at the top and 3 mm. deep at the bottom ; this was afterwards further adjusted by hand filing until, when the normal current was flowing, the equipotential surfaces were planes perpendicular to the axis of the instrument. To minimize the effect of the leading-in terminals, the moving system was placed at one-third of the length from the closed end. Magnetic impurity in the materials of which the moving coils were

constructed at first caused great difficulty, but eventually special coils of 0·2 mm. silver wire, silk covered, were employed.

An instrument on similar principles was independently devised and patented in July, 1911, by A. E. Moore, and described by him before the Institution of Electrical Engineers in 1917.* In this instrument a number of radial loops or alternatively solid concentric cylindrical conductors form the main current element. In the latter case the outer conductor is made like a cylindrical box in two halves, which are bolted together by a substantial flange right round the circumference. The central conductor is tubular and can be water-cooled, passes clear through the centre of one end, and makes metallic contact with the other. Originally Moore used a double circular pressure-coil system somewhat like the Agnew instrument, but eventually he developed a double **D**-shaped coil arrangement in which the two **D**-coils are supported in an ebonite ring with their straight sides horizontal, one above and the other below the central conductor. If $2l$ is the length of the horizontal straight side and l_1 the vertical distance of this side from the axis of the central conductor, he shows that the best relation, as far as the constant of the instrument is concerned, is when $l/l_1 = 1{\cdot}73$, for then the torque is practically independent of the angular relation between the magnetic axes over a range of about 80°, and a practically uniform scale over this range results.

For zero reading torsion instruments another type of moving system was designed which would be insensitive to vertical and lateral displacements of the coils. In this a light framework is arranged to support four rectangular coils, with their planes horizontal, in pairs symmetrical with the rotational axis above and below the central conductor. For an instrument intended to carry 300 amperes in the main current element the vertical distance between the planes of each pair was 2·4 cm., and the distance between the centre of each coil and the vertical axis of rotation was 4 cm., the coils themselves being square in form, having sides 3 cm. long ; the inner and outer sides of each coil were bent up sharply at right angles to the plane of the coil, each coil was 50 turns of No. 36 S.W.G. wire, and the total inductance of all four in series 0·9 millihenry, and the total weight of movement 8 grammes. A change in vertical position of 5 mm. was found to only affect the force about the axis of rotation by about 0·8%.

* *Journ. I.E.E.*, vol. iv, pp. 380–402, May, 1917.

Dr. Sumpner's Iron-cored Dynamometers and Induction Instruments

A very ingenious series of dynamometer instruments devised by Dr. W. E. Sumpner was constructed by the General Electric Company.* Their salient feature was the employment of shunt electromagnets on alternate current circuits, thus giving a strong alternating flux corresponding to the field of a permanent magnet instrument.

The inventor's object was, in fact, to construct instruments for alternate current working corresponding to the permanent magnet moving-coil instrument used on continuous currents. For this purpose he used a moving coil in the gap of either a "series" or a "shunt" magnet. Figs. 8.45 and 8.46 show the schematic

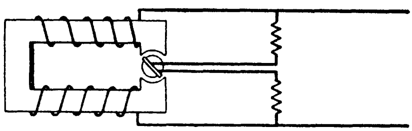


FIG. 8.45.—Sumpner series magnet wattmeter.

connections of two instruments on these lines. Fig. 8.45 is a watt meter with a "series" magnet in which the iron is merely used to intensify the field of the ordinary main coils.

In Fig. 8.46 we have the application of the "shunt" magnet to a wattmeter. The magnet is connected directly to the mains

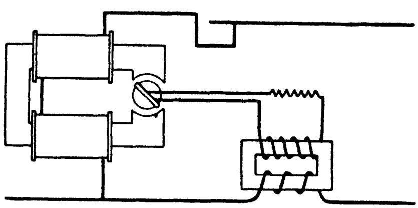


FIG. 8.46.—Sumpner shunt magnet wattmeter.

as in the voltmeter, but the moving coil, instead of being connected across a resistance, is connected to the secondary of a small transformer or mutual inductance of which the primary is in series with the mains.

* *Journ. I.E.E.*, vol. xxxvi, p. 421, Oct., 1905, and xli, p. 227, March, 1908.

The theory of the "shunt" magnet instrument may be briefly put as follows: If A is the area of the magnet cross-section, $\bar{B}$ the R.M.S. value of the flux density, and N the total turns on the core, we have for the induced E.M.F.—

$$V = cA\frac{dB}{dt} \times 10^{-8}, \text{ or } \bar{V} = +j\,\omega Na\bar{B} \times 10^{-8}$$

from which $$\bar{B} = j\frac{10^8\bar{V}}{\omega NA}.$$

The induction $\bar{B}$ is thus inversely proportional to the frequency, and lags 90° behind the P.D. if the resistance and core loss effects are negligible.

In a condenser the current $\dot{I} = C\,\omega V$, so that in Fig. 8.46 the torque, which is proportional to $Vi \propto \frac{C}{NA}V^2$, which is independent of frequency, and hence also of wave form.

In the wattmeter the secondary E.M.F. in the coil is

$$V_2 = M\frac{\delta I}{\delta t} = -j\,\omega MI$$

and the current in the moving coil is proportional to $\frac{V_2}{r}$, hence the torque is proportional to

$$j\frac{10^8 V}{\omega NA} \times j\,\omega MI \propto \frac{10^8 M}{NA} - VI$$

which is again independent of frequency.

The theory of this wattmeter is, however, more akin to that of the induction wattmeter, which will be given in a subsequent chapter.

Wattmeter Errors

The phase, frequency, and inductive errors have already been considered when dealing with the theory of the instrument, but, in addition to these, we have errors due to friction, heating and stray fields.

Friction

The frictional error becomes relatively important in this type of instrument on account of the smallness of the available torque, and in order that this shall be reduced to a practical minimum great care is required in the pivoting, while the weight of the moving system must be reduced as far as possible. A not infrequent cause of comparatively large friction error is to be traced to a want of correct alignment of the two pivots, for since it is imperative to remove all metal, as far as possible, from the fields of the coils, the pivots are generally supported from the winding or the insulating former on which the moving coil is wound; in either case warping may take place after the instrument is completed, which may throw the pivots out of true centre relatively to one another, causing the movement to bind in its jewels. For this reason the use of a continuous spindle right through the moving system is to be advocated, and this, under ordinary circumstances, should not cause any appreciable error.

In horizontal instruments inwardly-projecting pivots give better results than two independent outside ones, since the points of support come nearer to the centre of the coil, and warping, therefore, produces less relative movement; and, moreover, the top pivot bears practically all the weight, so that distortion will, in general, only affect the position of the pivot in the lower jewel and seldom produce binding.

Heating

The heating error is the same as that in voltmeters, since the pressure circuit carries a current proportional to the P.D. at its terminals. It is imperative that the resistance of the circuit should remain sensibly constant at all temperatures within the working range. Usually the error is relatively small, since it is necessary to reduce the inductive error by winding the pressure coil with as few turns as possible, and at the same time reducing the current to the lowest possible value consistent with obtaining the required torque. Hence the ratio of swamping resistance to copper resistance is usually very large, and by a suitable choice of material for winding the resistance the error becomes entirely negligible. Moreover, the subdivision of the series resistance in order to reduce its capacity effect is beneficial also from the point of view of heating, since in such resistances a much larger radiating

surface is available than would occur with windings of the ordinary type.

Stray Fields

External fields may cause serious errors in the ordinary types of non-astatic dynamometers unless shielded, either by means of an iron case or by a laminated shield, as already described. The presence of such stray fields may be detected by connecting the pressure circuit of the instrument to the supply, when, with no current flowing in the main circuit, a deflection will be produced. In wattmeters, and particularly those intended for heavy currents, the incoming leads to the main coil may seriously affect the working fields of the coils, and too much care cannot be exercised in bringing in these leads to an unshielded instrument.

If the iron cover is employed and it is sufficiently distant from the coils, as it usually is in modern instruments, this, together with the low conductivity of the material, will render the effect of any eddy currents likely to be set up negligible, and a test at full current and zero power factor with the cover on and off will at once reveal the effect, if it is present.

On the other hand, the presence of the cover tends to increase the working fields, and the instrument reads higher with it in position, and this fact must be taken into account in the final calibration, and any permanent magnetism in the cover will be revealed by a disagreement in the readings for the same load on alternating and direct current supply. If well carried out, the iron case appears to be capable of reducing the stray field to under 1% in fields likely to occur in ordinary practice.

Design of Dynamometer Instruments

When special care is given to the form and dimensions of the coils, dynamometer instruments are susceptible of having their constants calculated with a high degree of accuracy, and as such they are the ultimate standards for the determination of current. From this point of view they will be discussed later. But the construction of an absolute standard instrument is entirely unsuitable for ordinary indicating instruments, as it involves very large coils and special methods of mounting. On the other hand, with small and light coils it is not easy to make calculations with

any high degree of precision, but a rough approximation can be arrived at which will serve in designing the system.

Fundamental Principle

As shown in Chapter 4, the fundamental relation underlying the behaviour of dynamometer instruments is given by the formula $T = I_1 I_2 \frac{\delta M}{\delta \theta}$, where T is the torque in dyne-cm., I_1 and I_2 the currents in the two coils in C.G.S. units, M the coefficient of mutual inductance in cm., and θ the deflection in radians. From this we have $T = \frac{10^9}{981 \times 100} I_1 I_2 \frac{\delta M}{\delta \theta}$, where T is in gm.-cm., I_1 and I_2 in amperes, and M in henries, or $T = 10{,}200\, I_1 I_2 \frac{\delta M}{\delta \theta}$. Hence, if we can calculate the rate of change of mutual inductance with angle, or $\frac{\delta M}{\delta \theta}$ for a system, we can immediately deduce its torque.

Experimental Example

As an example of this relation, the following is a test of a Weston dynamometer voltmeter, the coefficient of mutual induction of which was experimentally determined for various positions of the moving coil as indicated by a scale divided in degrees fixed in front of the pointer. The resistance of the instrument was 2,084 ohms.

TABLE XXXIV.

Volts.	$\theta°$.	M (millihenries).	C (milli-amps.)	$\frac{\delta M}{\delta \theta}$	Torque. Calculated.	Torque. Observed.
0	0	−5·374	0	0	0	0
43·5	11·5	−4·257	20·85	0·097	0·0264	0·0265
60·2	25	−2·742	28·9	0·112	0·0581	0·0572
78	43·5	−0·3975	37·4	0·1266	0·104	0·1
94	61·75	+1·93	45·1	0·1276	0·152	0·142
107	75	+3·5	52·5	0·1185	0·191	0·172
120	84·5	+4·624	57·5	0·1185	0·228	0·194

Although the agreement in this case is not as close as might be desired, it is ample for the justification of the formula.

Approximate Calculation of Mutual Inductance

There are two simple cases in which the coefficient of mutual inductance can be approximately calculated with ease, and these serve as a guide to us in other instances.

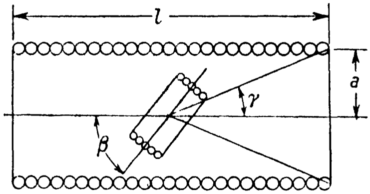


FIG. 8.47.—Calculation of mutual inductance between coils.

(1) *Long Helix and Internal Moving Coil.*—When a current of strength I_1, circulating in the long helix (Fig. 8.47) of N_1 turns, in a length of l cm., the field in the centre of the coil is uniform and is

$$H = \frac{4\pi I_1 N_1}{l}$$

If this passes through a moving coil of area A and N_2 turns, the plane of which is inclined at an angle β to the axis of the helix, the number of linkages,

$$\Phi N_2 = HAN_2 \sin\beta = \frac{4\pi A I_1 N_1 N_2 \sin\beta}{l}$$

and the coefficient of mutual inductance $M = \frac{\Phi N_2}{I_1}$, or $M = 4\pi\frac{A}{l} N_1 N_2 \sin\beta$ C.G.S. units, and $\frac{4\pi}{10^9}\frac{A}{l} N_1 N_2 \sin\beta$ henries, from which we have

$$\frac{dM}{d\beta} = \frac{4\pi}{10^9}\frac{A}{l} N_1 N_2 \cos\beta$$

In actual practice the helix is never extremely long in comparison with its diameter, and the field inside it is therefore smaller and non-uniform, but the value of H at any point is $\frac{I_1 N_1}{l}\Omega$, where Ω is the solid angle subtended by the surface of the winding at the point considered. At the centre of the helix $\Omega = 4\pi\cos\theta$, where θ

is the angle of the cone which the end of the helix makes at the centre, and hence

$$\frac{dM}{d\beta} = \frac{4\pi}{10^9}\frac{A}{l} N_1 N_2 \cos\theta \cos\beta$$

$$= \frac{4\pi}{10^9}\frac{A}{\sqrt{l^2 + 4a^2}} N_1 N_2 \cos\beta$$

where a is the radius of the helix.

(2) *Large Circular Coil.*—The field at the centre of a large circular coil of radius a and N_1 turns, carrying a current I_1, is

$$H = \frac{2\pi I_1 N_1}{a}$$

If the moving coil is relatively small, so that the field passing through it is approximately uniform, then the number of linkages

$$\Phi N_2 = \frac{2\pi A I_1 N_1 N_2 \sin\beta}{a}$$

$$M = \frac{2\pi}{10^9}\frac{A}{a} N_1 N_2 \sin\beta$$

from which $$\frac{dM}{d\beta} = \frac{2\pi}{10^9}\frac{A}{a} N_1 N_2 \cos\beta$$

It will be seen that this formula is equivalent to that obtained for a helical fixed coil if the length is small compared with its diameter, and that both formulae reduce to

$$\frac{dM}{d\beta} = \frac{2\pi}{10^9}\frac{A}{D} N_1 N_2 \cos\beta$$

where D is the oblique distance from the centre of the moving coil to the edge of the winding of the fixed coil.

In the case of coils with large winding dimensions in comparison with the radius the formulae are of course very complex, but it will serve for rough purposes of design to take D as the oblique distance to the centre of the edge of the winding, and A as the mean area of the moving coil.

In order to verify the above conclusions experimentally, two solenoids, each having a mean diameter of 5·4 cm., were wound with six separate sections, each of 100 turns, so that when brought

end to end they formed a continuous solenoid of 22 cm. length and having 1,200 turns.

At the place where the two coils joined a spindle was arranged to which a search coil could be attached with its plane vertical to the axis of the fixed solenoid. The angular position of this coil relatively to the axis of the fixed coil could be observed on a horizontal scale of degrees. Search coils of various diameters could be employed, and using the fixed coil as primary and the moving coil as secondary, the mutual induction between the two was measured by direct comparison with a Campbell variable standard of mutual induction, using an alternating current and a vibration galvanometer as a detector.

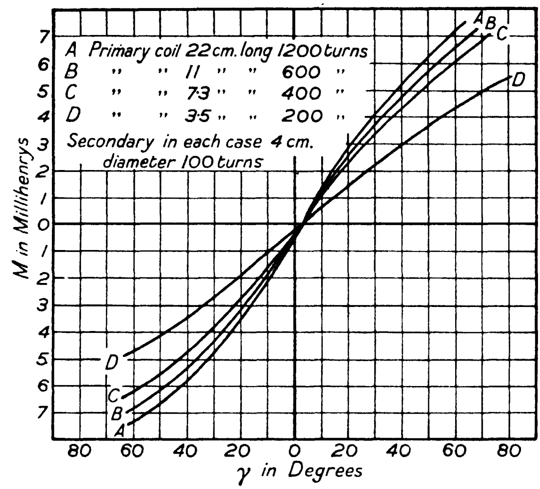


FIG. 8.48.—Curves giving relation between mutual induction and angle for concentric coils, varying length of primary.

The curves shown in Figs. 8.48 and 8.50 were obtained in this way, and show the variation of M with the inclination of the plane of the search coil. Fig. 8.48 shows the effect of increasing the length of the primary or fixed coil, the search coil being 4 cm. in diameter and kept constant.

Fig. 8.50, on the other hand, gives the effect of reducing progressively the moving coil diameter in a fixed coil of constant dimensions, and Fig. 8.49 shows the effect of separating the two halves of the fixed coil from 5 to 60 mm.

Even under these widely varying conditions the above formula is justified by the experimental results, as shown in Table XXXV (page 492).

The agreement between the observed and calculated results is fairly good in most cases, and where considerable discrepancy occurs this is often due to the difficulty of satisfactorily determining the tangent to the curve.

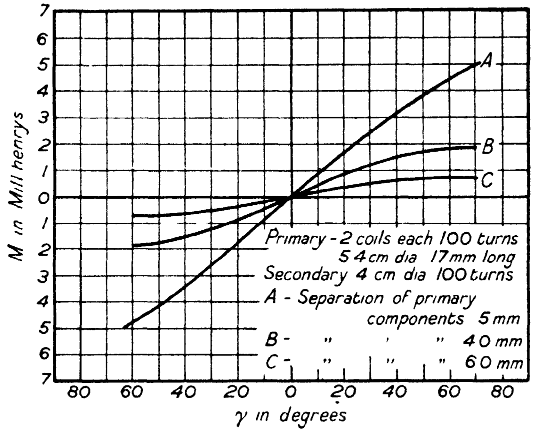


FIG. 8 49 —Curves showing relation between M and angle for concentric coils with increasing separation of two halves of primary.

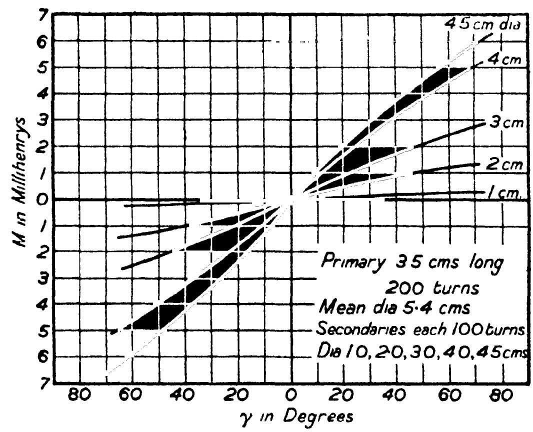


FIG. 8.50.—Curves showing relation between M and angle for concentric coils, varying diameter of secondary

An interesting point arises in connection with the above where an open short coil is carrying current as to the distribution of flux within the area of the fixed coil.

TABLE XXXV.

Arrangement of coils.	Length of primary.	Values of δM millihenries. 10° inclination. Calculated.	10° inclination. From curve.	60° inclination. Calculated.	60° inclination. From curve.
Increasing primary length. Secondary 4 cm. dia. (Fig. 8.51)	22 cm.	8·23	7·9	4·18	4·05
	11 ,,	7·61	7·4	3·86	4·05
	7·3 ,,	6·55	6·65	3·48	3·95
	3·5 ,,	4·83	4·45	2·46	3·3
	Dia. of secondary.				
Increasing secondary diameter (Fig. 8.52)	4·5 cm.	6·1	5·6	3·1	3·2
	4·0 ,,	4·825	5·05	2·45	3·4
	3·0 ,,	2·72	3·0	1·38	1·75
	2·0 ,,	1·21	1·5	0·615	0·75
	1·0 ,,	0·31	0·45	0·535	0·2
	Separation				
Separating primary components (Fig. 8·53)	5 mm.	4·67	4·55	2·36	3·4
	40 ,,	3·39	3·2	1·72	0·9
	60 ,,	2·86	1·3	1·45	0·55

It is possible, by the use of elliptic integrals, to calculate this for a simple ring coil, but the form of the curve can be very easily obtained experimentally. Fig. 8.51 is such a curve obtained

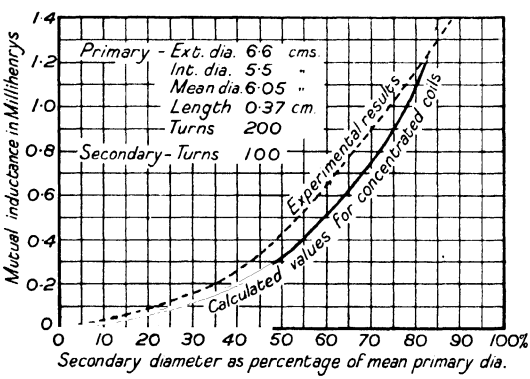


FIG. 8.51.—Experimental and calculated values of M for coaxial concentric coils varying secondary diameter.

by using a single fixed primary of 200 turns and a set of secondary coils, each of 100 turns, wound on a stepped cone, varying in diameter from 0·65 cm. to 5·2 cm., in eight equal steps. The largest diameter would just pass into the primary. Each of these coils was brought centrally into the primary, and the mutual induction between them was measured, and the curve indicates how rapidly the field increases as we pass from the axis toward the inner face of the fixed coil.

With regard to the best shape of coil for use in dynamometer instruments, Young has shown that for the same length of wire the ratio of the average torque of a circular coil to the average torque of a square coil is equal to the fraction $\frac{4}{\pi}$, or there is approximately a 27% increase in torque for the circular coil in a uniform field, the square coil giving a maximum torque when compared with any other rectangular form of coil.

Example of the Design of a 120-Volt 10-Ampere Single-phase Wattmeter

As an example, let us take the design of a dynamometer wattmeter of the simple circular coil type and of the above range.

In order that it shall be sufficiently robust, and to keep the frictional error within moderate limits, we may take the maximum torque as 0·2 gm.-cm. (for a 90° scale), and the ratio of torque to weight as 0·05, from which the weight of the whole moving system should not exceed 4 grammes, of which 2 grammes may provisionally be taken as the weight of the wire on the coil. If we also assume a periodic time τ of 1 second, then since $\tau = 2\pi\sqrt{\frac{I}{\psi}}$, where I is the moment of inertia in gramme cm.^{2} and ψ the torsional moment in dyne-cm. per radian, we have $I = \frac{\tau^2}{4\pi^2}\psi$.

In this case $\psi = \frac{2}{\pi} \times 0{\cdot}2 \times 981 = 135$ dyne-cm. per radian, and $I = \frac{1}{4\pi^2} \times 135 = 3{\cdot}4$ gm.-cm.^{2}, of which we may provisionally take the inertia of the coil itself as 2 gm.-cm.^{2}

The moment of inertia of a ring coil $I = 1/2\ ma^2$, where m is

the mass and a the radius of the coil, so that the mean radius $a = \sqrt{\frac{2I}{m}} = \sqrt{\frac{2 \times 2}{2}} = 1{\cdot}4$ cm., or the mean diameter of the moving coil should be 2·8 cm.

To fix the winding on the moving coil, we have to consider the maximum allowable current, the maximum time constant of the circuit for accurate indication on low power factors, and the amount of swamping of the temperature variation of the coil resistance. Let us take the maximum shunt current as 0·03 ampere at 120 volts, equivalent to an expenditure of 3·6 watts in the shunt circuit, which means a total shunt circuit resistance of 4,000 ohms. If, further, the greatest error in the power factor is not to exceed 0·001 at 100 periods per second, then, as we have seen, there must be 628 ohms for every millihenry of shunt circuit inductance, and the latter must not, therefore, exceed 4,000/628 = 6·4 millihenries. The volume of copper on the coil is 2/8·9 = 0·225 c.c., which, with a mean diameter of 2·8 cm., or periphery of 8·8 cm., gives a cross-section of copper of 0·0255 sq. cm. Taking the space factor of the windings as 0·2, this gives a winding area of 0·125 sq. cm., or 3·5 mm. square = 0·137 inch square.

The ratio of $\frac{b}{a}$ for the coil is therefore 0·35/1·4 = 0·25, and the inductance of the coil $L = KN^2a \times 10^{-9}$, where $K = 28{\cdot}5$.

$$\text{Consequently } N = \sqrt{\frac{L \times 10^9}{Aa}} = 1{,}000\sqrt{\frac{6{\cdot}4}{28{\cdot}5 \times 1{\cdot}4}} = 400$$

turns, or 20 layers of 20 turns each, and the covered diameter of the wire must not exceed 137/20 = 6·3 mils. The moving coil can, therefore, be wound with 400 turns of No. 43 S.W.G. wire, 3·6 mils. diameter bare or 5·5 mils. double silk covered. The periphery of the coil is 8·8 cm. or 3·5 in., so that the total length of wire is $\frac{3{\cdot}5 \times 400}{36} = 39$ yd., which at 2·36 ohms per yard gives a total copper resistance of 92 ohms, and a weight of 2·1 gm. of copper. The ratio of copper resistance to total resistance is 92/4,000 = 0·09% per 1° C.

Coming next to the main coils, since the external diameter of the moving coil is 28 + 3·5 = 31·5 mm., we may make the internal diameter of the main coils 35 mm. If we also take the total axial length of the fixed coils as about twice the diameter of the moving

coil, or say 60 mm., to secure a moderately strong and uniform field, then from the relation

$$T = 10{,}200\, I_1 I_2 \frac{dM}{d\beta}$$

we have

$$\frac{dM}{d\beta} = \frac{T}{10{,}200\, I_1 I_2} = \frac{0{\cdot}2}{10{,}200 \times 0{\cdot}03 \times 10}$$
$$= 6{\cdot}5 \times 10^{-5} \text{ henries per radian,}$$

and since $\frac{dM}{d\beta} = \frac{2\pi}{10^9} \frac{A}{D} N_1 N_2 \cos\ \beta$, where A is the area of the moving coil $= \pi \times (1{\cdot}4)^2 = 6{\cdot}2$ sq. cm., and D, the oblique distance is $\sqrt{3^2 + 2^2} = 3{\cdot}6$ cm., $N_2 = 400$ and $\cos\beta = 0{\cdot}707$ at $45°$, therefore, we have $N_1 = \frac{10^9}{2\pi} \frac{D}{AN_2} \cos\ \beta \frac{dM}{d\beta} = \frac{10^9}{2\pi} \cdot \frac{3{\cdot}6 \times 6{\cdot}5 \times 10^{-5}}{6{\cdot}2 \times 400 \times 0{\cdot}707}$ $= 21{\cdot}2$, or, say, 22 turns or 220 ampere turns; if, therefore we divide the main coil into two sections, each will have 11 turns.

Taking the wire as No. 12 S.W.G. 104 mil. bare and 110 mil. covered, this gives an axial length for 11 turns in one layer of 1·21 in. or 31 mm. The mean diameter of the coil is $35 + 2{\cdot}5 = 37{\cdot}5$ mm., from which the length of wire is 11·8 cm. or 4·65 in. per turn, or 2·84 yd. total, which at 2·83 ohms per 1,000 yd. gives a resistance of 0·008 ohm, or, say, 0·01 ohm with connections.

The drop of potential in the main coil is therefore 0·1 volt, and the power consumption 1 watt at 10 amperes, which is quite satisfactory. The electrical conditions are, therefore, completely satisfied.

As regards the mechanical design, the same considerations hold as in permanent magnet moving coil instruments, except that we must fall back on air or liquid damping instead of magnetic damping.

THE ASTATIC DYNAMOMETER

In the previous case the dynamometer is non-astatic. The design of an astatic system is more difficult, and some of the forms do not lend themselves easily to calculation. It is fairly obvious also that the effectiveness of an astatic system is always less than that of a non-astatic one.

The simplest astatic system is that used by Duddell and Mather in their wattmeter (Fig. 8. 19), where two simple non-astatic systems are mounted one above the other. In this case calculation

is similar to that just given, but it is evident that the power loss in both the main and pressure circuits is much increased for the same weight of moving system. This arrangement is also unsuitable for polyphase wattmeters, as the axial length of the whole system would become excessive.

Next in simplicity, probably, comes the astatic arrangement in the Drysdale wattmeter (Fig. 8.18). This is similar to the simple non-astatic form, but one main coil is reversed, and the moving coil is divided into two halves wound in opposite directions. Here the main field is reduced for the same number of ampere turns, owing to the opposing action of the two halves of the main coil.

To reduce this it is advantageous to separate the two components of the main core more widely and to make the moving system in the form of two rectangular coils, as shown. If the coils of the current circuit are also rectangular in form, the sides being of length b and b_1 respectively, then at an axial distance D

$$\Omega = \frac{2b}{\sqrt{D^2 + \frac{b^2}{4}}} \arc\tan \frac{b_1}{2D}, \text{ or } = \frac{2b_1}{\sqrt{D^2 + \frac{b_1^2}{4}}} \arc\tan \frac{b}{2D} \text{ approxi-}$$

mately (Fig. 8.52). Hence the field at the centre of such a coil of N_1 turns and axial length l is

$$H = \frac{N_1 I}{l}\{4\pi - 2\Omega\} = \frac{4N_1 I}{l}\left\{\pi - \frac{b}{\sqrt{l^2 + b^2}} \arc\tan \frac{b_1}{l}\right\}$$

If we have a second coil of similar shape, the axial distance between the centres of the coils being D_1, then we have for the field due to the second coil at the centre of the first,

$$H_1 = \frac{N_1 I}{l}\left\{\frac{2b}{\sqrt{\left(D_1 - l/2\right)^2 + \frac{b^2}{4}}} \arc\tan \frac{b_1}{2D_1 - l} - \frac{2b}{\sqrt{\left(D_1 + l/2\right)^2 + \frac{b^2}{4}}} \arc\tan \frac{b_1}{2D_1 + l}\right\}$$

$$= \frac{2bN_1 I}{l}\left\{\frac{\arc\tan \frac{b_1}{2D_1 - l}}{\sqrt{(D_1 - l/2)^2 + \frac{b^2}{4}}} - \frac{\arc\tan \frac{b_1}{2D_1 + l}}{\sqrt{(D_1 + l/2)^2 + \frac{b^2}{4}}}\right\}$$

Hence the total field at the centre of one coil is

$$H \pm H_1 = \frac{4N_1I}{l}\left\{\pi - \frac{2 \text{ arc tan } \frac{b_1}{l}}{\sqrt{l^2 + b^2}} \pm \left(\frac{\text{arc tan } \frac{b_1}{2D_1 - l}}{\sqrt{(D_1 - l/2)^2 + \frac{b^2}{4}}} - \frac{\text{arc tan } \frac{b_1}{2D_1 + l}}{\sqrt{(D_1 + l/2)^2 + \frac{b^2}{4}}}\right)\frac{b}{2}\right\}$$

the + sign being for assisting fields and the − sign for astatic combination.

If A is the total area of the rectangular moving coil (the central conductors in the astatic form being neglected as being near the axis and in a weak field),

$$M = \frac{4N_1N_2A}{l}\left\{\pi - \frac{2b \text{ arc tan } \frac{b_1}{l}}{\sqrt{l^2 \times b^2}} \pm \left(\frac{\text{arc tan } \frac{2D_1 - l}{b_1}}{\sqrt{(D_1 - l/2)^2 + \frac{b_2}{4}}} - \frac{\text{arc tan } \frac{2D_1 + l}{b_1}}{\sqrt{(D_1 + l/2)^2 + \frac{b_2}{4}}}\right)\frac{b}{2}\right\} \sin \beta$$

This expression seems somewhat complicated, but in reality it is only slightly less simple than that given for the circular coil. If we draw the plan and elevation of the system as in Fig. 8.52 it is evident that the values under the square root signs, are simply the diagonal distances D_2, D_3 and D_4, and the arc tangents are the angles θ, θ_3 and θ_4.

The expression therefore becomes

$$M = \frac{4N_1N_2A}{l}\left\{\pi - \frac{\theta}{D_2}b \pm \left(\frac{\theta_3}{D_3} - \frac{\theta_4}{D_4}\right)\frac{b}{2}\right\} \sin \beta$$

and $$\frac{dM}{d\beta} = \frac{4N_1N_2A}{l}\left\{\pi - \frac{\theta}{D_2}b \pm \left(\frac{\theta_3}{D_3} - \frac{\theta_4}{D_4}\right)\frac{b}{2}\right\} \cos \beta$$

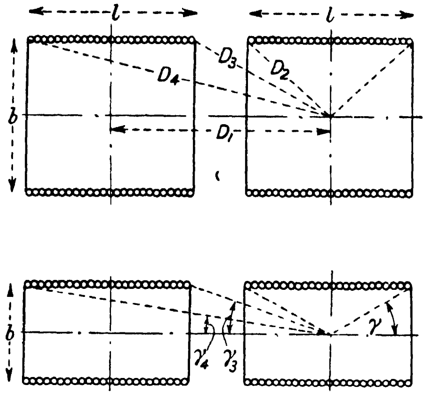


FIG. 8.52.—Plan and elevation of astatic coil system.

Thus, for example, if $D_1 = 5$, $l = 3$, $b = 4$, $b_1 = 5$, $N_1 = 100$, and $N_2 = 100$,

$$\frac{dM}{d\beta} = \frac{4 \times 100 \times 100 \times 15}{3} \left\{ \pi - \frac{0\cdot93}{2\cdot5} 4 \pm \left(\frac{0\cdot524}{4} - \frac{0\cdot29}{6\cdot8} \right) 2 \right\} \cos \beta$$

$$= 2 \times 10^5 \{ \pi - 1\cdot49 \pm (0\cdot131 - 0\cdot043)2 \} \cos \beta$$

$$= (3\cdot3 \pm 0\cdot36)\, 10^5 \cos \beta.$$

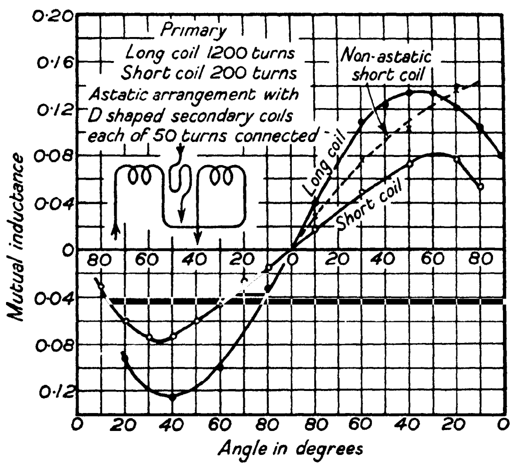


FIG. 8.53.—Curves showing relation between M and angle for circular primary and double D-shaped secondary astatically connected.

From this it appears that the effect of the second coil in this case is only 10% of that of the first, so that the astatic arrangement only gives about 20% less torque for the same turns as the non-astatic arrangement.

These conclusions were experimentally verified with the model used in the previous experiments. In this case one half of the primary or fixed coil was reversed, and the moving coil constructed of two D-shaped coils, each of 50 turns, connected in series. The experimental values of M were plotted in Fig. 8.53 against the angle, and the non-astatic equivalent is also plotted with them for purposes of comparison, and it will be seen that the loss due to the astatic arrangement is very marked in the case of the short fixed coil practically over the whole range, whilst with the long primary coil the curves are practically coincident over half the range but a rapid departure occurs at each end.

Table XXXVI contains some experimental data for some typical dynamometer instruments.

CHAPTER 9.

HOT-WIRE INDICATING INSTRUMENTS

ALTHOUGH inferior to other types of instruments as regards accuracy and ease of construction, hot-wire instruments have always had a considerable vogue, and this has been increased of recent years by the demand for instruments for use in radio-telegraphic measurements, for which purpose hot-wire instruments are indispensable. In principle they are the most perfect instruments for alternating current measurement, as their indications depend upon the heat developed by the passage of the current through a conductor, and are thus practically rigidly dependent on the mean square of the current ; and they are more free than any other current-measuring instrument from inductive errors, as the current simply passes through a straight conductor.

Against these advantages, however, must be set the disadvantages of their being somewhat complicated to construct on account of the necessity for magnifying the very small expansions produced by the heating, and they are usually somewhat sluggish and indefinite in their indications, besides taking a considerable amount of power ; and owing to the fact that the working wire has to be run at a fairly high temperature, they have very little overload capacity. They are also liable to errors from changes of external temperature, against which they must be compensated, and they are, of course, affected by currents of air unless effectively screened. Care must also be taken always to use the instrument in the position in which it was calibrated, as otherwise its indications will be altered by the difference in the convection currents of the surrounding air.

Hot-wire instruments may be divided into two classes :

(1) *Expansion Instruments*, in which the deflection is produced by the linear expansion of the current-carrying conductor (Cardew, Hartmann & Braun, Holder, Whitney, etc.).

(2) *Thermo-junction Instruments*, in which the indication depends upon the E.M.F. produced by a thermo-junction heated by the thermal conduction or radiation from the current-carrying conductor.

In addition to these, other devices are possible, as, for example, the heating of a thermometer bulb by the passage of the current,

but none other than those mentioned above appear to have reached a practical stage, mainly owing to the difficulty of compensating the changes in the external temperature.

Elementary Theory of Hot-wire Instruments

When a current I amperes passes through a conductor of resistance R ohms, the power expended

$$P = I^2R \text{ watts, or } I^2R \times 10^7 \text{ ergs per sec.}$$

and since the mechanical equivalent of heat $J = 4{\cdot}2 \times 10^7$ ergs per calorie (gramme °C.), heat developed in calories per sec.

$$h = \frac{I^2R \times 10^7}{J} = 0{\cdot}24I^2R$$

In order to secure sufficient flexibility the conductor is always made of small area, and consequently attains a steady temperature very soon after the current is flowing. When this is the case the whole of the heat developed in the wire is lost by conduction, convection and radiation, and if the wire is long and fine the last two factors only are of importance.

If s is the area of surface of the wire in square centimetres and θ the rise of temperature above the surrounding air, it is customary to represent the loss of heat by Newton's law of cooling as $Us\theta$, where U is the coefficient of emissivity, or loss of heat in calories per sq. cm. per sec. per 1° C. excess of temperature above the atmosphere. The coefficient U is unfortunately only constant for a few degrees rise of temperature, but taking it as constant, we have, as soon as the temperature is steady,

$$h = 0{\cdot}24I^2R = 0{\cdot}24\frac{V^2}{R} = Us\theta$$

from which

$$\theta = \frac{0{\cdot}24R}{Us}I^2, \text{ and } \theta = \frac{0{\cdot}24}{UsR}V^2$$

or the rise of temperature, if small, is proportional to the square of the P.D. or to the square of the current, assuming U and R to be constant.

(1) Expansion Instruments

In the case of instruments in which the expansion of the conductor is utilized to produce the deflection, the change of length

$\Delta l = l_0 a_H \theta$, where l_0 is the original length and a_H the coefficient of expansion of the conductor.

Hence
$$\Delta l = l_o\,\theta a_H = \frac{0{\cdot}24 l a_H R}{Us}\,I^2 = \frac{0{\cdot}24 l_o a_H}{UsR}\,V^2$$

so that the increase of length of the conductor is proportional to the square of the current or P.D.

With a rapid periodic alternation of the supply, the expansion is therefore proportional to the mean square of the current, so that if the instrument is calibrated with direct currents, it will indicate the square root of the mean squares of an alternating current or P.D.*

In the first instance, therefore, we should desire $l_o\,\theta a_H$ to be as large as possible, but unfortunately coefficients of expansion are not large, one of the largest, that of zinc, being only 29·2 millionths of its length per 1° C. If we increase l_o unduly the instrument becomes unwieldy, and θ cannot be pushed too far without loss of elastic properties in the working wire, oxidation, limit of overload capacity, and other objectionable features. It is therefore obvious that a practical, compact instrument must necessarily employ some device which will, for a comparatively short wire, magnify the small increase in length and at the same time convert it from a linear into a rotary motion. There is also the further difficulty of obtaining a sufficiently high internal resistance in voltmeters, and of yet having a sufficiently large current to produce the required temperature rise.

The reduction in diameter of the working wire can only be carried to a practical minimum, since, apart from the loss in mechanical strength, the handling of very fine fibres would become unduly expensive. On the other hand, it is imperative that the heat capacity should be reduced to its smallest possible value, in order that we may obtain quick response to variations in the measured quantity.

These considerations all tend to make this type of instrument very wasteful of power, and the "watts lost" in the instrument are therefore extremely high.

* It should be observed, however, that the correctness of indication of a hot-wire instrument with alternating current does not depend upon any particular law of cooling, provided that the frequency of alternation and the heat capacity of the wire are sufficiently high for there to be no sensible fluctuation of temperature during the period. In this case the form of the scale depends upon the law of cooling, but if the instrument is calibrated with direct current it will read correctly with A.C. supply.

Again, since the heating varies as the square of the working current, the scale is necessarily one which closes very rapidly towards zero, and, under these circumstances, we require great certainty of zero in order to obtain accurate low readings. This is, unfortunately, not the case in hot-wire instruments, since both local internal heating from the working wire and external changes of temperature will cause expansion of the supports of the wire itself, altering the normal zero tension and thus producing a shift of zero. Every instrument must, therefore, possess some device by which the zero may be readily set, and some form of compensation is usually attempted by making the supports in such a way that their coefficient of expansion is the same or neutralizes that of the working wire. But this expedient is not always quite satisfactory where rapid changes of temperature are likely to occur, since the thermal capacity of the supports is, in most cases, enormously greater than that of the working wire, and consequently they lag very considerably behind it in taking up the new temperature.

Magnification

However long the wire may be made, or however high its temperature, the expansion is too small to actuate the pointer spindle directly, and recourse must be made to some magnifying device. The following are the chief devices of this kind, and their employment may serve as a classification for the various instruments :

1. Wheel and pinion (Cardew).
2. Lever (Vulcan, Chauvin & Arnoux, Whitney).
3. Sag—Single (Cie des Compteurs, Holden)—Double (Hartmann & Braun).

The Cardew Wheel and Pinion Hot-wire Voltmeter

One of the first practical instruments of this type was invented by Major Cardew, and described in 1887. He employed a platinum-silver wire about 396 cm. long and 0·0635 mm. in diameter. This long wire was strung backwards and forwards on a system of pulleys in the manner indicated diagrammatically in Fig. 9.1. The central loop was carried by a little pulley *p* held by a cord, which was fastened to a drum *d* mounted between pivots. A tension on this cord and pulley, and therefore on the working wire itself, was

maintained by a second cord fastened to the drum and terminating in a spiral spring *s*, so that the two cords, leaving the drum tangentially, tended to rotate it in opposite directions. If, now, the working wire expanded, the little pulley in the loop would ascend and the slack in its cord would be taken up by the rotation of the drum due to the tension produced by the fixed spring. The drum

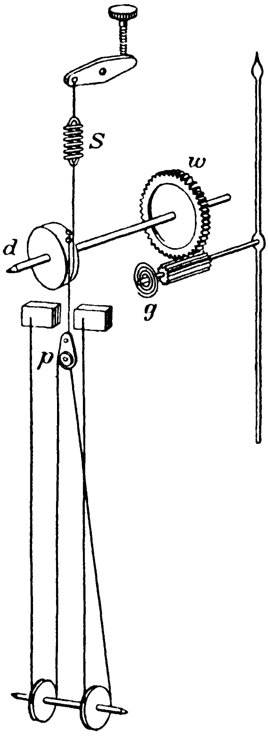


FIG. 9 1.—Principle of Cardew voltmeter.

spindle carries a spur wheel *w* gearing into a pinion on the pointer shaft, which is further provided with a control spring *g* to take up any backlash in the gearing.

The movements of the little pulley *p* are thus magnified up and a long circular scale results. It will be observed, however, that these movements are due to the expansion of only half the total length of the working wire, but by utilizing the double arrangement the internal resistance of the instrument is increased and the

rapidity of response to change of temperature greatly improved. The range may be doubled by employing a second similar wire stretched in the same way as the working wire and connected in series with it, but not so as to contribute anything to the movement of the mechanical system. This wire, therefore, simply acts as a dead resistance, but it changes in resistance in exactly the same way as the working wire, and is therefore always the same multiple of it. Compensation for changes of temperature were effected by making the rod or tube supports of the pulley system partly of brass and partly of steel, so that their mean coefficient of expansion was equal to that of the working wire. Zero setting was accomplished by mounting the spring *s* on the end of a little rocking lever, which can be tilted by means of a screw pushing against the other end and operated from outside the case.

In one type of instrument the wire was strung on the frame, the side rods of which were made compound in the manner indicated above, while in the other case the tube itself formed the support of the lower pulleys and was therefore compound. The presence of this long tube was a serious defect, making the instrument unwieldy and difficult to mount satisfactorily on the switchboard.

Owing to the fact that instruments of this type will indicate equally well on direct or alternating current circuits, and that they are free from the errors of frequency, wave form, hysteresis, and stray fields, efforts were made to reduce them to a more compact form for switchboard work, and several types were made available which operate with a working wire short enough to be contained in the usual sizes of case employed for switchboard instruments.

Lever Hot-Wire Instruments.—Fig. 9.2 shows diagrammatically a Chauvin & Arnoux type of instrument. In this the metal framework is constructed in two parts, consisting of a lower inverted **T**-shaped piece *A*, the central leg of which supports at its upper end the crossbar *B*, which can rock upon it. These two are held in contact by the rod and spring *C* attached to one end and the compensation wires *D* at the other. The working wire *E* is attached to the upper bar *B* at the point *F*, and terminates at the crossbar *G*, which rocks about the pivot extension in *A*. At the other end of *G* is attached a fibre which, after taking a turn round the little pulley on the pointer spindle, is anchored by means of a light spring to a second extension on the upright of *A*. Temperature compensation is effected by means of the wires *D*, which are of

the same material as the working wire, and therefore have the same coefficient of expansion, and the zero adjustment is made by means of the nut and screw to which C is attached. Since D consists of a number of fine wires, it readily responds to changes of external temperature, and consequently the lag of the compensation is greatly reduced and zero changes become less troublesome.

For voltmeters the working wire is placed in series with a non-inductive resistance, which limits the current through the combination to 0·1 ampere for full-scale deflection, and for ammeters

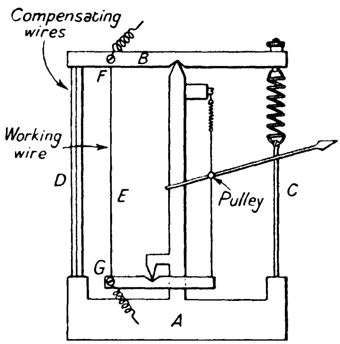


FIG. 9.2.—Principle of Chauvin & Arnoux hot-wire voltmeter.

the wire is shunted with a low resistance which gives a drop of 0·15 volt for currents above 4 amperes for full-scale deflection. The shunts are separate from the instruments for ranges above 10 amperes, and are made interchangeable.

Another form of instrument devised by Roller, and made by the Whitney Company, is shown diagrammatically in Fig. 9.3. In this the working wire is arranged in the form of a loop which, at its upper end, passes round a pulley on a horizontal pivoted shaft, whilst the ends are secured to a metal plate below in such a way that one end is in metallic connection while the other is insulated. This lower plate is attached to a spring which exerts an equal tension on both sides of the loop, and the attachment plate is so guided that the direction of the pull is always at right angles to the axis of the upper pulley shaft. This also carries a light arm, forked at its lower end and counterweighted at its upper one.

Between the limbs of the fork is a second pivot axis which carries

a little pulley, and a silk fibre attached at each end to the extremities of the fork takes a turn round the pulley, whose shaft also carries the instrument pointer as shown. The limbs of the fork are made sufficiently springy to keep the silk fibre taut enough to prevent slip on the pulley surface.

If current is now sent up one side of the loop, say in at the anchor plate and out at the upper pulley, one side will be heated and expand, and the resulting difference in tension between the two

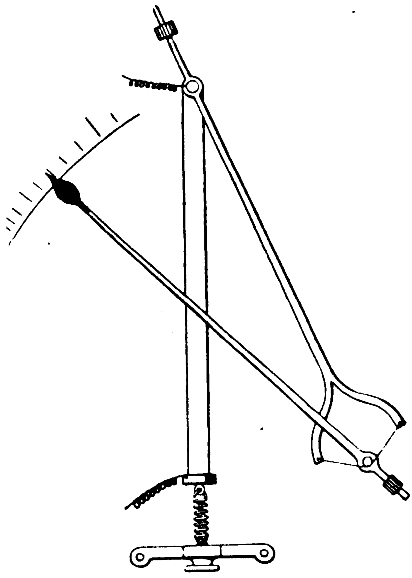


FIG. 9.3.—Principle of Whitney hot-wire voltmeter.

sides is equalized by rotation of the upper pulley; this causes the forked arm to move through a similar angle and rotate the pointer axis by means of the silk fibre.

Compensation for external temperature change should here be practically perfect, since any change in external temperature will affect both sides of the loop equally, and therefore no rotation should result, since the compensation wire is the unused side of the loop, and is identical, therefore, with the working wire both in coefficient of expansion and in heat capacity.

The entire movement is mounted on a base plate which is carried on a heavy bearing, and is capable of rotation through a small

angle about the centre coincident with that of the pointer spindle, and by this means the pointer may be adjusted to zero.

Sag Type Instruments

Instead of directly utilizing the increase of length of the heated wire, the ends may be permanently attached to two supports and the increase in sag employed to actuate the indicating mechanism, for it is obvious that for a small increase in length there will be a very appreciable increase in the sag.

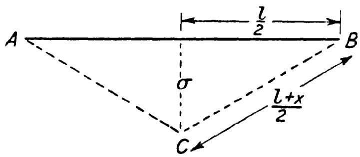


FIG. 9.4.—Diagram to illustrate amount of sag.

Consider the case shown in Fig. 9.4, and let us suppose the working wire when cold is stretched between the points A and B, and at this temperature has a length l. If, now, at the point C a fibre is attached and kept under tension, then when the wire is heated it increases its length to $l + x$ and is pulled into the form shown in the figure and we have

$$\left(\frac{l+x}{2}\right)^2 - \frac{l^2}{4} = \sigma^2, \text{ from which } \sigma^2 = \frac{1}{2}lx + \frac{1}{4}x^2$$

Now x is very small compared with l, so that the last term may be neglected and we may write

$$\sigma^2 = \frac{1}{2}lx$$

and since the expansion

$$x = \Delta l = \frac{0{\cdot}24\ l_o a_H R}{Us} I^2 = \frac{0{\cdot}24\ l_o a_H}{UsR} V^2$$

$$\sigma^2 = \frac{0{\cdot}24\ a_H R}{Us} l^2 I^2 = \frac{0{\cdot}24\ a_H}{2UsR} l^2 V^2$$

and

$$\sigma = \sqrt{\frac{0{\cdot}2\ 4a_H R}{2Us}}\, lI = \sqrt{\frac{0{\cdot}24\ a_H}{2UsR}}\, lV$$

From this we have the interesting conclusion that the sag is directly proportional to the current or P.D., and if the fibre is wound round a pulley the angular deflection will be proportional to the current or P.D., and the scale will be uniform.

The advantage gained by this method can be seen at once by a numerical example. Suppose that the working wire is of glatinum silver 20 cm. long and having a coefficient of expansion a_H of 0·0000162, and that its working temperature is 100° C., the air temperature being 15° C., then the total expansion $x = 20 \times 0{\cdot}0000162$ $(100 - 15) = 0{\cdot}0275$ cm., and $\sigma = \sqrt{\tfrac{1}{2}lx} = \sqrt{\tfrac{1}{2} \times 20 \times 0{\cdot}0275} =$ 0·525 cm., and the magnification of the expansion is $\dfrac{0{\cdot}525}{0{\cdot}0275} = 19{\cdot}1$ times.

Hence, by this simple device a 20-cm. wire will produce the same movement as about 4 metres of straight wire, with the advantage of a more even scale.

Unfortunately, however, these advantages cannot be fully realized in practice, because the pull of the control spring on the fibre produces an initial sag in the wire, and there is a tension in the working wire which is magnified to the same extent as the motion, and thus may produce a very considerable stress in so fine a wire.

If σ_0 is the initial sag in the wire and σ_1 that after heating,

$$\sigma_0{}^2 = \tfrac{1}{2}lx_0, \text{ and } \sigma_1{}^2 = \tfrac{1}{2}lx_1,$$

from which $$\sigma_1{}^2 - \sigma_0{}^2 = \tfrac{1}{2}l(x_1 - x_0) = \tfrac{1}{2}lx,$$

and $$\sigma = \sigma_1 - \sigma_0 = \sqrt{\tfrac{1}{2}lx + \sigma_0{}^2} - \sigma_0.$$

Therefore in the above example, with an initial sag of 0·1 cm.,

$$\sigma = \sqrt{\tfrac{1}{2} \times 20 \times 0{\cdot}275 \times 0{\cdot}1^2} - 0{\cdot}1 = 0{\cdot}435 \text{ cm.},$$

and the magnification is only $\dfrac{0{\cdot}435}{0{\cdot}0275} = 15{\cdot}8.$

At the same time the divisions become crowded at the lower end of the scale. Fig. 9.5 shows the effect of different initial sags of the working wire on the maximum reading and scale.

In the first case if the sag were communicated to the pointer shaft by a cord passing completely round a pulley on it, then the diameter d of this pulley would have to be for a 90° deflection

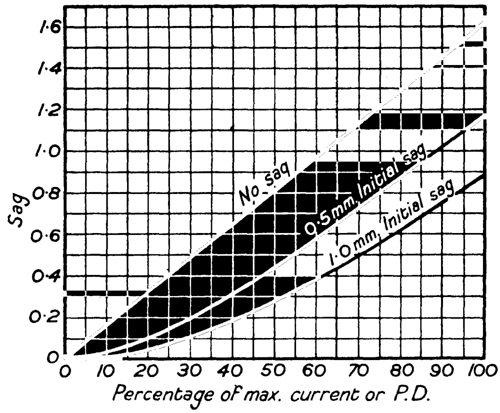


FIG. 9.5.—Curves showing effect of different initial sags.

$d = \frac{4 \times 0{\cdot}525}{\pi} = 0{\cdot}669$ cm., or $\frac{4 \times 0{\cdot}435}{\pi} = 0{\cdot}555$ cm., in the second instance.

The Compagnie pour la Fabrication des Compteurs constructed an instrument on this principle, the working parts of which are

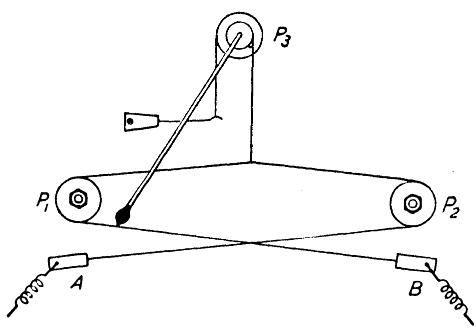


FIG. 9.6.—Principle of Cie. des Compteurs hot-wire voltmeter.

shown diagrammatically in Fig. 9.6. The working wire is fixed at the points A and B to two terminal pieces, and then passes round two fixed pulleys P_1 and P_2; at a point midway between these pulleys a fibre is attached to the working wire, and is carried off at right angles to a pulley P_3 on the pointer shaft. Tension is

kept on this fibre by means of a second thread fastened to a second pulley on the pointer shaft at one end, and held taut by a flat spring to which the other end is attached.

Temperature compensation was partly effected by mounting the fixed supports of the wire on a plate having the same coefficient of expansion as the working wire, and also by making the terminal tongues *A* and *B* of an alloy with a larger coefficient, so that by suitably choosing their lengths they expand inwards an amount equal to the outward expansion of the plate due to the local heating of the wire. The working wire in this case was of nickel-steel, and the working current for full-scale deflection 0·15 to 0·2 ampere. The fibres transmitting the motions of the working wire to the pointer shaft must be carefully chosen, since ordinary silk and cotton threads and cords are very susceptible to humidity, and will untwist and lengthen as the humidity increases. In some instances a hair has been employed, but it has very doubtful advantage. In this particular instrument the motions are transmitted to the pointer shaft by a fine metallic strip which is free from these objections, but involves additional insulation of the working parts.

Zero setting is effected by a screw operated from outside the case, which produces a slight deformation of the supporting plate.

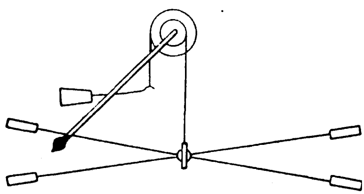


FIG. 9.7.—Principle of Cie. des Compteurs hot-wire ammeter.

The ammeter systems are arranged as shown in Fig. 9.7. Since it is necessary to reduce the P.D. to the lowest possible value across the working wire these are arranged in cross fashion. Thus the current enters the cross junction in the centre by means of a flexible lead and passes outwards to the terminals so that the wires are all in parallel, and the movements of the junction are communicated to a little lever to which the strip passing to the pulley on the pointer shaft is attached. Otherwise the details of the transmission arrangements are identical with the voltmeter. The working

wires in this case are an alloy of low resistivity, and the drop required for full-scale deflection is about 0·3 volt. The range is extended by the use of shunts or series transformers. The former are arranged as shown in Fig. 9.8, so that by cutting a broad flat plate in zigzag fashion the required drop is obtained with a reasonable span between the terminals.

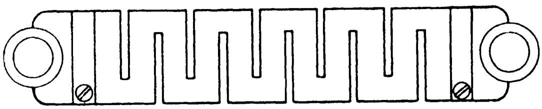


FIG. 9.8.—Shunt for Cie. des Compteurs hot-wire ammeter.

The Marconi Company also devised an ingenious form of hot-wire instrument, which probably should be classified as a single-sag instrument, the arrangements being shown in Fig. 9.9. In the smaller sizes the whole of the working parts were enclosed in a glass bulb which was exhausted ; two wires were employed, stretched vertically and parallel through the bulb, each joined at its lower end to a leading-in wire in the same fashion as the filament of an ordinary lamp, whilst, at their upper end, they terminated on a metal cross arm which was pivoted on a vertical axis, the jewels being carried on a glass framework sealed inside the bulb. The rotational axis was controlled by a spiral spring in the usual way.

The instrument worked on the bifilar principle : thus, consider two vertical parallel wires supporting a mass of material ; if a twist is given to the suspended mass, the fibres or wires exert a restoring couple on it, but in twisting the mass the suspension raises it by a small amount, and in effect, therefore, the suspending wires are shortened a little.

In the instrument under consideration the two wires attached to the cross arm were twisted through an angle, the torque produced being balanced by the control spring ; if now they became heated they elongated, and therefore the couple they exerted was reduced, and the spring therefore took the cross arm to a new position.

The ends of the cross arm are extended beyond the points to which the wires are attached, and are then bent sharply down at right angles and shaped to form pointers which nearly touch the inner surface of the cylindrical enclosing bulb, and the scale of the instrument is carried on a band around the bulb.

The exhausted enclosure had many obvious advantages, particularly as far as zero-keeping qualities are concerned, and the energy consumption was reduced to about one-third that of an equivalent non-enclosed type.

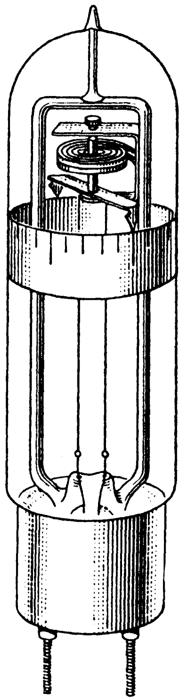


FIG. 9.9.—Marconi vacuum enclosed hot-wire voltmeter.

Thus an instrument having a cold resistance of 12·4 ohms required 0·11 ampere at 1·44 volts for full-scale deflection, the energy consumption being 0·158 watt; the smallest measurable current was 0·02 ampere, and the overload capacity 0·125 ampere. For an instrument of 30·2 ohms resistance the corresponding figures were 0·035 ampere at 1·25 volts, the energy consumption 0·043 watt, with a minimum current reading of 0·007 ampere.

Double Sag Instruments

The sag principle may be carried further, and a still greater magnification of the expansion of the working wire obtained by

utilizing a second similar system at right angles to the first, as in Fig. 9.10. Then if $\sigma = \sqrt{\frac{1}{2}l\,\Delta l}$ as before, and we attach a second

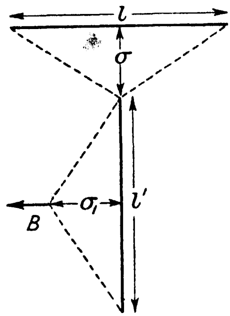


FIG. 9.10.—Principle of double-sag instrument.

system as shown in the figure, then we have by the same reasoning

$$\sigma_1 = \sqrt{\tfrac{1}{2}l_1\sigma} = \sqrt{\tfrac{1}{2}l_1}\,\sqrt[4]{\tfrac{1}{2}l\,\Delta l}$$

and if l_1 is equal to l this becomes

$$\sigma_1 = \sqrt[4]{\tfrac{1}{8}\,l^3\,\Delta l}$$

Thus, assuming the conditions of the former example, the movement of the point B is nearly 2·29 cm., and the magnification is approximately 83 times.

Were it not, therefore, for other considerations—as, for instance, the uncertainty of zero—we could reduce the working temperature of the wire, with the advantage of a smaller working current and expenditure of power in the instrument, whilst the emissivity would also be reduced and more constant and the mechanical condition of the wire improved.

It must, however, be remembered that the mechanical strain on the working wire is greatly increased by this magnification, and that, therefore, the considerations concerning the initial sag of the wire mentioned on page 509 apply with even greater force in this case. Were it not so, the scale of the double sag instrument would actually be more open at the bottom.

The double sag principle is realized in the Hartmann & Braun type of instrument which is illustrated in Figs. 9.11 and 9.12. In the voltmeter the working wire is stretched horizontally,

and is about 16 cm. long and 0·06 mm. in diameter. The tension employed in very considerable, experiment showing it to be of the order of 30 grm. weight; this means a stress in a working wire 0·06 mm. in diameter of 1,060 kg. per sq. cm., or 6·7 tons per sq. in. At a point usually about 6·5 cm. from one end a fine metallic wire is attached to the working wire and

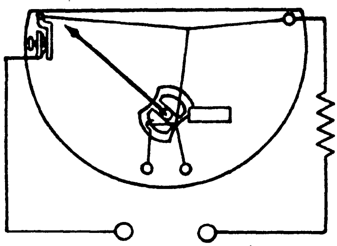


FIG. 9.11.—Principle of the double-sag hot-wire voltmeter.

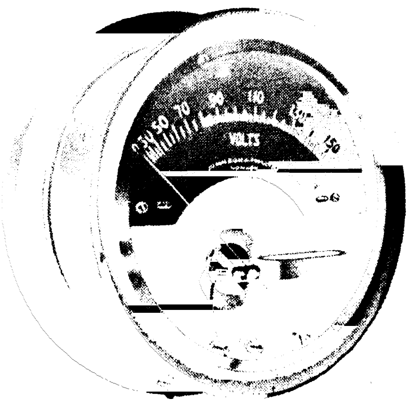


FIG. 9.12.—Example of double-sag hot-wire meter (Johnson and Phillips).

carried down to an insulated post on the lower part of the supporting plate. Sufficient tension is maintained to cause a slight dip in the working wire by attaching a non-conducting thread about 5·5 cm. below the point of intersection. This thread is carried out horizontally and approximately parallel to the working wire, and after taking a turn round a little double grooved pulley on the pointer spindle passes to a flat spring, which keeps the whole system taut in most instances; to prevent the possibility of slipping, the

cord is attached to the pulley, by means of a little screw, which grips it where it passes from one groove to the other.

Upon the same axis as the pulley is mounted an aluminium damping disc which moves between the poles of a permanent magnet. Since the system is taut and only acquires its final position of equilibrium slowly, the functions of this damping arrangement are of rather a different character than usual; for the tendency to overshoot with sudden change of load, and the vibration of the wire system from mechanical shock, are expended in the wire themselves, and set up in them additional stresses. The damping arrangement, therefore, acts as a safety brake to the moving parts, preventing them from acquiring, by reason of their inertia, a momentum which would throw serious additional stresses into the already heavy mechanically-loaded system.

In order to reduce the variation in the air currents in the neighbourhood of the hot wire a metal baffle plate is fixed immediately above it and extends over the whole length.

The wire current in voltmeters of this type is usually about 0·20 to 0·25 ampere, but the wire resistance varies considerably in various instruments. The following table contains particulars of some of the instruments examined :

TABLE XXXVII.—*Particulars of Double Sag Hot-Wire Voltmeters.*

Range (volts.)	Total resistance (ohms, cold.)	Wire resistance. Cold.	Wire resistance. Hot.	Current.	Watts.	Volts across wire.
120	573·5	17·25	17·55	0·209	25·1	3·58
50	223·5	31·9	33·4	0·222	11·1	7·45
25	118·9	16·9	17·12	0·209	5·24	3·75
10	50·6	21·4	21·75	0·1965	1·96	4·29
3	13	12	12·2	0·246	0·681	3

Zero adjustment is provided by means of the spring piece at the left-hand end of the wire. The nut on the end of the pinching screw can be turned from outside the case by means of a suitable screwdriver, and the tension on the working wire, which is maintained by means of a three-leaf flat spring, can be varied by slackening or tightening on the pinching screw.

The series resistance is wound on an open framework and mounted in the recessed back of the instrument, openings being

provided in the edges of the base plate and metal cover plate to secure efficient ventilation.

The whole of the working parts are mounted on a compensation plate of bronze which has a coefficient of expansion equal to that of the working wire. This plate is circular in form and about 1·65 mm. thick, and is stiffened by a horizontal web across the back to lessen the liability of distortion to alter the normal tension in the wire; in the centre of this web a boss is cast, and the whole plate is supported by this from the back plate of the instrument. Thus it is rigidly fixed only at its centre, and is free to expand in all directions. Compensation is nevertheless very far from

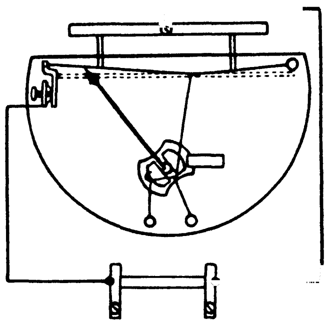


FIG. 9.13.—Principle of double-sag hot-wire ammeter.

perfect, because of the very serious lag produced by the heat capacity of the plate.

In ammeters of this type for small currents a similar arrangement to the voltmeter is adopted, except that a larger wire is used to produce the initial sag.

Thus in a 2-ampere instrument the wire has a diameter of 0·46 mm., and is shunted with a resistance of about 1 ohm, but except for this shunt the instrument is identical with the voltmeters. When heavier currents are to be measured the working wire is not only increased in diameter to 0·3 or 0·4 mm., but it is also divided into sections which are arranged in parallel across a suitable shunt.

The sectioning of the wire is accomplished by fastening to it, in symmetrical positions, strips of silver foil, which are then curved back to an insulated bus bar connected to the terminals of the shunt. Fig. 9.13.

The following table gives some particulars of ammeters of this type:

TABLE XXXVIII.

Maker.	Range.	Resis-tance.	Watts.	Drop in volts.	Time for steady reading (secs.)	Time to return to zero. (secs.)
Johnson, Phillips	2	0·3075	1·23	0·615	33	90
Ditto	1	1·122	1·122	1·122	23	40
,,	10	0·0427	4·27	0·427	25	45
Trub Tauber	1	2·02	2·02	2·02	22	47
,, ,,	10	0·05	5	0·5	17	15

In the later forms of Hartmann-Kempf instruments considerable modification in design was effected, with a view to obtaining better compensation, quicker action, and more reliable performance generally.

The compensation plate is in two sections, as shown in Fig. 9.14. The larger part is of thin iron, having a coefficient of ex-

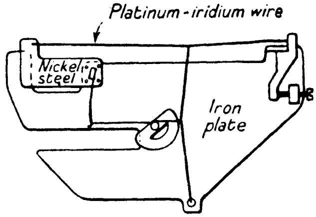


FIG. 9.14.—Modified arrangement of Hartmann-Kempf hot-wire instruments.

pansion of 0·000012, and the top left-hand corner is cut away as shown, and an extension of nickel steel with a coefficient of expansion of 0·0000008 is riveted on and carries the support of a platinum-iridium wire 0·03 mm. in diameter. By substituting this alloy for the original platinum-silver wire a higher working temperature is possible with a finer wire, so that, although the coefficient of expansion of the platinum-iridium is smaller than that of platinum-silver, there is no loss in expansion, while the higher working temperature renders the instrument less susceptible to external changes of temperature. Thus, with a wire about 20 cm. long and 0·03 mm. diameter, the temperature at full load is about 300° C., whilst in the older instruments, with platinum-silver wires, it is usually more nearly 150° C. The maximum working

current is about 0·15 ampere, and the actual extension is about 0·2 mm. This is magnified 30 times by the transmission system to the pointer shaft.

The post from which the tension spring is mounted is placed on the junction between the iron and nickel-steel pieces, and the spring points downwards instead of upwards. By adopting this arrangement the spring, therefore, is relatively always in the same position with respect to the wire, and is independent of expansions of the compensation plate. The normal tension is, therefore, not altered by changes in the plate itself. The second sagging wire of phosphor-bronze is attached in the usual way, but the non-metallic thread which communicates its movements to the pointer shaft is attached to the larger diameter section of a double-grooved pulley on the pointer shaft, whilst the spring cord acts on the section of smaller diameter. The damper is semicircular and swings between the poles of a thin laminated permanent magnet, which is supported on the base independently of the compensation plate, which is cut away in the manner shown in Fig. 9.14 to accommodate it, and the compensation plate is therefore freed from the load and constraint which in the earlier forms was liable to occur. Thus, since the compensation plate carries nothing but the working parts it can be made much lighter, and therefore its heat capacity considerably reduced.

As already mentioned, the great field of the hot-wire instrument is in the measurement of high-frequency currents, and the arrangements described above, although fairly satisfactory for the measurement of currents of moderate frequency, are quite unsuitable at very high frequencies.

In instruments in which the working wire is sectioned as described above, and which are sometimes misnamed "unshunted" ammeters, several serious sources of error are likely to arise, for although it is true that the working wire has only a very small impedance, this constitutes only a small part of the whole impedance of the various circuits through the instrument, and changes of current distribution in the whole system may be very considerable, tending to make such instruments read high as the frequency increases. The symmetrical arrangement of the leading-in wires and the employment of material of high resistivity for the working wire will however reduce the error.

Nearly all ammeters for more than six amperes are shunted, and

in many cases one component of the shunt acts as the working wire or operates the indicating mechanism. An example of an internal shunted meter is shown in Fig 9.15.

Under all circumstances where high-frequency currents are employed the question of current distribution has to be carefully considered. In solid conductors at high frequencies the current does not distribute itself uniformly over the cross section, but

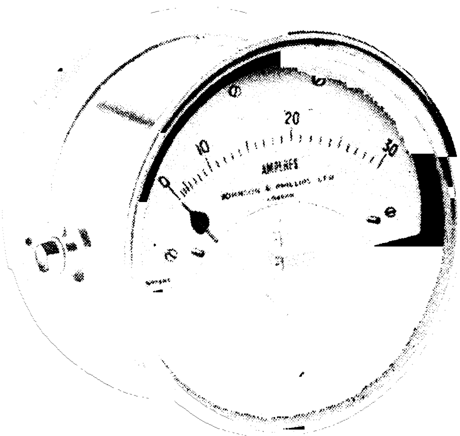


FIG. 9.15.—Hartmann and Braun hot-wire ammeter.

confines itself to the outer regions, and this effect is equivalent to an increase in resistance as the frequency increases.

Thus, if R_f is the resistance of the conductor at a frequency f, and R_o its resistance to steady currents, then

$$R_f = R_o\left[1 + \frac{1}{12}\left(\frac{2\pi f l \mu}{R_o \times 10^9}\right)^2 - \frac{1}{180}\left(\frac{2\pi f l \mu}{R_o 10^9}\right)^4 + \cdots\right]$$

where l is the length of the conductor and μ the permeability of its material, and since $\frac{1}{R_o}$ is proportional to the area of cross section, it will be seen that R_f depends on this and also upon the frequency, and hence, to reduce "skin effect," the cross section should be kept small. A still further decrease in the change of resistance is effected by the employment of a material of high specific resistance. Thus a No. 42 S.W.G. copper wire will have an apparent increase in resistance of 0·75% at a frequency of one million cycles

per second, while in a similar wire of constantan the increase will be less than 0·001%.

Where, therefore, high-frequency currents are to be measured, the first step must be to eliminate resistance changes by reducing the conductors to a series of fine filaments of high specific resistance in parallel with one another; but this alone is insufficient if the frequency is very high, for then the distribution of currents among the individual filaments may not be equal, even though the resistance of each is identical, owing to the mutual induction between the various pairs of them; and if, therefore, the current is incurred from the heat produced in any one, considerable errors may arise. Thus, with several wires arranged between terminal blocks, equidistant from one another, but in one plane, it will be found that the reactance of the outer wires will be less than that of the inner ones, and hence the outer conductors will carry a larger proportion of the current as the frequency increases, even though the greatest care is taken to make their steady current resistances identical.

Thus Dellinger finds in a three-wire instrument consisting of wires 10 cm. long, 0·08 mm. diameter, and spaced 4 mm. apart, that at a megacycle/sec the increase of current in the outer wires is some 2·8%, and the decrease of current in the centre one is 5·3%; or, again, in a seven-wire instrument with six wires 8·6 cm. long and 0·15 mm. diameter, spaced 0·19 cm. apart, and the seventh acting as the indicating wire 9·5 mm. from the nearest of the other six wires and parallel to them, the readings were 10% high at 100,000 cycles, and 46% high at 750,000 cycles:

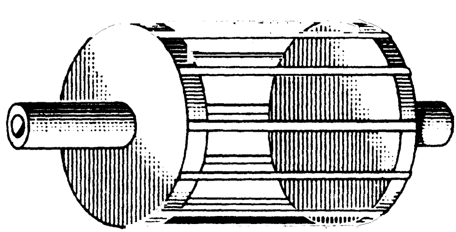


FIG. 9.16.—Arrangement of wires or strips in high frequency hot-wire ammeter.

This distribution error may be avoided by arranging the wires at equidistant intervals round a cylinder, as in Fig. 9.16, and we then arrive at theoretically the most perfect arrangement. The terminal blocks in this case are in the form of two discs with their

planes vertical and a series of fine high resistance wires, or in some cases, very thin high-resistivity strips are attached between them at equal intervals round the circumference, forming a kind of squirrel-cage arrangement.

The current leads are taken straight to the centre of each disc, so that everything is perfectly symmetrical about the central axis. The mutual induction of each wire or strip with respect to the others is now the same, and the chief difficulty becomes identity of resistance, which demands skill and patience in assembly. An arrangement of this kind has been manufactured by Hartmann & Braun.

For high-power radio communication work the substitution of very thin metal strip of high resistivity for fine wires in parallel is now becoming general. But it is usual, in order to overcome the difficulty of the want of mechanical homogeneity in the material of the plate, to cut slits in it parallel to its length, and a narrow strip is then employed to actuate the indicating mechanism. With very thin metal of very high resistivity the error due to change of resistance can be rendered negligible, but the change in current distribution in the plate may be appreciable; this, however, is less harmful than in the case of parallel wires, because heat convection and conduction tend to equalize the temperature over the whole plate.

The current distribution in the massive terminal block will, however, seriously alter that in the plate unless special precaution is taken in shaping them so that the effect is reduced.

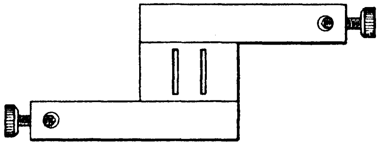


FIG. 9.17.—Arrangement of shunt for high frequency hot-wire ammeter.

Probably the best arrangement is due to F. W. Roller, and is shown in Fig. 9.17. In this the thin metal strip is connected between two identical and parallel metal lugs, and the current enters and leaves the arrangement at opposite corners.

Care must always be taken in employing strip instruments to use them in the position in which they were calibrated, as otherwise

the alteration of convection may introduce errors which may amount to as much as $\pm$ 10% between the vertical and horizontal dial positions.

It can be claimed that the hot-wire principle is free from the errors of magnetic hysteresis, wave form, and, with certain reservations, stray field and frequency (for in order to eliminate the effects of mutual induction between the leads and the working parts of the instrument at high frequencies the former should be brought straight into the instrument and not allowed to run parallel to the working parts). Again, when the tension springs are of steel, the proximity of a powerful magnet pole will cause them to deflect, and thereby increase the normal tension on the system and thus produce serious errors. Nevertheless, there are some very grave disadvantages in this type of instrument.

The power consumption is necessarily very high, and voltmeters particularly are very liable to suffer from the effects of momentary overloads, for since the heat capacity of the wire is purposely made small in order to obtain quick action, it rapidly reaches a high temperature, and hence only very brief overload may raise the temperature above the point of fusion and destroy it. Attempts to protect by means of fuses are seldom entirely effective. In some cases the heat capacity of the fuse is greater than that of the wire, and it is therefore quite ineffective, since it lags considerably behind the wire in attaining the fusing temperature. In some types of instrument protection of a much more satisfactory character can be afforded by employing a short-circuiting device across the wire which is closed automatically by the wire expanding to a predetermined limit.

The most persistent cause of error in all instruments of the hot-wire type is their uncertainty of zero with a scale closing very rapidly at its lower end. The heating error is naturally very complicated, and the study of the behaviour of one of the early forms of Hartmann & Braun voltmeters under various conditions is interesting.

In the first place, what may be called the normal heating error of the instrument is surprisingly high, being of the order of 1·5% for a 120-volt instrument, and the constant value is attained in about half an hour, as shown by the curve A in Fig. 9.18. If, now, the instrument is deflected well up to its scale when enclosed in an oven which can be gradually heated, the curve B is obtained

for the instrument when subjected to a rise of external temperature at the rate of about 0·25° C. per minute, the final temperature being 40° C. The lag of the compensation plate is well shown by the rapid rise of this curve at first, but after twenty-five minutes

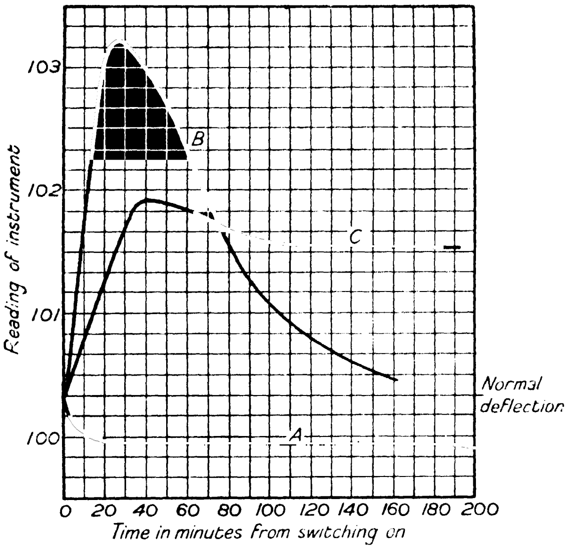


FIG. 9.18.—Temperature error curves for hot-wire voltmeter.

have elapsed it begins to overtake the wire ; even after a lapse of two hours, however, it has not fully corrected. If the instrument is subjected to a similar rise of external temperature, but is only switched on at intervals just long enough to obtain the reading, the zero being adjusted for each observation, the curve *C* is obtained.

Fig. 9.19 shows diagrammatically an attempt to compensate for this uncertainty of zero. The working wire *w* and a compensating wire *cw* are joined at one end together, and the common junction is then attached to a spring which keeps both in tension ; the other end of the compensating wire is fixed rigidly to the support plate, while that of the working wire is attached to one end of a rocking lever capable of small angular movement about the point *o*. To the other end of this lever the actual sagging wire, which carries no current, is attached, and at its other end is rigidly fixed.

It is therefore obvious that the expansion of the working wire, due to the heating of the current flowing in it, will cause the lever

to move outward at the end to which it is attached, so that the end to which the transmission wire is fastened moves inwards, producing a sag which is communicated to the pointer shaft by cord and pulley in the usual way. Any expansion produced by external rise of temperature will, however, cause no movement of the lever,

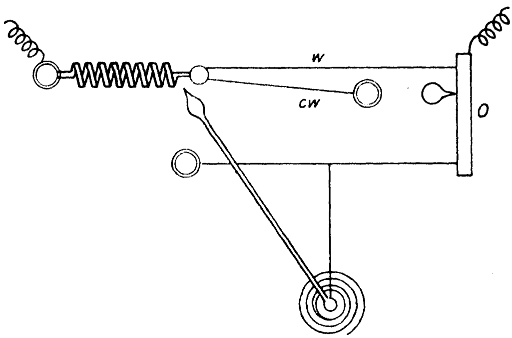


FIG. 9.19.—Principle of temperature compensation for hot-wire voltmeter

since it affects both the working and compensation wire equally, and their common junction only moves under the action of the tension spring. The coefficient of expansion of the compensation wire must be sufficiently large to allow for its own shortened length and the expansion of the sagging wire if complete correction is to be attained.

Design of Hot-wire Instrument of the Double Sag Type

As an example of design let us take a voltmeter with a maximum range of 120 volts.

In order that the movement may be contained conveniently in the ordinary-size case, let us provisionally make the working wire 12 cm. long and pulled down so that the initial sag is 5 millimetres. If we allow a temperature rise of 150° C. and use a platinum-silver wire 2 mils in diameter (0·00508 cm.), and having a linear coefficient of expansion of 0·0000162, then

$$\Delta l = 12 \times 0{\cdot}0000162 \times 150 = 0{\cdot}0292$$

and the increase in sag at maximum temperature will be

$$\sigma = \sqrt{\frac{12}{2}} = 0{\cdot}029 + 0{\cdot}25 - 0{\cdot}5 = 0{\cdot}1525$$

If we attach a second sagging system 8 cm. long at right angles to this, if it were attached at the middle point of the first wire, and we allow an initial sag of 5 mm. in it also, then the movement of the pull-off point in the middle of this would be

$$\sigma_2 = \sqrt{\frac{8}{2}} \times 0{\cdot}1525 + 0{\cdot}25 - 0{\cdot}5 = 0{\cdot}427 \text{ cm.}$$

But suppose the pulley on the pointer axis round which the cord is taken has a diameter of 5 mm., then for a 90° deflection the travel required would be $\frac{\pi 0{\cdot}5}{4} = 0{\cdot}393$.

Now, since we must keep the pointer axis on the centre line of the instrument, it is convenient to make the point of attachment of the second system to one side of the centre line, so that it will come down conveniently beside the pulley ; and since we only require a movement of 0·393 cm., and we have a maximum movement of 0·427 cm., we can conveniently do this.

The locus of any pull-off point on the working wire is an ellipse whose semi-axes are approximately $\frac{l}{2}$ and σ_{max}, and therefore the movement of the point of attachment required to give a travel of 0·393 cm. is 0·137 cm. instead of 0·1525, so that the point of attachment will be

$$\frac{12}{2}\sqrt{\frac{(0{\cdot}6525)^2 - (0{\cdot}637)^2}{(0{\cdot}6525)^2}} = 1{\cdot}3 \text{ cm.}$$

reckoned from the centre line, which will bring the second system down conveniently beside the pulley shaft.

Now the heat generated in the wire is $0{\cdot}24I^2R$, and for a steady temperature we have $0{\cdot}24I^2R = U\pi dl\theta$ where U is the emissivity, d the diameter, and θ its temperature,

$$\text{or watts} = I^2R = \frac{U\pi dl\theta}{0{\cdot}24}$$

Ayrton and Kilgour have given the following expressions for the emissivity of very thin wires at various temperatures :

At 100° C. $U = 0{\cdot}001036 + 0{\cdot}0120776$ (d^{-1} mils).
200° C. $U = 0{\cdot}0011113 + 0{\cdot}0143028$ (d^{-1} mils).
300° C. $U = 0{\cdot}0011353 + 0{\cdot}016084$ (d^{-1} mils).

From these the value of U for our case will be for a 2-mil wire approximately 0·008.

Hence the watts $= \dfrac{0{\cdot}008 \times \pi \times 12 \times 150 \times 0{\cdot}00508}{0{\cdot}24} = 0{\cdot}957.$

The specific resistance of platinum silver is 31·6 microhms at 0°C., and its temperature coefficient 0·027 per cent. per 1° C.

Hence the resistance of the wire will be

$$R = \frac{32{\cdot}92 \times 12 \times 4}{10^6 \times \pi \times (0{\cdot}00508)^2} = 19{\cdot}5 \text{ ohms}$$

at the full working temperature, and therefore the current for maximum deflection is

$$I = \sqrt{\frac{0{\cdot}957}{19{\cdot}5}} = 0{\cdot}222 \text{ ampere}$$

and the total resistance of the instrument must be $\dfrac{120}{0{\cdot}222} = 541$ ohms, and the resistance in series with the wire will be $541 - 19{\cdot}5 = 521{\cdot}5$, and the total power expended in the voltmeter is $(0{\cdot}222)^2 \times 541 =$ 26·6 watts.

THERMO-JUNCTION INSTRUMENTS

Thermo-junction instruments depend upon the following principle: The current to be measured is sent through a fine wire resistance or heater, and the heat generated in it is communicated to a thermo-junction, which is in turn connected to the indicating device, usually a sensitive moving coil permanent magnet instrument which indicates the increase in E.M.F. of the junction. We may therefore broadly classify such instruments into two groups:

Firstly those in which the heat is communicated to the thermo-junction by radiation and convection, and secondly, those in which the heat is imparted by direct conduction.

In the first group there is practically only a single self-contained representative, this being the Duddell Thermo Ammeter (Fig. 9.20).

In this instrument the movement is practically identical with the ordinary moving coil instrument, except that the ends of the moving coil, instead of being attached to conducting springs, are brought out axially below the coil and are soldered to two little bars of special alloys, which in turn are closed by means of a horizontal silver receiving plate, thus forming a complete thermo-junction circuit.

If this silver plate is warmed the junction of the alloys produces a current which, circulating round the coil windings in the field of the permanent magnet, produces a deflection which is a function of the heat received, and hence is proportional to the square of the current in the heater.

To allow of the junction being carried on the axis of rotation the coil is provided with inside pivots, and since the control springs have only a mechanical function to fulfil, one only is necessary, and this can be of a material chosen without reference to its conductivity.

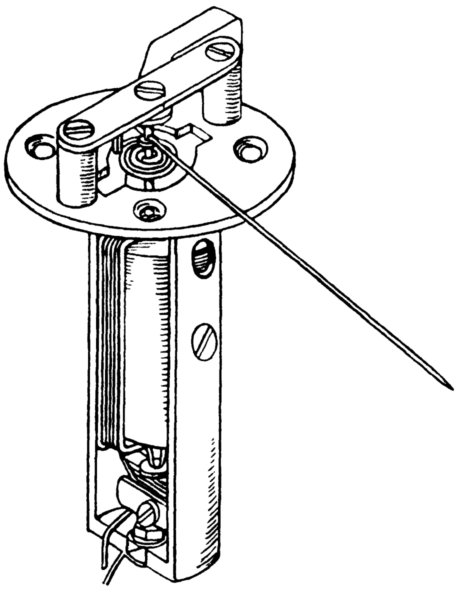


FIG. 9.20.—Principle of Duddell thermo-ammeter.

The heater in low-current instruments consists of a small plate of platinized mica. The platinum is scraped away on this in zigzag fashion, and by this means a resistance of several hundred ohms can be attained with some 8 or 9 bends in the zigzag, and the area of the heater even then does not exceed 0·2 sq. cm. For currents greater than 20 milliamperes a wire grid replaces the platinized mica.

The heater is mounted immediately below the receiving plate of the junction, to which it communicates its heat by radiation and convection.

In order that the instrument shall give quick response the heat capacity of the junction must be small, and this involves a very thin receiving plate and small elements to the thermo-junction. The resistance of the copper coil must also be kept low, and it is usual in making measurements with systems of this type to make the resistance of the indicating instrument a little higher than that of the junction.

No statement is made by the designer in his original paper as to the materials employed for the elements of the junction, except that they were such that the thermo-electric power rises with the temperature, and this effect is utilized partially to balance the increase of resistance of the coil with rise of external temperature, and leads to a smaller temperature error than would otherwise obtain. Thus, when used as an ammeter the temperature error is about 0·01 per cent. per 1° C., and when employed directly as a voltmeter without series resistance the error is 0·1 per cent. per 1° C. The power consumption is 0·015 watt, and an overload capacity of three times the normal current is claimed.

Like every type of thermal instrument zero creep is troublesome, particularly if heating is uneven, and there is no effective method of compensating for this. And again, in common with every hot-wire instrument (except, perhaps, the vacuum enclosed thermo-junction), the calibration is affected by position; since the transference of heat from the heater to the receiving plate is largely due to convection, the amount of heat received will be less when the axis of rotation is horizontal than when it is vertical. This difference can, of course, be reduced to a minimum by bringing the heater very close to the receiving plate.*

Another scheme, somewhat analogous to the above, is sometimes used for high frequency measurements. It consists of employing a short heating wire through which the current to be measured is passed; above this a thermo-junction, or, in some cases, a series of thermo-junctions, are mounted as close as practicable, and these are connected to a sensitive moving coil instrument, and the arrangement is then calibrated with direct current. The junction and heater must be enclosed very carefully to exclude, as far as possible, the effects of draughts, and where interchangeable heaters are

* For more refined and delicate work Duddell devised the thermo-galvanometer, which is a modified form of Boys' radio micrometer, and will be described when dealing with reflecting instruments.

employed for extending the range these must be provided with a means of exactly defining their position with respect to the junctions.

In all thermo-junction devices, however, the most satisfactory results are attained by enclosing both heater and junction inside an exhausted bulb, thus entirely eliminating the cooling effects of convection currents and draughts, and protecting the junction from oxidation.

The exhaustion must be carried below 0·1 mm. of mercury pressure before good results are attained, and at 0·0001 mm. no further advantage occurs. For general purposes an exhaustion to 0·01 mm. seems to be the best.

To secure high sensitivity and sharpness of reading the junction should contain as little metal as possible, and the component wires should be of the smallest diameter. Electric fusion or welding are usually better than soldering if properly carried out. One simple and satisfactory method of making the junctions is to twist the component wires together for a short distance at their middle point; they are then put under tension across the thin edges of a pair of adjustable electrodes, and current is sent across the twisted portion until fusion takes place, when the two junctions will separate. Another method is to butt-weld, holding the wires in suitable grips, one of which is provided with a fine screw feed. The oxy-hydrogen flame may also be applied to the twisted ends, but, however made, it is essential that the fusing temperature is reached rapidly in order to minimize the possibility of oxidation.

W. H. Wilson and Miss Epps have shown that it is possible to construct satisfactory junctions by depositing copper, or, better still, silver, electrolytically on a core of constantan so as to form a sheath along part of the length of the wire. In this way a series of junctions can easily be formed by winding the constantan wire in the form of an open spiral on an insulating support. The spiral is then immersed in the plating bath so that the electrolyte reaches a level corresponding to half the periphery, and thus half of each convolution receives a deposit. By this means two lines of junctions are formed, and if one of these is exposed to the heater a neat and compact thermo-pile is formed. The investigators found that silver was the most satisfactory material for the sheath, and that its thickness should be such that its area of cross-section is from 30 to 35 per cent. that of the constantan core.

The employment of independent junctions has several important

advantages over the direct conduction types, for although there is some loss of sensitivity on account of the junction not being in actual contact with the heater, there is an entire absence of reversal error in the direct current calibration and a freedom from capacity current error, and the possibility of extended range with interchangeable heaters.

Direct Conduction Instruments

In this case the junction is directly attached to the heating wire, and for small capacity arrangements the heater and junction may be included in the indicating instrument case and an appropriate scale provided. In constructing the apparatus care must be taken that the junction itself is directly on the wire and is symmetrical in other respects to it if the reversal error on direct current is to be reduced to a minimum.

As in the previous case, vacuum enclosure is an advantage, but this necessarily limits the system to the measurement of comparatively small currents. Usually the heater and junction are arranged at right angles to one another, and the four leading-in wires are brought up at right angles to the plane in which the heater and junction are arranged, and in all the various modifications of this arrangement it is better to reduce the inductive action of the leads to the heater when the wires lie in one plane by bringing them up at right angles to that plane.

If the system is utilized for the measurement of currents above, say, 1·5 amperes, vacuum enclosure is not easily possible, and a multiple wire heater must be employed; but the current distribution troubles already discussed on page 521 are equally applicable here, and if a single junction on one wire is employed, very serious errors will arise at high frequency. Attempts to reduce this by utilizing the total heat are sometimes employed, and for this purpose the thermo-junction arrangement is made by bridging consecutive pairs of wires with alternate pieces of the junction materials, so that there is a junction on each wire, but each alternate one is reversed. The better form of parallel wire arrangement is that of spacing the wires round the edges of two parallel upright discs, as already described, and Fleming employed this method, placing a junction on one wire.

As far as the materials of the junction are concerned, iron constantan, copper constantan and manganin constantan seem to

be most generally employed. The first of these (iron constantan) has the advantage that the thermo E.M.F. is almost exactly proportional to the temperature difference over the usual temperature range, and the temperature coefficient of its voltage sensitivity is very small; but the high thermal conductivity and magnetic properties of the iron may, under some circumstances, give trouble. In fact Schering has stated that the combination is useless because it cannot be calibrated with direct currents. The actual thermo-E.M.F. is about 51 to 53 micro-volts per 1° C.; a copper constantan couple is in some respects better, although the thermo-E.M.F. is 20 per cent. lower. Schering in 1906 employed manganin constantan with excellent results, for manganin does not differ greatly from copper in thermo-electric character, but its specific resistance and small resistance variation with temperature are greatly in its favour.

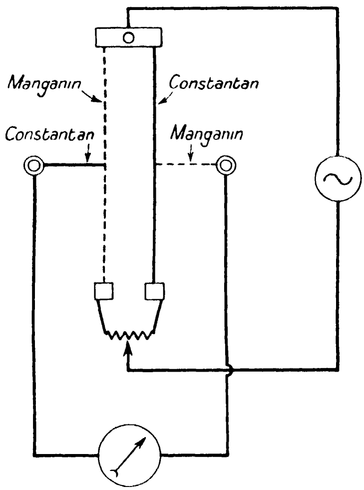


FIG. 9.21.—Thermo-junction arrangement for measuring small alternating currents.

The Reichsanstalt arrangement is indicated in Fig. 9.21, in which a fine wire of manganin and a similar one of constantan are stretched between terminal blocks parallel to one another; the lower pair of terminals are joined through a resistance, and a sliding contact from the source of supply makes contact on to this, the other terminal of the supply being connected to the common junction at the other end. Thus the wires are in parallel, and, by adjusting the position of the slider, can be made to carry exactly equal currents.

To their centre points a second wire of the other material is attached with a minute trace of solder, and is then carried out to a suitable terminal and then to the galvanometer ; thus the two junctions are in series. It is claimed for this arrangement that the temperature coefficient and Peltier effect is of no consequence, and no difference is shown on direct or alternating current. The apparatus will show 0·24 volt, and a current of 0·05 ampere gives full scale deflection of the indicating instrument, the resistance of the apparatus being 4·8 ohms.

In 1906 Prof. J. A. Fleming used a bismuth-tellurium couple for the measurement of antenna currents which were passed through a fine constantan wire across which the junction was arranged, the whole being in a vacuum. The high thermo-electric power of tellurium was thus taken advantage of. In 1911 Austin described a tellurium platinum junction for high frequency working in which two copper wires are supported in insulating material ·3 mm. apart. To one is attached a short piece of 0·8 mm. platinum wire which had been made white-hot and inserted in a tellurium bead, thus supporting it with a practically resistanceless joint. The other copper lead carries a short length of platinum wire 0·02 mm. diameter, which is bent round into contact with the tellurium and the junction welded by means of a spark ; preferably this should be done in a non-oxidizing atmosphere.

In use the high frequency current is sent round the junction, to the ends of which a sensitive moving coil instrument is attached. Owing to its inductance the galvanometer only registers the current due to the heating of the junction. It is obvious that this arrangement is only useful for alternating currents, and could not be calibrated with direct current.

Various other modifications and arrangements are employed, but as they are exclusively for high frequency working and not direct indicating, they will be referred to under the section devoted to the special type of work to which they are adapted.

HOT-WIRE WATTMETERS

There have been several notable efforts to utilize the hot-wire principle for the measurement of electrical power.

The first was due to M. B. Field, who in 1898 showed that if an instrument is made which contains an element which responds to the square of the sum of instantaneous values of the current and

voltage of a circuit, and another which behaves similarly to the square of the difference of these quantities, and these two are arranged so that they subtract the difference from the sum, the residual effect is proportional to four times the true power in the circuit. Thus, if I is the instantaneous current and V the instantaneous voltage $(I + V)^2 - (I - V)^2 = 4VI$. Essentially Field's proposal was to construct a hot-wire instrument containing two wires arranged so that the pointers indicate the difference in their expansions.

A potential transformer having two "step down" secondaries wound in opposite directions is connected with its primary across the mains, and its secondaries, each with one element of the hot-wire instrument in series, across a non-inductive low resistance in one of the mains, as shown in Fig. 9.24, In practice a double hot-wire instrument with two pointers was employed, and so arranged that the mechanism moving the pointers was capable of independently returning to zero, while the pointers were held clamped in their deflected positions. This was accomplished by a lever at the side of the instrument which, when operated, held the pointers and at the same time shunted the hot wires. As the instrument was practically aperiodic the simultaneous sum and difference could be thus

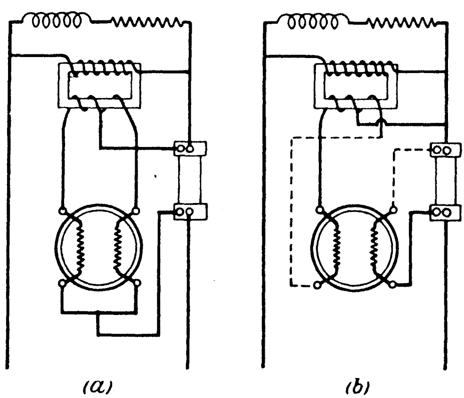


FIG. 9.22.—Field's arrangement of hot-wire wattmeter.

observed. The connections shown in (*a*) (Fig. 9.22) are employed when measuring power ; if a change-over switch is arranged so as to convert the connections to those shown at (*b*)(Fig. 9.22), then the right-hand element of the double instrument will read the current

and the left-hand the supply voltage, and thus in a single instrument the power and power factor could be easily measured.

Field points out that the method is capable of considerable accuracy. The inductance of the pressure circuit is of no consequence; on the other hand, the resistance in series with the mains must be truly non-inductive, and should give a drop of, say, 1·5 volts.

R. Bauch, in 1904, also made use of a differential hot-wire principle for measuring alternating current power. In one of his arrangements two hot wires of platinum silver, 125 mm. long and 0·15 mm. in diameter, are arranged so that they are under equal tension when the instrument is at zero (Fig. 9.23). One wire carries a

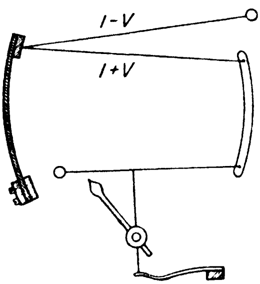


FIG. 9.23.—Principle of Bauch hot-wire wattmeter.

current proportional to $I - V$, and the other a current proportional to $I + V$; this latter is attached to a lever capable of motion in the plane of the wires about an axis a short distance from the end, and to the other end of the lever the ordinary sagging wire arrangement is attached for magnifying and indicating the expansion. The difference in the expansion of the two working wires produces the sag in the third mechanically. The currents in the two hot wires were derived from a transformer similar to that described above. The power lost in the transformer and instrument at no load was 3·69 watts, while at full load (500 watts) it was only 4·73 watts.

Roller also described an adaptation of the principle, which is illustrated in Fig. 9.24. The two parallel wires W_1 and W_2 are joined at their upper end to a conducting lever hung from its central point by a ligament which, in its turn, is attached to the end of a second lever similarly centrally suspended, and carrying at its other end a spring and adjusting nut; both wires in series are shunted across a resistance in series with one of the mains, whilst

a high resistance is connected between the common junction of the wires and the other main, so that one wire has a current proportional to $I + V$ and the other $I - V$. There will then be a difference in heating, and the lever to which they are attached will turn about

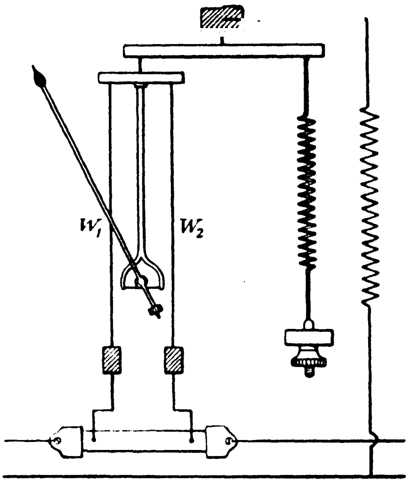


FIG. 9.24.—Principle of Roller hot-wire wattmeter.

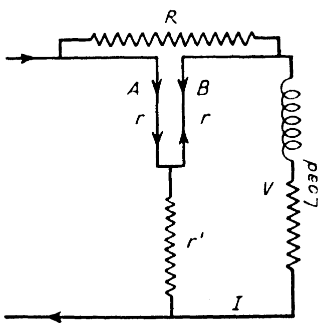


FIG. 9.25.—Diagram showing principle of Irwin hot-wire wattmeter.

its point of support. To the under-side of this lever is rigidly attached the bifurcated arm, like that already described in the Whitney instruments, the little bow-string passing round a pulley on the pointer shaft, and by this means the tilt of the lever is indicated on the instrument dial.

In 1907 Irwin described his arrangement, which is diagrammatically illustrated in Fig. 9.25. The two wires are connected to

a high resistance at their common junction, their other ends spanning a non-inductive resistance in series with the main, so that differential heating results exactly as above.

If i_1 and i_2 are the currents in the two wires, I the main current (assumed large compared with i_1 or i_2), r the resistance of each strip, R the parallel resistance of the shunt and strips, r^1 the series

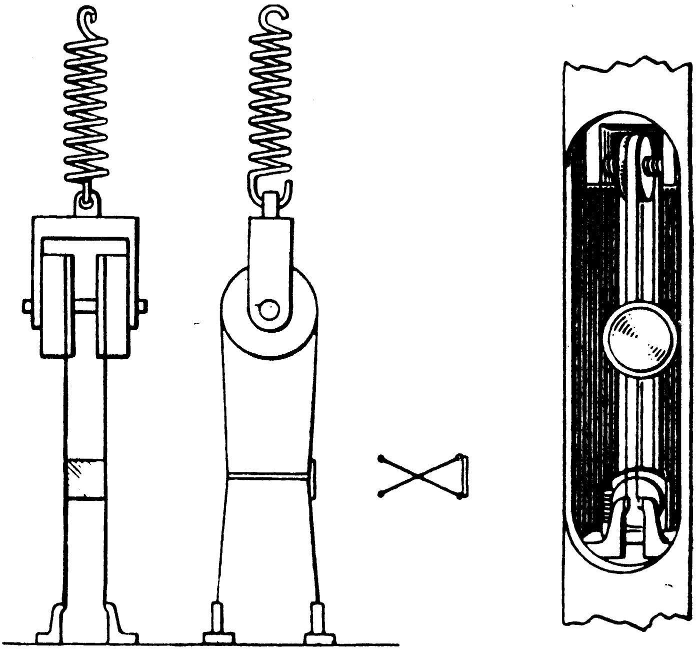


FIG. 9.26.—Irwin hot-wire wattmeter.

resistance, it can be easily proved that the deflection which is proportional to

$$i_2^2 - i_1^2 = \frac{2R}{(r + 2r^1)^2} VI + \frac{R}{r(r + 2r^1)} RI^2$$

and this equals kVI if R is small compared to r and r^1.

For the greatest difference of heating it should be arranged that the current derived by the wires from the shunt should be equal to the pressure current; then at full load unity power factor these two currents cancel in one strip, and the other only will be heated.

The expansion of the strips is rendered evident by arranging them as indicated in Fig. 9.26; each strip is looped over an insulated

pulley and anchored to a metal terminal; the pulley frame is carried by a spring which keeps the loops taut. Each element of the loop is then tied back by means of a non-conducting fibre to the opposite member of the adjacent loop, that is, they are cross-tied, as shown in the diagram. A mirror is mounted between the two loops, and the result of their unequal expansion will have the effect of rotating this mirror by an amount proportional to the difference in the rise of temperature of the two wires.

In the instrument described by Irwin the effective length of each strip was 10 cm., and the diameter 0·005 cm.; they were mounted 0·15 cm. apart, and were of platinum silver. The normal tension of the spring was 8 oz., and reducing this to half increased the sensitivity 15%. With a view to still further increasing the sensitivity the enclosing case was exhausted, and it was found that very little is gained until the pressure falls to 50 mm. of mercury, after which there is a very rapid rise in sensitivity, the form of curve being such as to show that unless it were possible to keep the degree of exhaustion very constant, the sensibility would vary very considerably.

It may be pointed out that all these hot-wire wattmeters are essentially merely modifications of Prof. Ayrton and Perry's three-voltmeter method of measuring power in alternate current circuits, the two hot wires taking the place of two of the voltmeters, whilst the third voltage is the drop across the shunt.

CHAPTER 10

ELECTROSTATIC INSTRUMENTS

ALL electrostatic instruments are essentially voltage measuring devices, and their action is based upon the attraction or repulsion exerted between two electrified conductors. Such instruments, therefore, have the advantage that they take no current from the circuit; they are free from the errors of magnetic hysteresis, wave form and frequency; they have no heating error, the loss of energy in the instrument is negligibly small, and they read equally correctly on alternating or direct-current circuits.

Lord Kelvin classified such devices into three groups, viz. (1) Repulsion, (2) Symmetrical, and (3) Attracted disc types.

The simplest representative of the first group is the original gold-leaf electroscope, and although this instrument has been considerably improved, and is capable of giving metrical results, it has not been adapted to ordinary commercial measurement. A modification of the principle has, however, been employed by Hartmann & Braun for switchboard work, the scheme of which is shown in Fig. 10.1.

Two inner rectangular metal plates are mounted, with their inner faces inclined at a small angle to one another, a short distance apart; between them is a third light plate, of similar area, which is suspended by means of two flexible metal strips from a horizontal rod above, and in its zero position this movable plate lies close to, and parallel with, one of the fixed ones to which it is electrically connected, whilst the other fixed plate is insulated from both of them. A non-conducting fibre is attached on the middle line, but well below the centre of the moving plate, and passes horizontally out through a suitable hole in the fixed plate to the indicating mechanism. This consists of a horizontal pivoted spindle carrying a pointer in the usual manner. The fibre is attached to an arm on this spindle, at a short radius to the axis of rotation, and is kept taut by a spring controlling the shaft.

The action of the instrument is obvious: since the two initially parallel plates are connected to one pole of the supply they become similarly charged, and the moving plate is therefore attracted

towards the second fixed one, which is charged in the opposite sense by connection to the other pole of the supply. The movement of the central plate is communicated to the pointer shaft by the thread attached to it; the control is, of course, gravity, and the arrangement is suitable only for use on high-voltage circuits.

Practically all indicating instruments, however, belong to the second and third groups of Lord Kelvin's classification.

For low and intermediate pressures a modification of the symmetrical electrometer is very usual. We may regard instruments of this type as a system of three conductors, two of which are fixed and the third movable. These are arranged so that the moving conductor constitutes one element of a condenser with each of the

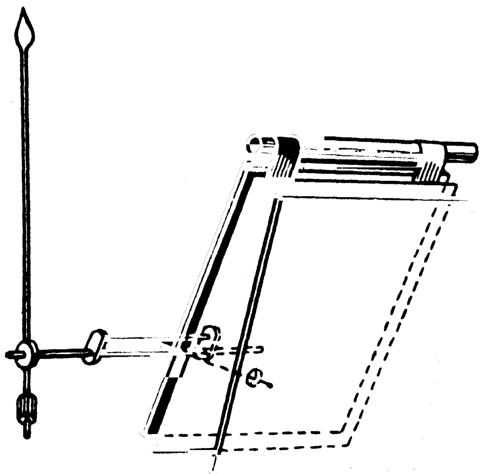


FIG. 10.1.—Principle of Hartmann and Braun electrostatic voltmeter.

fixed members. If V_1 is the difference of potential between the moving element and one of the fixed quadrants, and C_1 the capacity of the arrangement, and similarly if V_2 and C_2 are the potential difference and capacity between the element and the other quadrant, then the energy is

$$E = \tfrac{1}{2}(V_1^2C_1 + V_2^2C_2)$$

and the torque

$$T = \frac{dE}{d\theta} = \tfrac{1}{2}\left(V_1^2\frac{dC_1}{d\theta} + V_2^2\frac{dC_2}{d\theta}\right).$$

Now if the system is symmetrical

$$\frac{dC_2}{d\theta} = -\frac{dC_1}{d\theta}$$

and the torque therefore is

$$T = \tfrac{1}{2}\frac{dC}{d\theta}(V_2^2 - V_1^2) = \frac{dC}{d\theta}\frac{V_1 + V_2}{2}(V_2 - V_1)$$

If a is the radius of the edge of the element and ζ the clearance between the needle and the quadrants, $C = \frac{s\varepsilon}{4\pi\zeta}$, where s is the extent of the opposing surfaces and ε the dielectric constant ; then

$$\frac{dC}{d\theta} = \frac{\varepsilon}{4\pi\zeta}\frac{ds}{d\theta} = \frac{\varepsilon}{4\pi\zeta}\,\tfrac{1}{2}a^2,$$

and if both ends of the needle are effective we have

$$\frac{dC}{d\theta} = \frac{\varepsilon a^2}{4\pi\zeta}$$

and the torque therefore is

$$T = \frac{\varepsilon a^2}{4\pi\zeta}\frac{V_1 + V_2}{2}(V_2 - V_1)\text{ dyne cm. per E.S.U.}$$

or if T is measured in gramme cm. and V in volts, then

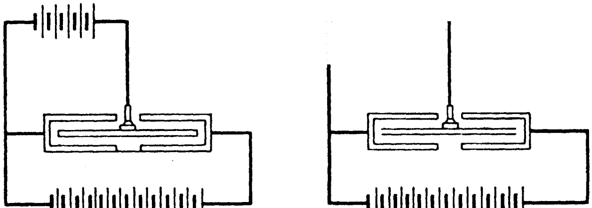
$$T\text{ gm. cm.} = \frac{\varepsilon a^2}{1{\cdot}11 \times 10^9\zeta}\frac{V_1 + V_2}{2}(V_2 - V_1)$$

If the element is enclosed so that both sides of it are effective, as is usually the case, then the torque is doubled.

When the element is separately charged and kept at a high and constant potential, while the P.D. to be measured is applied to the quadrants, the instrument is said to be heterostatically arranged ; on the other hand, we may connect the element to one of the fixed quadrants, and when this is done the arrangement is idiostatic. The scheme of connection for each of these cases is shown diagrammatically in Fig. 10.2.

In most commercial instruments it is usual to employ the idiostatic method of connection, one terminal of the supply being connected to the moving system, and the other to the fixed member.

One of the chief difficulties encountered in constructing commercial instruments on this principle for low voltages is to get sufficiently large working forces to allow of a reasonably robust method of suspension. For a given potential difference we can only increase the torque by increasing the capacity of the system, either by multiplying the number of elements acting together, or increasing the areas ; in either case the weight of the moving element increases very considerably, with the result that pivot friction becomes of considerable importance. We might, on the other hand, reduce the distance between the fixed and moving conductors, but a limit



(a) Heterostatic (b) Idiostatic

FIG. 10.2.—Alternative methods of connecting electrostatic instrument.

is imposed here, both by the mechanical difficulty of construction and the tendency to spark over.

The sparking distances in air between different forms of electrode are shown in the curves in Fig. 10.3, and the curve (Fig. 10.4) is derived from actual practice, and gives the relation between the clearance in centimetres, and the voltage, plotted to a logarithmic scale, for actual instruments.

These may serve as a guide as to limiting distance, but such curves should only be employed in practice with large factors of safety, in order to guard against the want of uniformity in the electrostatic field due to the shape of the conductors employed, and which, in most cases, is quite beyond predetermination.

In design, anything in the nature of a sudden change in curvature of the surfaces of the conductor should be carefully avoided, all sharp edges, corners and points entirely eliminated, and the surfaces worked to a high degree of finish.

The employment of compressed gas as a dielectric has definite advantages, and Watson had devised a high-potential instrument of this type of very compact dimensions. In other cases the

tendency to brush discharge is reduced by entirely immersing the live parts in an insulating oil.

In connection with the formation of brush discharge, it is well to remember that there is less tendency for this to be produced if the electrode with the smaller surface is connected to the negative

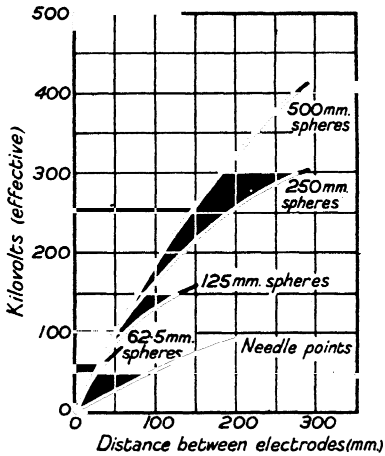


FIG. 10.3.—Sparking distance in air for different electrodes.

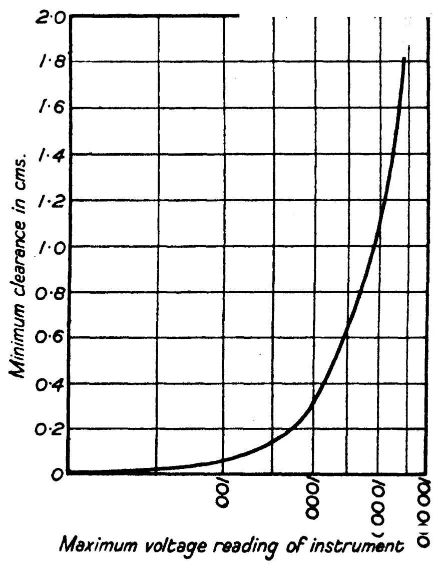


FIG. 10.4.—Relation between voltage and clearance in electrostatic instruments.

terminal of the supply. Thus, in the extreme case, if we were to connect a point to the positive and a plate to the negative of a unidirectional supply, we should produce a fine brush discharge which, on reversal, would practically disappear. Where such brush discharge does occur ozone may be produced, and this will attack and corrode such metals as silver, aluminium, brass, zinc and iron ; on the other hand, copper and nickel resist the action well.

Moreover, every electrical circuit is subject to surges ; the mere opening or closing of a switch may, under certain conditions, produce a momentary rise of pressure considerably above the normal value, and sufficient to break through the insulation of the instrument. In some cases the insulation is reinforced against both brush discharge and surge by covering the surfaces of the conductors with thin sheets of mica, or by coating them with thin layers of highly insulating varnish, but it is essential, if this device is to be at all effective, that the protective coatings should be extremely thin and electrically strong, and if they are not applied with the greatest care they may become a source of weakness rather than of strength. There is also considerable risk of their acquiring irregular surface charges which may make the readings of the instrument ambiguous.

Protection by means of a safety spark gap is of doubtful utility, as not infrequently when these operate an arc follows which may seriously damage the instrument, and it would seem inconsistent to build an instrument with a higher general insulation than that which the actual gap provides, so that the only function of such a gap is the localization of the point at which the discharge will pass.

Fusing the instrument is also a doubtful form of protection, since in most cases the time lag of the fuse is too great to offer adequate protection, particularly with suspended instruments, and the comparatively slow break often results in a destructive arc unless the fuse is very carefully designed and enclosed. One of the best methods of protection is to employ a very high resistance in series with the instrument. Thus, a resistance of several megohms in series will not appreciably affect the capacity current, but in the event of a flash-over or short-circuit in the instrument, the current traversing this resistance will cause sufficient drop to prevent the formation of a persistent arc.

With instruments for comparatively low voltage, or where high sensitivity is desired over a definite range, it is often found that

thermo and contact E.M.FS. are troublesome, and for this reason it is often considered desirable to make the entire working parts of the same metal, with as few joints as possible in the construction; and in instruments intended for high frequency measurements there is an additional argument for the reduction of the number of joints, since their resistance may seriously affect the capacity current.

It does not, however, follow that making the inductors and element of one metal will safeguard against high contact forces, and cases are on record where instruments so constructed show reversal errors greater than 2·5%, while, on the other hand, instruments using dissimilar metals like brass and aluminium show errors less than half of 1%.

Thus, it is frequently found that even when considerable care is taken in construction, instruments of this type exhibit a definite

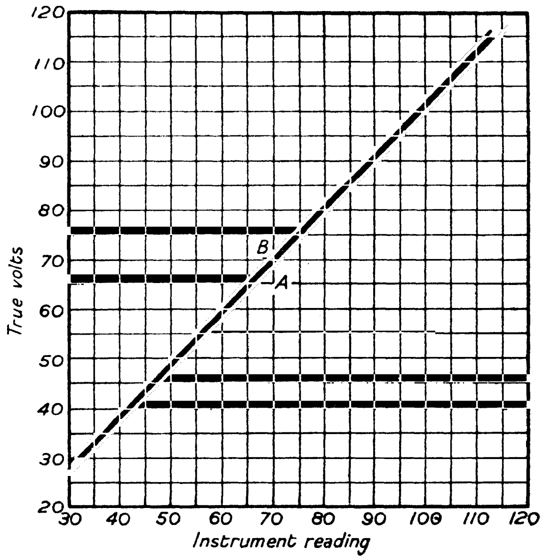


FIG. 10.5.—Calibration of electrostatic voltmeter, showing effect of reversing P.D. at instrument terminals.

polarity. The curve shown in Fig. 10.5 is an example, the calibration showing a constant difference when the P.D. at the terminals of the instrument is reversed. This is usually most marked in new instruments and often diminishes with time. Treatment by boiling the parts in a solution of soap and washing-soda has been found

effective in reducing it, but it is always best to delay the final calibration for some considerable time after the completion of construction, and it appears that this time is somewhat reduced if the instruments are kept fully deflected, preferably with an alternating P.D., and at a moderately high temperature.

The employment of dissimilar metals in the construction of the element may greatly exaggerate the effect and cause it to vary with temperature, while soldered joints require very careful workmanship, and should, wherever possible, be entirely eliminated. Unless very carefully made they are often the seat of very persistent E.M.FS., so that it is better to employ well-made lap joints, screwing or riveting them securely together. To finish by gilding or plating the entire movement after construction is an advantage in some types.

In the final stages the movement should be handled as little as possible with the bare hand, so that the possibility of moisture from the skin entering the joints is reduced to a minimum.

To increase the torque of low-reading instruments, Lord Kelvin constructed them in the form of a series of simple elements acting together, so that each element consisted of a fixed cell acting on its own moving vane mounted on a common shaft, and to this type he gave the name " multicellular."

In such instruments the fixed inductors consist of two aluminium castings in the form of a series of equally-spaced, triangular, horizontal shelves projecting from a stout back plate. These are mounted in a supporting framework, opposite one another, as in Fig. 10.6, and are both in metallic contact with the insulated terminal of the instrument ; the surfaces are finished bright and smooth, and all edges and corners rounded off to reduce the possibility of brush discharge.

The upper plate of the supporting frame is insulated from the remainder by ebonite washers and bushes, and this supports the movable system, which consists of a series of light aluminium vanes clamped between spacing pieces on a central rod, so that each vane occupies a symmetrical position with respect to the corresponding cell of the fixed system when it is suspended. The vanes are of the double-sector form, punched from thin aluminium foil, with ridges pressed up in the surface to stiffen them.

The spindle passes up through a hole in the upper insulated plate, on which are two vertical supports attached at their upper end

to the circular plate carrying the torsion head for setting the zero, which consists of a worm-wheel operated by a screw projecting through the upper part of the case.

Over the upper end of the spindle, where it emerges through the plate, a safety collar is slipped; this is of sufficient diameter so

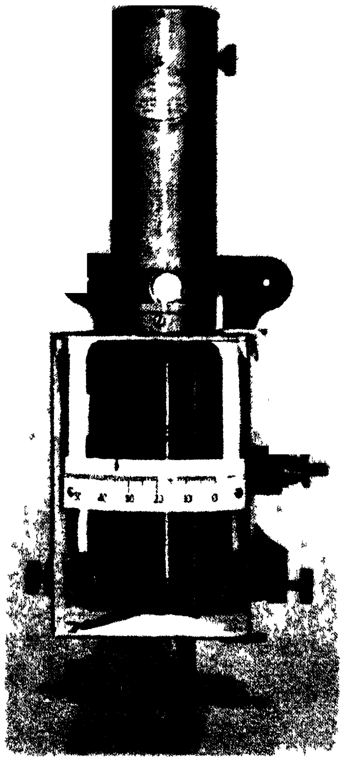


FIG. 10.6.—Kelvin multicellular electrostatic meter.
(Photo by courtesy of the Director, Science Museum).

as not to pass through the hole in the plate, and it therefore limits the travel of the suspended parts in a downward direction.

The suspension is a fine wire or strip of phosphor bronze or platinum iridium, and is soldered between the upper end of the moving spindle and the torsion head, but to guard against breakage by accidental shock it is usual to interpose a "light coach" spring between one end of the fibre and its connection. In some cases

this is at the upper end, just below the torsion head, but in the horizontal dial pattern it is carried on the upper end of the moving spindle. If, therefore, the instrument receives a severe shock, causing the heavy vanes to descend, the spring, by reason of its resilience, gives until the safety collar comes into contact with the upper plate of the framework, and the suspension is thus protected against a load in excess of its normal.

At the lower end the moving spindle terminates in the oil-damping device. This is usually either a horizontal disc carried on a vertical stem, which can be hooked on to a pin at the lower end of the spindle, or is a vertical rectangular vane pierced with a number of holes ; in either case the damper is entirely immersed in a glass vessel containing mineral oil, which is secured to the base plate of the instrument by a collar and bayonet joint.

The pointer is attached to the upper part of the moving shaft just below the level of the upper plate of the supporting frame, and moves over a horizontal or edgewise scale.

As we have already seen, the torque is proportional to the square of the difference of potential between the fixed and moving systems, and hence the scale will necessarily be crowded at its lower values ; some improvement in this respect is obtained by using two metallic guard plates along the face of the fixed cells (from which they are insulated) in such a position that when the moving element is resting in its zero position one edge is close to each plate.

These guard plates are in electrical connection with the element, so that when connected to the supply both element and plate are at the same potential ; the element is thus screened from the effect of the adjacent quadrant, and at the same time a force of repulsion is exerted between plate and element which is greatest near zero and diminishes as the deflection increases, and the lower values of the scale are therefore opened out a little.

In some of the later instruments these guard plates are thin metallic strips supported from the base of the instrument, while in other cases they are constructed by fixing between the upper and lower plates of the cells a strip of ebonite ; a fine wire is then laid along the face of this strip which is connected to the moving system, and over the wire a strip of tin-foil is fastened so as to give a broadened metallic surface.

The clamping device consists of a metallic spring-piece attached at one end to the framework, and forked at the other so as to embrace

the moving spindle ; a milled head screw passes through the base under this spring which, when screwed home, raises the forked end against a collar on the shaft, and lifts the whole moving system until the conical end of the collet to which the pointer is attached presses against the coned hole through which the spindle passes in the upper plate, thus holding the element rigidly while the suspension is entirely freed from load.

The moving system is sometimes not brought directly to the terminal, but is connected to a little switch arm which is capable of making contact with either terminal, so that in one position the switch connects the whole system of vanes, quadrants and case together, and in the other position the vanes are connected to one pole of the supply ; the operation of this switch, therefore, provides a means of checking the zero of the instrument without disconnection, and ensures that during the operation the potential of the whole system is the same.

In Hartmann & Braun's form of this instrument the moving element is modified in form, being given a practically straight side where it comes against the guard plate, and the movement is suspended and provided with a damping disc and magnet at its lower end.

For potentials above 1,500 volts a modification of the above type of instrument was devised. In this, on account of the higher voltage, fewer cells are required ; the axis of rotation is horizontal, and the moving system is supported on knife edges, on the Kelvin system already described, and is gravity controlled. Two sets of three parallel, fixed quadrant plates are supported from the base of the instrument symmetrically about the axis of rotation of the element, which consists of two parallel aluminium blades which move between the quadrants. In order to lighten the element these blades are perforated with numerous holes, which effects a considerable saving in weight without materially affecting the capacity.

The movement is screened by circular plates mounted at the back and front of the working parts, the front plate being provided with a wide slit half-way across, so that the position of the element may be observed. The sensibility and counter-weights are carried on a cross arm at right angles to the axis of rotation, and the knife-edge supports are fixed on the disc screen plates, which are at the element and case potential, while the fixed quadrant plates constitute the insulated member.

This type of instrument, with small modifications in the shape

of the various parts, has been very widely adopted. In most cases one set of fixed plates only is employed with an unsymmetrical moving element.

The form shown in Fig. 10.7 is such an instrument (in the figure, three movements are assembled together as a 2,000-volt three-phase leakage indicator). The fixed conductors are curved plates having a radial width of about 2·5 cm., and are supported from their upper end about 1·27 cm. apart. The needle consists of a corresponding number of aluminium vanes carried on a horizontal shaft; they are stiffened by pressed-up ridges, and are so arranged that in the zero position they are just under the influence of the sector plates,

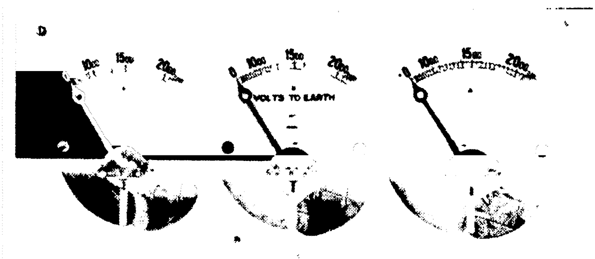


FIG. 10.7.—Everett Edgcumbe electrostatic leakage indicator

between which they are drawn as the potential rises. The pivot friction of the arrangement is reduced by employing vertical pivots in polished steel cups, carried on brackets attached to a horizontal supporting rod; the pivots are therefore at right angles to the axis of rotation. At the back of the jewel bracket a second little right-angled piece is fixed which is capable of vertical adjustment; this is arranged just above the needle shaft at each end, and is for the purpose of clamping the needle when the instrument is being moved.

The clamping device consists of a broad, brass forked piece mounted on a rocking insulated lever fixed below the movement, and operated by a pushing screw worked from outside the case. If this is screwed home, one end of the insulated lever is raised whilst the other is depressed against the flat spring, which also bears against the bottom of the case; this action lowers the forked piece, and the needle descends with it until it engages with the steel jewels;

if, on the other hand, the screw is withdrawn, the rocking lever is tilted in the opposite direction by the flat spring, the fork is raised, and lifts the needle spindle off its jewels and presses it against the angle-pieces on the brackets and holds it firmly. In the single switchboard instruments the whole movement is effectively screened by enclosing in a metal inner casing (carried on the same insulator as the needle terminal), which is provided with an opening through which the scale and pointer may be viewed. The only other opening in the screen is the clearance hole, through which the support of the fixed inductors passes.

Some continental makers also make voltmeters of the above type, arranging the needle and fixed inductors above the axis of rotation, and pivoting the system in the ordinary way, and in most cases an effective air-damper is also fitted.

For still higher voltage Lord Kelvin further simplified the multicellular construction, as illustrated in Fig. 10.8. In this only two quadrant plates are mounted parallel to one another, of the form shown in the figure ; between these a large paddle-shaped aluminium plate swings on a knife-edge arrangement. Below, the plate is prolonged into a narrow curved arm to which a second screwed arm is attached, carrying the zero and sensibility weights (see Fig. 2.24, page 64). The range is extended by hanging auxiliary weights from a hole provided at the end of the curved arm. At the other end of the plate a fine pointer is attached which moves over a vertical scale. No permanent damping is provided, but a stiff horizontal wire is suspended by silk threads behind the pointer below the scale level, and by means of a bent lever outside the case this wire may be moved so as to touch lightly against the pointer and damp the movement by mechanical friction, final readings being taken when the wire is moved out of contact.

The shallow metal case acts as a screen, and to prevent the glass of the front from acquiring a disturbing potential, a strip of tin-foil is attached to its inner surface and put into connection with the metal case. As in the previous instruments, the moving vane and case are in metallic connection, and the quadrant supports are brought through the back of the case with ebonite washers and bushes.

Ferranti Ltd. have produced a range of miniature electrostatic voltmeters employing quadrants and a flat vane for the moving element. Fig. 10.9 shows an instrument of this type with a full

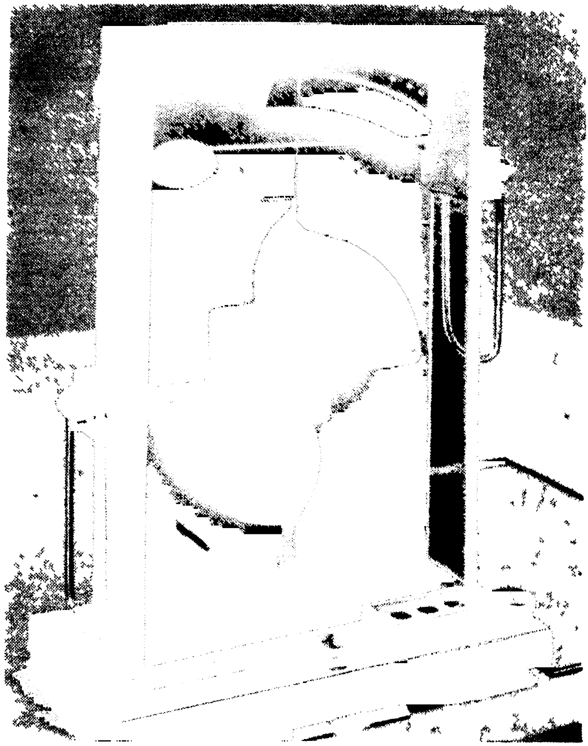


FIG. 10 8.—Kelvin electrostatic voltmeter for 24,000 volts
(Photo by courtesy of the Director, Science Museum)

scale deflection of 300 V. By careful design it has been possible to make the full scale deflection as low as 150 V. in a portable instrument. For higher voltage ranges the single vane construction is used, as shown in Fig. 10.10

Profs. Ayrton and Mather in their designs discarded the flat plate for one of cylindrical form moving between fixed inductors, which were also segments of cylinders concentric with the axis.

The cylindrical form has several advantages, particularly in low-reading instruments, since for a given mass of material it has greater strength and is less liable to deformation ; its moment of inertia is less than the equivalent flat plate, and a smaller clearance is more easily attained, and this is unaffected by vertical displacements, which seriously alter the constant when the flat horizontal plate is employed.

The effect of surface charges has already been mentioned, and in electrostatic instruments this may become particularly troublesome. With all instruments in which the controlling forces are small it is quite easy to produce sufficient electrification on the glass of the observation window by friction with a dry rubber to fully deflect the pointer to its stops, although the terminals may be

FIG. 10 9—Ferranti electrostatic voltmeter for 300 V.

FIG. 10 10—Interior of Ferranti electrostatic voltmeter for higher voltages.

entirely disconnected, and hence comparatively small surface charges may produce serious errors by causing electrostatic deflection of the pointer.

Prof. Ayrton proposed to overcome this difficulty by coating the inside surfaces of the glass with a transparent conducting varnish, made in the following way :

Dissolve ¼ oz. of transparent gelatine in 1 oz. of glacial acetic acid by heating together on a water-bath. To this solution add half the volume of dilute sulphuric acid, prepared by mixing 1 part of strong acid to 8 parts of distilled water by volume. Apply while still warm to the clean warm glass surface, and, when hard, varnish with anti-sulphuric enamel.

Alternatively, thin the gelatine solution, as prepared above, by the addition of acetic acid (say 20 vols. of acid to 1 vol. of solution). Float the thinned solution over the cold clean glass, driving off excess of acid by warming ; allow to cool, and refloat two or three times. Thin the antisulphuric enamel with ether and float this over ; expel the ether by gently warming and repeat.

Such screens are perfectly transparent and durable, when carefully applied, against electrostatic trouble ; their successful application, however, requires some skill and experience.

Carpentier also employed a cylindrical moving element and inductor, as shown in Fig. 10.11. The fixed inductors consist of two concentric cylinders which are split longitudinally across two diameters at right angles so as to form four pairs of segmental plates. The element is a rectangle of thin aluminium sheet, whose long sides are also curved to cylindrical form, and this is mounted with axis horizontal, so as to swing symmetrically in the annular space between the fixed inductors, the movement being gravity controlled. The whole of the working parts are assembled on a base plate which slides in guides between the poles of a large horseshoe permanent magnet, which provides the damping torque by setting up eddy currents in the element, and renders the movement aperiodic without adding anything to the weight or inertia of the moving system.

The Stanley Company of America have also constructed an instrument with cylindrical element and fixed inductors, these being embedded in suitable insulating material.

With very high pressures the difficulties of constructing a deflectional instrument are very great, for, owing to the brush discharge

from the inductors which ionizes the air and thus reduces the effective insulation, it is impossible to construct an instrument of reasonable dimensions with a gaseous dielectric at ordinary atmospheric pressure. It is usual, therefore, to immerse the whole movement in an insulating oil which serves to improve the insulation, reduce the discharge, and, by reason of its higher specific inductive

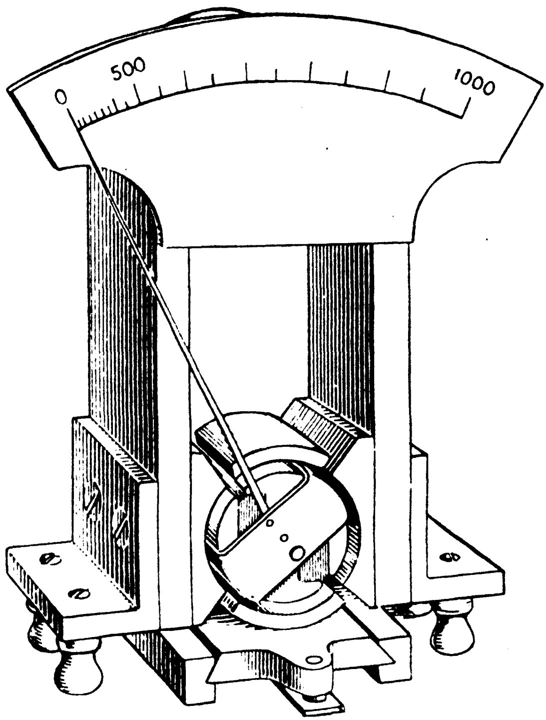


FIG. 10.11.—Principle of Carpentier electrostatic voltmeter.

capacity, reduce the dimensions of the inductors for a given torque.

There are, however, several disadvantages in the use of oil, among which may be mentioned the sluggish movement which often results with complete immersion, the necessity of recalibration with any change of oil, and the greatly increased weight of the instrument and its unportability.

A. E. Watson has surmounted these difficulties by taking advantage of the increased electric strength of air under considerable pressure. In this instrument the moving member is hammered

from copper sheet to a form which presents no sharp edges, and is mounted on a vertical pivot spindle bearing between jewels which are inserted in the ends of a steel tube projecting downward from the ebonite bushing of the needle terminal, the control being provided by a coaxial helical spring.

The fixed inductor presents a continuous internal contour, and both elements are shaped so as to, as far as possible, conform with the condition that the angular rate of change of capacity shall be inversely proportional to the deflection.

The outer casing is of metal of sufficient thickness to withstand the high internal air pressure to be used (100 to 200 lb. per square inch), and is flanged and faced to give good pressure-tight joints, which are made with rings of composition sheeting placed in grooves so that they cannot squeeze out.

The connection to the moving element is brought out at the top of the instrument by a long ebonite bush, supported by a large porcelain insulator. The connection to the fixed inductor is similarly brought out below the instrument.

The clearance between the two members when fully deflected is about 2 cm., and since at 200 lb. per square inch the electric strength of air is 400,000 volts per centimetre, there is an ample factor of safety in the instrument, which is intended to indicate 200,000 volts. The deflections are observed by means of a long horizontal pointer moving over a vertical bent scale.

Another form of deflectional voltmeter for very high pressures has been constructed by the Westinghouse Company.

In this instrument the element is hung from a rectangular stirrup by means of a corrugated insulating rod which is attached to the lower horizontal side of the stirrup, whilst the pivot spindle just passes through the corresponding upper side. The element (Fig. 10.13) consists of two hollow cylindrical metal conductors whose ends are closed by hemispherical metal caps; the cylinders are attached to a thin metal cross-piece which is mounted on the lower end of the insulating rod, so that the axis of each cylinder is parallel to and symmetrical about the axis of rotation. The whole system is supported from a vertical jewel bracket, and swings between two curved fixed inductors which form the terminals of the instrument.

The element thus becomes charged by induction, and the form of the fixed inductor is so chosen that as the moving system turns the distance between the terminal plate and the element diminishes,

or, in other words, the system tends to increase its electrostatic capacity. The movement is controlled by means of a spiral spring, and its deflection is indicated by means of a pointer moving over an edgewise scale. The whole of the working parts of the system are immersed in an oil tank, above which the pointer and scale-box are mounted, the terminals being also brought out at the top by means of two corrugated insulating pillars which are curved outwards.

The pointer and element are carefully counterbalanced in order that the instrument shall be independent of levelling, and the

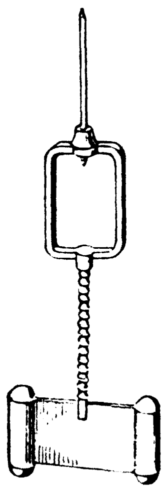


FIG. 10.12.—Vane of Westinghouse oil-immersed electrostatic voltmeter.

buoyancy of the hollow cylinders is adjusted to, as far as possible, remove the load from the jewels. Owing to the form of the moving member employed it acts as a very effective damper.

In calibration the neutral position of the movement is first determined by applying a potential to the instrument, and noting the direction in which the element tends to turn when slightly displaced, and when this is found a slight advance in the required direction is given to the element. Electrostatic forces between the pointer, scale and cover are avoided by connecting them metallically together, while the instrument is effectively screened by the metal tank for containing the oil. Fig. 10.13 shows diagrammatically the arrangement of the instrument, but as actually constructed the edges

of the fixed inductors are rolled over so as to reduce, as far as possible, edge effect. Fig. 10.14 is a calibration curve and the resultant scale. As usually constructed the scale carries two sets

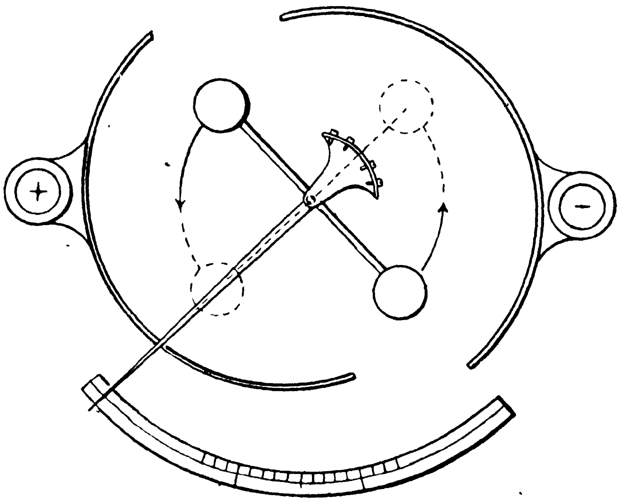


FIG. 10.13.—Diagrammatic arrangement of Westinghouse high voltage electrostatic voltmeter.

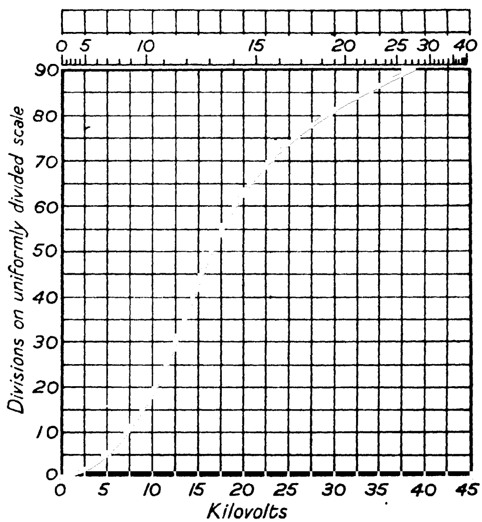


FIG. 10.14.—Calibration curve for Westinghouse high voltage voltmeter.

of divisions, one for direct reading, and the other a scale of even divisions which is useful in calibration and allows of the instrument being used with other elements to extend its range.

The scale form given in the figure is for 40,000 volts, and corresponds to 81° of arc, the movement being advanced 10° from its neutral position.

The overall dimensions of the instrument are 12 × 12 × 10 in., and when filled with oil it weighs about 45 to 50 lb.

The range of these instruments is sometimes extended by employing condensers in a condenser terminal which can be thrown into circuit or short-circuited as desired, the operation being effected by means of a silk cord.

This method, originally suggested by Prof. Ayrton, of extending the range of an electrostatic instrument by condenser multipliers is applicable to all cases, and it has the advantage of giving a certain amount of protection to the instrument, since a well-designed condenser will not break down under at least three times its working voltage. The theory of this method of increasing the range of electrostatic instruments is fully dealt with in the chapter on increasing the range of A.C. instruments in Part II.

With potentials exceeding 8,000 to 10,000 volts it is more usual to employ instruments based on the principle of the Snow Harris Electrometer, in which one conductor in the form of a small plane disc was suspended from one arm of a balance over a similar disc fixed below it. If a high difference of potential was applied to these two, a force of attraction was exerted between them which could be counter-balanced by weights at the other end of the beam.

This principle was extended and perfected by Lord Kelvin in his absolute electrometer, to be described later, and in a modified form is the basis of several commercial instruments. If the conductors are in the form of two flat horizontal plates, it has been proved in Chapter IV, page 152, that the force exerted is $F = \frac{\varepsilon A V^2}{8\pi D^2}$, where ε is the dielectric constant of the medium between the plates, A the area of the moving element, and D their separation. The forces available when very high pressures are to be measured are quite considerable. For instance, the dimensions of an instrument intended for a 200,000-volt circuit designed by E. Jona are as follows: Diameter of suspended disc, 50 mm.; separation, 280 mm.; medium paraffin oil, $\varepsilon = 2$.

Then $$V^2 = \left(\frac{2 \times 10^5}{3 \times 10^2}\right)^2 = 0{\cdot}445 \times 10^6$$

and

$$F = \frac{2 \times 19{\cdot}66 \times 0{\cdot}445 \times 10^6}{8\pi 784 \times 981} = 0{\cdot}91 \text{ grammes weight}$$

It should, however, be remembered that the above formula only holds strictly when the diameter of the moving electrode is not less than half that of the guard-ring, and the separation is not greater than four times the diameter of the moving plate.

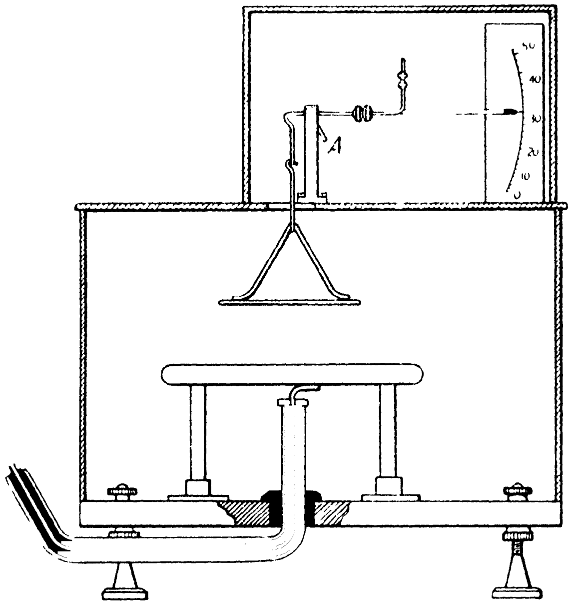


FIG. 10.15.—Early form of Kelvin volt balance.

An early form of Kelvin volt balance is shown in Fig. 10.15. In this instrument the fixed electrode is a horizontal metal plate with curved edges, which is supported on the hole, slot and plane principle on three glass pillars attached to the slate base plate. Contact to this plate is made by a spring piece which is connected to a heavily insulated lead, which passes out through a central hole in the plate in a suitable bent glass tube, which also supports, at its outer end, the terminal connection.

The moving electrode is a smaller disc of aluminium, suspended by metallic links which pass out through a slit in the top of the outer metal cover to one arm of a balance beam, the knife edges of which are supported by two brass pillars erected on the top of the cover. The other arm of the beam is provided with counterweights, and is prolonged into the pointer of the instrument, which moves over a vertical scale arc. The range is varied by hanging weights on the short stirrup arm A, which projects at an angle from the axis of rotation between the supporting pillars, and the balance arrangement and scale are covered by a metal case, provided with an observation window for observing the deflections.

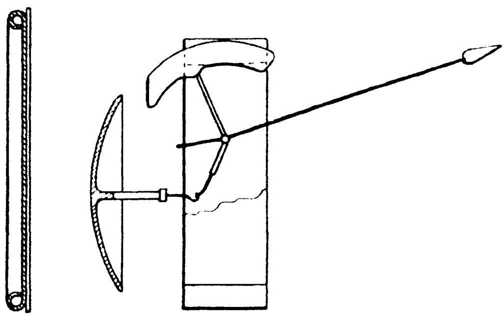


FIG. 10.16.—Later form of Kelvin volt balance.

A later form of instrument is shown diagrammatically in Fig. 10.16, which embodies exactly the same principles, but is modified in mechanical details. The attracted plate is worked to a spherical curvature, and is hung from a short arm on the pointer axis, so that it practically counterbalances the pointer and a thin aluminium damping sector which moves between the poles of a permanent magnet. The scale is arranged in the same manner as in most switchboard instruments, over the movement, and to minimize the brush discharge the fixed plate is covered by a sheet of mica.

A further modification is shown in Fig. 10.17, where, in order to obtain a more proportional scale, a flat endless spring is attached to the centre of the moving electrode in such a way as to give stability against the increase of the force of attraction as the distance between the electrodes diminishes. This spring is then used as the electrical connection to the moving plate through its

fixed connection, and the supports to the pointer shaft are of insulating material, so that all but the two electrodes may be independently earthed.

Fig. 10.18 is another form of instrument on the same principle, designed by E. Jona for direct reading up to 200,000 volts. The moving electrode is suspended by means of a light metallic chain

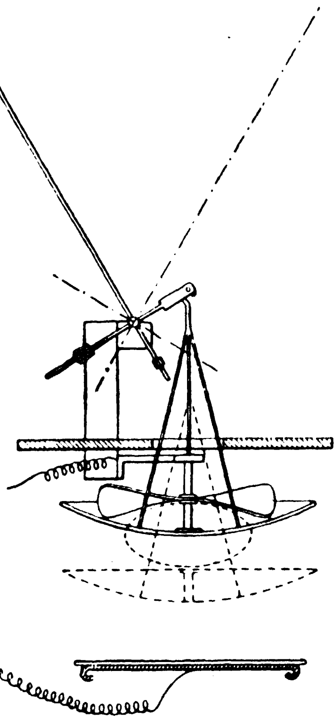


FIG. 10.17.—Further modification of the volt balance.

which passes through the centre of a long insulating column, which supports at its upper end the pointer axis. This suspension is fastened to a short arm at right angles to the axis, and three other arms, all mutually at right angles, carry the sensibility and counter-weights. The former are adjustable, and the range can be altered by using the different weights provided. The lower end of the support pillar projects well down into the glass oil container and terminates in a cylindrical metal petticoat which completely sur-

rounds the moving electrode, and is in electrical contact with it and the pointer gear by means of a metallic connection in the central hole of the insulating column.

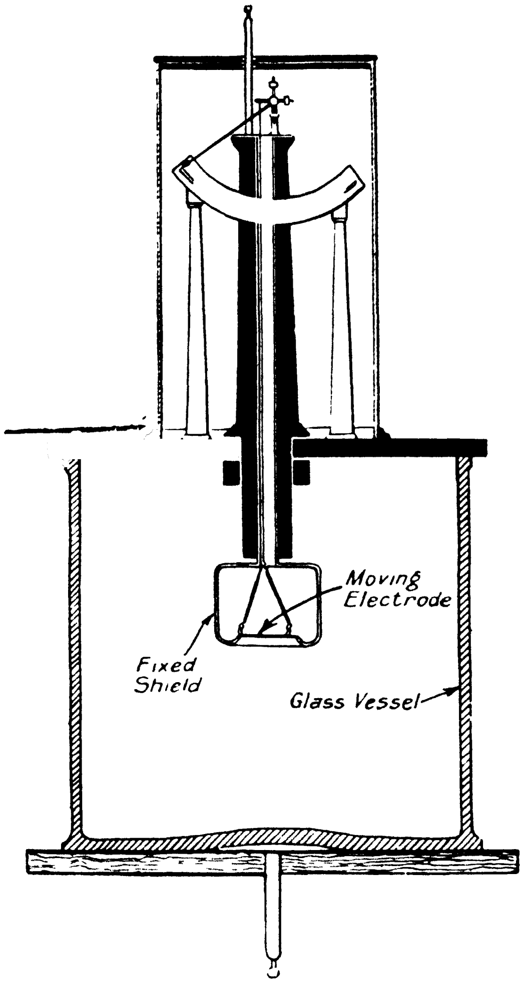


FIG. 10.18.—Jona's attracted disc voltmeter.

The fixed electrode is a sheet of tin-foil lining the bottom of the oil vessel, and this receives a charge by induction from another tin-foil sheet on the outside of the glass bottom, which is in metallic connection with the terminal of the instrument. The whole

electrode system is immersed in paraffin oil, which serves both to increase the strength and capacity of the system, and at the same time damps the motion by fluid friction. The scale arc is carried on pillars from the insulating cover, and a circular glass cover encloses the scale and pointer movement, the terminal for connecting the moving electrode being brought out through the top.

Messrs. Siemens & Halske have designed a similar instrument, the construction of which is shown in Fig. 10.19. The moving

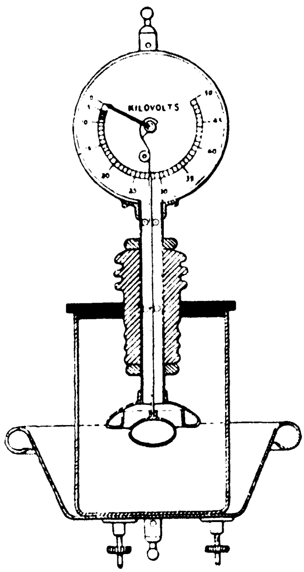


FIG. 10.19.—Siemens and Halske high voltage electrostatic voltmeter.

electrode in this instrument is a closed hollow spheroid attached to the lower end of an aluminium rod which moves vertically between guide pulleys carried inside an outer metal tube, which is held in a long corrugated porcelain insulator passing through the insulating cover of the oil vessel. This tube forms the support of the metal dial-box at its upper extremity, while its lower end is expanded into a hemispherical petticoat whose lower edges are turned over in the manner indicated in the diagram, and this surrounds the moving electrode and forms a guard shield. At the upper end of the aluminium rod attached to the moving conductor a metallic filament is attached, which passes over a guide pulley to a specially

shaped cam on the pointer shaft, which is controlled by a spiral spring in the usual manner, connection to the terminal being made, through the spring.

The fixed electrode is in the form of a deep metal pan whose upper edge is turned over in a broad roll; the sides are carried up so that the top edge is practically level with the lower edge of the guard shield when the oil vessel is in position in it. The pan is provided with levelling screws, and when intended to work on an entirely insulated system it is supported on porcelain insulators, or when employed with one terminal grounded an iron stand is provided, and the dial-box is covered by a glass bell jar through which the upper electrode terminal passes. It will be seen from the construction that very efficient screening is obtained by using the deep pan electrode and the petticoat, and the forms of the electrodes are such that any sharp edges or points are eliminated. The whole of the metal parts supported by the porcelain insulator, which passes through the cover of the oil chamber, are in metallic contact with the other, and are therefore all at one potential, whilst the employment of the cam device permits of a long circular scale (about 270°) which is proportional over 70% of its length.

It should, however, be noted that the placing of the electrode outside the glass vessel in this and similar forms of instrument is open to some objections, as there is a liability to disturbing charges remaining on the glass, which may lead to errors in the indications of the instrument despite the screening.

In the instrument designed by Prof. A. Grau a rather different arrangement of conductors is adopted, as shown diagrammatically in Fig. 10.20. A hollow metallic cylinder is supported on a glass platform near the bottom of a glass oil-containing vessel, and forms the electrode connection, which is effected by means of an insulated wire which passes down through the insulating cover of the vessel.

The moving system consists of a hollow metal cylinder with hemispherical ends, which is mounted above a smaller cylindrical leaden weight; below the weight the axis is continued by a glass rod which passes between three guide pulleys set at 120° to one another on the glass supporting platform over a hole bored through it, which allows the rod to pass freely through. Above the hollow cylinder the axis is continued by a glass tube which, at its upper end, terminates in a metallic fibre, passing over a pulley on the pointer shaft to a counter-weight which is suspended by it on the

other side. The pulley is also counter-weighted to give the necessary control, and carries a pointer, as shown in the figure, which moves over a vertical scale. Damping is increased by a circular horizontal mica disc which is mounted on the central spindle between the cylinder electrode and the leaden sinker.

An instrument of somewhat different form, but involving the same principle, has been devised by Abraham. In this instrument

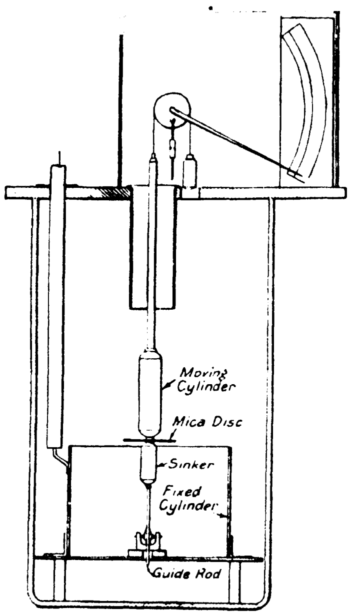


FIG. 10.20.—Grau's high voltage electrostatic voltmeter.

the moving disc and attracting plate are separately supported from an iron base on insulating pillars with their planes vertical. The moving electrode is a comparatively small disc carried on the end of a horizontal rod, which is slung by two vertical fibres within a hollow metal sphere. This sphere has a circular aperture in it which is surrounded by an annular guard-ring, and the moving disc in its zero position is flush with the surface of the guard-ring and practically fills the aperture.

The attracting plate on its separate support moves in guides on the base in front of the sphere, so that the distance between the parallel planes of the moving disc and attracting plate can be altered

at will, and hence the range of the instrument may be altered. A scale on the base gives the range for definite positions of the attracting plate. It will thus be seen that the force of attraction between the conductors tends to draw the moving disc forward through the aperture in the sphere, and when this force is removed the disc falls back under the action of gravity to its normal zero position. To indicate the movement a rigid arm is carried at right angles to the horizontal rod to which the moving disc is attached, and to the end of this arm a light cord is attached and passes round a pulley on a pointer shaft which magnifies the small movement in the usual way. To damp the motion the inner end of the horizontal rod carries a second disc with upturned edges, which fits with small clearance a circular chamber, and thus acts as a short-range piston air-damper. The edges of the disc, plate and guard-ring are worked back to a fair curvature to minimize the effect of brush discharge, and the sphere forms a most efficient electrostatic screen.

There are several excellent designs of instrument intended for the measurement of high potential which, however, cannot be regarded as strictly indicating instruments, inasmuch as they require a certain amount of manipulation before a reading is obtained.

E. R. Wolcott has described an attracted disc voltmeter intended for accurate reading up to 75,000 volts, which is constructed on the Kelvin guard-ring electrometer principle, and is shown in Fig. 10.21. The moving plate is suspended by means of a long helical spring which passes up the centre of the long metal tube which, at its lower end, terminates in the guard-ring, and is supported from the levelling screws by means of three insulating columns. Near the lower end of the suspension tube are two opposite windows, and a beam of light from an auxiliary lamp casts a magnified shadow of the tip of one of the suspending loops on to a translucent screen mounted in one of the apertures, and which is ruled across with a horizontal line, and the position of the shadow with reference to the line is observed through an insulated tube carried on the guard-ring supports. A fine adjustment for the position of the attracted disc is provided at the upper spring attachment on the top of the suspension tube, and the calibration is effected by means of a little platinum spiral of known mass which can be added to or removed from the moving plate. This spiral is fixed below a

little light disc which has a central hole through which the supporting rod of the moving electrode can pass, until a coned shoulder on the rod engages with the disc and supports the mass. The weight is removed by raising a concentric tube through a rack and pinion

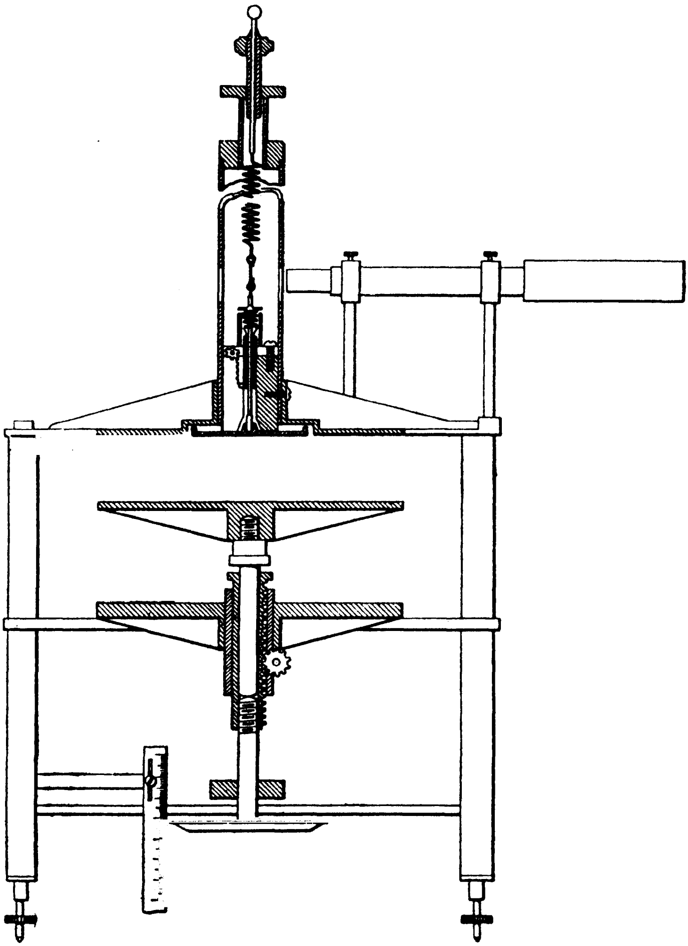


FIG. 10.21.—High voltage electrostatic instrument designed by E. R. Wolcott.

until it engages with the disc and lifts it off the shoulder, leaving the moving element quite free.

The attracting plate is carried by a central pillar on a platform below the guard-ring, and is capable of quick vertical movement

through a rack and pinion arrangement ; the final adjustments are, however, made by means of a slow-pitched screw provided with a divided head which reads against a vertical scale, and both may be calibrated directly in volts.

The method of operation is then quite simple. The instrument is first levelled until the moving plate swings freely in its guard-ring. The attracting plate is then moved up into contact with the underside of the guard-ring, and the scale and divided head are adjusted to read zero. The index shadow is then arranged to coincide with the line on the translucent screen. The attracting plate is now lowered, and the calibrating spiral let down on to its shoulder, and the tension on the supporting spring adjusted until the shadow of the index is again on the line. The weight is then removed and the moving plate ascends by a definite amount, and the instrument is then ready for measurement. When the P.D. to be measured is applied, the moving plate is attracted downwards and the attracting plate is adjusted until the index is on the line—that is, the force exerted is the same as the mechanical force of the calibrating weight, and the position of the lower plate can now be read off on the vertical scale and divided head. The instrument is therefore of a constant force zero reading type, and it is claimed that an accuracy of one-tenth of 1% is attainable.

A very excellent form of instrument was first described by A. Tschernyscheff, which is really a combination of an attracted disc electrometer with an electro-dynamometer, the arrangement of the instrument being shown in Fig. 10.22. The moving electrode is a flat disc closely fitting a surrounding guard plate. This disc is hung from the arm of a beam which rocks on pivots bearing in jewels. The other arm of the balance beam carries a suspended coil which is arranged centrally over a fixed coil on the insulating plate which supports the whole of the movement. These coils may be connected in series and to a source of current supply through an accurate current-measuring device. A fixed plate is carried by an ebonite insulator parallel to and below the moving electrode.

If a difference of potential exists between these two the moving plate will be attracted downwards, and this force can be counterbalanced by sending a measured current round the coils of the dynamometer in such a way that the moving coil is drawn down into the fixed one. The electrostatic attraction between the plates is thus counterbalanced by the electromagnetic attraction between

the coils, or, if k is the constant of the electrometer and k_1 the constant of the dynamometer,

$$k_1 I^2 = kV^2$$

and V is therefore obtained in terms of I—in fact the ammeter could

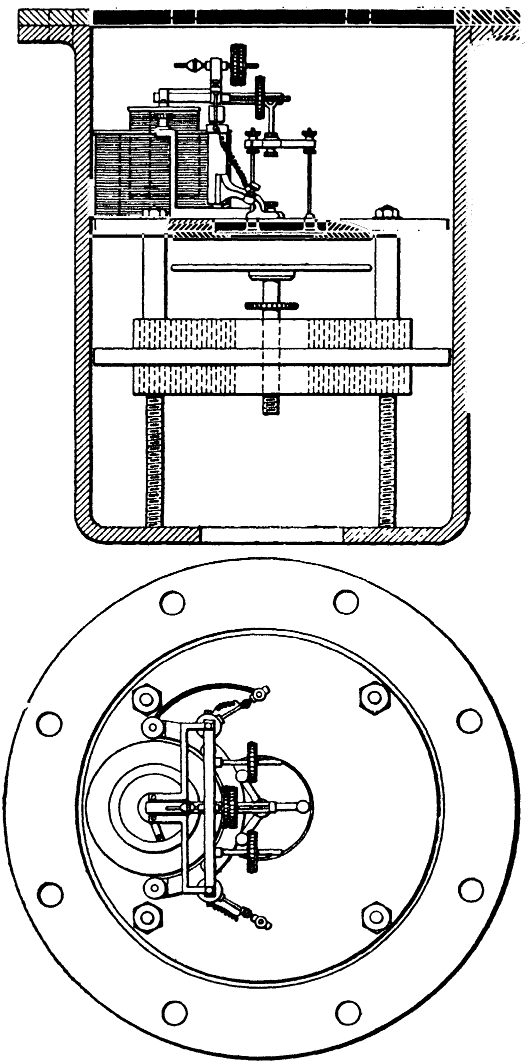


FIG. 10.22.—Tschernyscheff's attracted disc and dynamometer instrument.

be graduated to read volts directly. The whole instrument is enclosed in a metal casing, and can be supplied with compressed air when employed for very high potentials, and it is, of course, obvious that the fixed plate is the only highly insulated electrode, the rest of the apparatus being earth connected.

A. Palm* has described a similar arrangement, made by Messrs. Hartmann & Braun, and intended for calibrating high-pressure voltmeters. In this case the instrument is made in double form, the moving elements of the two electrostatic systems being disposed symmetrically about the axis of rotation, one above and the other below, as shown diagrammatically in Fig. 10.23, while the moving

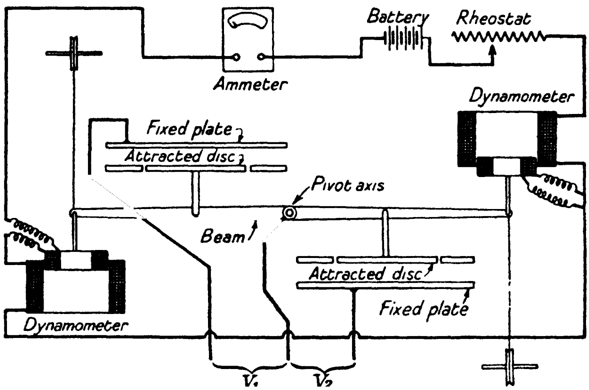


FIG. 10.23.—Double arrangement of attracted disc dynamometer voltmeter.

coils of the dynamometers are at the ends of the balance arms and are arranged to counteract the attraction between the elements of each electrostatic system. All four dynamometer coils are joined in series and connected to a source of direct current through an ammeter and adjustable resistance, so that the current may be regulated until the balance is in equilibrium, when the potential difference to be measured is applied to the electrostatic systems, this position being read by means of a mirror and telescope.

The whole arrangement is enclosed in a bronze chamber with stout walls having large cylindrical extensions on opposite sides, and, under working conditions, the chamber is filled with nitrogen under a pressure of 12 atmospheres, which, according to Peterson, has an electric strength of 240 kilovolts per centimetre, the ad-

* A. Palm, *Zeits. techn. Physik*, i, 7, pp. 137–146, 1920.

vantages claimed for this gas, as an insulation, being that the insulation is not deteriorated by flash-over, the dielectric constant is nearly unity, that ozone is not produced, and that there is no burning at the point of sparking.

The high voltage connections are introduced through the cylindrical extensions of the case by means of hollow corrugated insulators, so perforated that they are exposed, both internally and externally, to the gas pressure.

After a careful and complete study of the possible sources of error, Palm concludes that the accuracy of indication is within 1% at 50 and 100 kilovolts and 0·35% at 150 and 300 kilovolts, this latter being the highest pressure to which the instrument has been submitted. Moreover, since the electrostatic capacity with the series arrangement of the electrometer elements is only 10 cm., it is possible to employ the instrument for measurements on high-frequency circuits.

MOVING DIELECTRIC INSTRUMENTS

Electrostatic voltmeters, based upon the principle of a fixed system of conductors and a moving dielectric, have been suggested from time to time. The first of these is probably one due to Arno, exhibited at the Paris Exhibition in 1900. In this instrument three insulated segments of a metallic cylinder are connected to a three-phase supply, and are surrounded by a paraffin paper cylinder which is suitably pivoted and rotates with the field by reason of the lag in its polarization; the movements are controlled by a spiral spring, and the resultant scale had exactly even divisions.

R. Burrows revived the system in 1910, and proposed to construct a voltmeter in which the conductors were two coaxial metal cylinders with evenly spaced rectangular openings in the sides. In the space between these a cylinder of the dielectric substance is free to rotate about the common axis, and is provided with similar rectangular openings in its sides, and the variation of capacity is effected by the movement of this cylinder. In the zero position the openings in the moving cylinder are not coincident with those of the fixed ones, but on deflection they tend to move into a position of coincidence and thus increase the electrostatic capacity of the system.

Unfortunately, however, Dr. Thornton's experiments have shown that the dielectric constant of all the insulating substances which he examined, when under the influence of steady electrostatic fields,

is by no means constant. The curves in Fig. 10.24 are taken from his experiments and show how the constant varies with time. In general, it will be seen that during a short interval after the first application of the P.D. there is a rapid rise in the value of ε in most cases, followed by a slower increase until a saturation point is reached, which may not occur until the lapse of some hours.

Taking the intercept of these curves with the vertical, and calling this value ε_0, he compared it with the corresponding values obtained

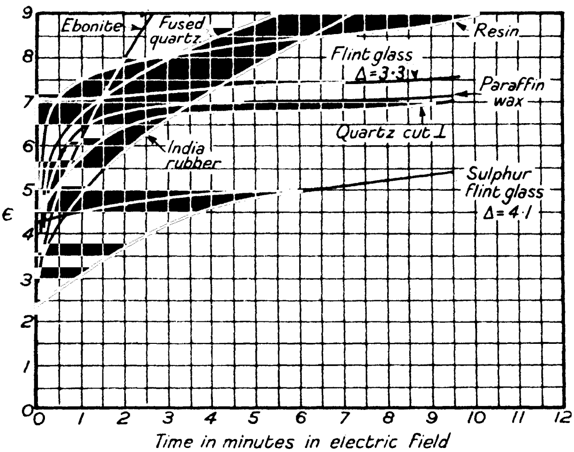


FIG. 10.24.—Dr. Thornton's curves, showing dielectric polarization.

in an alternating field of 80 cycles per second (ε_{80}), with the result shown in the table on p. 574.

From this it will be seen that the ratio $\varepsilon_0/\varepsilon_{80}$ is, in several cases, greater than unity, and hence these substances will have an appreciable frequency effect.

The research, therefore, indicates that an instrument constructed on the lines indicated above would have a torque which would gradually increase with time, and the pointer of the movement would therefore creep up since the capacity is dependent on the value of ε; and, moreover, that where the frequency effect was present an instrument for alternating current circuits would have to be calibrated at the special frequency.

From a study of Dr. Thornton's results, Burrows selected light flint glass of density 3·3 as the most likely substance for the moving

TABLE XXXIX

Material.	ϵ_{80}.	ϵ_0.	ϵ_0/ϵ_{80}.
Quartz cut parallel to optic axis	4·6	13	2·8
Flint Glass, Density 4·1	8·5	25	2·9
Resin	3·09	7	2·3
Canada balsam	2·72	19·5	6·9
Paraffin wax	2·32	5–6	2·4
Gutta-percha	4·43	7	1·58
Sulphur	4·03	4·38	1·07
India-rubber	3·08	3–4	1·1
Ebonite	2·79	4·2	1·5
Quartz cut at right angles to optic axis	4·54	4·55	1
Light flint glass, density 3·3	6·98	7	1
Amber	2·8	2·8	1
Sealing wax	4·56	4·6	1

cylinder, owing to the absence of the initial rapid rise, the high value of ϵ, the comparatively slow increase with time, and the absence of frequency effect. For a 300-volt instrument he worked out the dimensions as follows: outside diameter of glass cylinder 5 cm.; gap between metal cylinders 4 mm., each with three evenly spaced openings; working length 10 cm., clearance 0·25 mm. This gives a deflecting torque of 20 dyne-cm., and he proposed to suspend with a quartz fibre 10 cm. long and 0·1 mm. diameter, which would give a restoring couple of 30 dyne cm. for a 60° twist. He further proposed to overcome the square law of the scale by shaping the openings so that the working length varies as the ordinates of a rectangular hyperbola.

The proposition is in many ways tempting and full of possibilities, were it not for the uncertainty of the polarization effects in the dielectric, and in this respect the properties of light flint glass, examined by Thornton, are unique, no other substance as yet examined being anything like as suitable. But to work to the clearance given above in this material would be very difficult without very costly workmanship, and although the torque is greater than that given by an instrument of the moving-needle type of the same range, the weight of the moving element would be nearly

100 gm., and its moment of inertia about 500 gm.-cm.^{2}, so that the periodic time would be about 25 seconds and the problem of damping would be serious. Further investigations on the behaviour of other dielectric substances may, however, solve many of these difficulties.

The principle of the so-called " electric wind," which is produced by the mutual repulsion of the electrified particles of air and the charge on a pointed conductor, when it is raised to a high potential, has been developed by Siemens & Halske as a voltmeter. The pointed conductor is placed within the end of a tube, and the discharge produces an effect somewhat like that of an injector and causes air to be drawn through the tube. The air current so produced acts on a light piston in the other end of the tube, which is curved like the cylinder of an air damper to a circular arc. The motion is thus rendered rotational and communicated to a pointer shaft at the centre of curvature, which is controlled by a spiral spring the usual way.

DESIGN OF ELECTROSTATIC INSTRUMENTS

As shown in Chapter IV, formula 14, the fundamental relation underlying the action of electrostatic instruments is

$$T = \frac{1}{2} V^2 \frac{dC}{d\theta}$$

where T is the torque in dyne-cm., V the P.D., and C the capacity both in electrostatic units.

Since one electrostatic unit of P.D. is 300 volts, and the force of one gramme weight is 981 dynes,

$$T \text{ (gm.-cm.)} = \frac{1}{2 \times 981 \times 90{,}000} V^2 \frac{dC}{d\theta} = 5 \cdot 67 \times 10^{-9} \frac{dC}{d\theta} V^2$$

where V is in volts and C in electrostatic units. Or, since one microfarad is 9×10^5 electrostatic units,

$$T \text{ (gm.-cm.)} = 0 \cdot 0051 V^2 \frac{dC}{d\theta}, \text{ when } C \text{ is in microfarads}$$

To verify this relation the following tests were made on two electrostatic voltmeters of very different type and range, and although the agreement between the observed and calculated torques is not perfect, it is satisfactory when we take into account the small magnitude of the quantities involved.

TABLE XL.—*Tests on Electrostatic Voltmeters*

240-volt suspended multicellular.

Volts.	Angle (degrees).	Capacity ($\mu\mu$ farads).	Torque. Observed.	Torque. Calculated.
240	65·5	51·55	0·00235	0·00229
200	62·5	51	0·00224	0·0022
170	59	50·1	0·002016	0·00203

3,000-volt 12-inch Ayrton-Mather, pivoted.

Volts.	Angle (degrees).	Capacity ($\mu\mu$ farads).	Torque. Observed.	Torque. Calculated.
2,400	80	15·9	0·3435	0·318
2,200	60	12	0·286	0·2665
2,000	39	8·5	0·229	0·22

These experiments may, therefore, be regarded as justifying the fundamental formula, but at the same time they show clearly how small the forces are in actual electrostatic instruments. The problem of obtaining a satisfactory instrument resolves itself, therefore, into getting as high as possible a value of $\frac{dC}{d\theta}$ compatible with an adequate clearance to avoid sparking, and with sufficiently low values for the weight and inertia of the moving system.

As an example, we will take the design of a 3,000-volt electrostatic instrument suitable for a 2,200-volt supply, and giving a torque of 0·25 gm.-cm. at its maximum voltage, then we have

$$\frac{dC}{d\theta} = \frac{10^9\ T}{5{\cdot}67\ V^2} = \frac{10^9}{5{\cdot}67} \times \frac{0{\cdot}25}{9 \times 10^6} = 4{\cdot}9 \text{ cm. per radian}$$

and since for a parallel plate condenser,

$$C = \frac{\varepsilon A}{4\pi D} \text{ and } \frac{dA}{d\theta} = \frac{4\pi D}{\varepsilon}\frac{dC}{d\theta}$$

for a maximum P.D. of 3,009 volts, a distance D of 0·5 cm. is found to be suitable if an ordinary air dielectric is used, so that

$$\frac{dA}{d\theta} = 4\pi \times 0{\cdot}5\frac{dC}{d\theta} = 2\pi \times 4{\cdot}9 = 30{\cdot}8 \text{ sq. cm. per radian.}$$

Taking firstly a design of element and inductors of the ordinary quadrant electrometer form, we have

$$\frac{dA}{d\theta} = 4 \times \frac{1}{2}(a^2 - a_o^2) = 2(a^2 - a_o^2) \text{ sq. cm. per radian;}$$

taking the two surfaces of the two vanes of the element, where a is the outer radius of the needle, and a_o the inner radius of the quadrants (which we may take as 1 cm. to avoid sparking to the shaft), then

$$2a^2 - 2 = 30{\cdot}8, \text{ or } a = \sqrt{16{\cdot}4} = 4{\cdot}05 \text{ cm.}$$

If the vanes are made from 0·1 mm. aluminium sheet, with two wings each of angle 75°, its mass will be $\frac{1}{2}a^2\theta\Delta \times = 1{\cdot}31 \times 16{\cdot}4 \times 0{\cdot}01 \times 2{\cdot}57$, or, say, 0·6 gm. and its moment of inertia $I = \frac{1}{2}wa^2 = 0{\cdot}3 \times 16{\cdot}4 = 4{\cdot}92$ gm.-cm.^{2}

If, on the other hand, we make the element in the form of a single segment of a cylinder having an axial length of l cm., and radius a, then $\frac{dA}{d\theta} = la$, if the fixed inductor is on one side only, or $2la$ if both internal and external surfaces are used.

Taking the latter case, and with the same clearance of 0·5 cm., we have $\frac{dA}{d\theta} = 30{\cdot}8$ sq. cm. as before. Therefore,

$$2la = 30{\cdot}8, \text{ or } la = 15{\cdot}4 \text{ sq. cm.}$$

It is clear that in this case the inertia will be lower the less we make the value of a, or the greater we make l. Taking the maximum convenient value for the latter as 6·5 cm., so as to avoid making the instrument case too deep,

$$a = 15{\cdot}4/6{\cdot}5 = 2{\cdot}37, \text{ say } 2{\cdot}4 \text{ cm.}$$

In this case the range can be longer, the element having an angular breadth of about 120°, or approximately 2 radians. The total area of the cylindrical surface will then be 32·7 sq. cm., and its weight, if made of 0·1 mm. aluminium, is

$$32{\cdot}7 \times 0{\cdot}01 \times 2{\cdot}57 = 0{\cdot}84 \text{ gm.}$$

and its moment of inertia,

$$I = wa^2 = 0{\cdot}84 \times (2{\cdot}4)^2 = 4{\cdot}85 \text{ gm.-cm.}^2$$

A comparison of the two forms shows that for this particular range the only advantage of the cylindrical over the flat element is in the longer scale which is obtainable with the former type. In the case of the flat element it will be better to employ a symmetrical air damper, consisting of two paddle vanes moving in sector-shaped chambers, and a damping constant of about 40 is as much as we can expect without making the damper unduly large.

From Chapter 3, page 133, we have, for this type of damper,

$$D = \left(\frac{0\cdot 248}{t} + 0\cdot 169\right) ba^2$$

and if we work to a clearance of 0·4 mm. and make the radius of the vanes equal to that of the element, then $D = 1\cdot 58ba^2$, from which $b = \dfrac{40}{1\cdot 58 \times 16} = 1\cdot 6$ cm. Allowing 5 mm. in the centre of the box for the spindle to pass through, each vane will be $1\cdot 6 \times 3\cdot 75$ cm., with edges turned up to 3 mm., and if constructed of 0·1 mm. aluminium sheet, their weight will be 0·48 gm., and their moment of inertia 2·8 gm.-cm.^{2}

The pointer can be of aluminium tube weighing 0·006 gm. per cm., and if we allow a length of 10 cm., the mass of the pointer is 0·06 gm., and its moment of inertia $\times \cdot 06 \times 10^2 =$ 2·0 gm.-cm.^{2}

The moment of the pointer is $0\cdot 06 \times 5 = 0\cdot 3$ gm.-cm., and if we arrange a counter-weight at 2 cm. radius, its mass must be 0·3/2 = 0·15 gm., and its moment of inertia $0\cdot 15 \times 2^2 = 0\cdot 6$ gm.-cm.^{2}

Hence the total mass and inertia of the movement will be:

Flat Element.

	Gramme.	Inertia (gm.-cm.^{2}).
Weight	0·6	4·92
Vanes	0·48	2·8
Pointer	0·06	2
Counter-weight	0·15	0·6
Shaft	0·312	—
Total	1·602	10·32
Say	2 gm.	11 gm.-cm.^{2}

If the total angle is $65° = 1{\cdot}13$ radians, the torque per radian will be $245/1{\cdot}13 = 213$ dyne-cm. per radian, and therefore the undamped periodic time is $2\pi\sqrt{\frac{12}{213}} = 1{\cdot}5$ seconds, and the ratio of damping 0·255.

If, however, we adopt the cylindrical element construction, since the element is asymmetrical about the axis, we can, with advantage, employ an eddy-current damper as a counter-balance. Suppose, therefore, we fit to the shaft a 90° sector of aluminium 1 mm. thick and 2·4 cm. radius, and from the centre of its arc provide a pointer of aluminium tube 8 cm. long, making the radius of the scale 10·4 cm., then the mass of the sector is $\frac{\pi(2{\cdot}4)^2}{4} \times 0{\cdot}1 \times 2{\cdot}57 = 1{\cdot}16$ gm., and its moment is $1{\cdot}16 \times 1{\cdot}6 = 1{\cdot}86$ gm.-cm. If the pointer tube weighs 0·006 gm. per cm. as before, its total weight will be $0{\cdot}006 \times 8 = 0{\cdot}048$ gm., and its moment will be $0{\cdot}048 \times 6{\cdot}4 = 0{\cdot}307$ gm.-cm., so that the total moment of pointer and sector is $1{\cdot}86 + 0{\cdot}307 = 2{\cdot}17$ gm.-cm. Now, the moment of the element is $0{\cdot}84 \times 2{\cdot}4 = 2$ gm., so that allowing for the necessary arms, etc., we may consider this as counter-balancing the sector and pointer.

The moment of inertia of the sector is 3·34 gm.-cm.², and of the pointer 2·226 gm.-cm.² Hence the total mass and inertia of the movement is :

Cylindrical Element.

	Weight (gramme).	Inertia (gm.-cm.²).
Needle	0·84	4·85
Damping sector	1·16	3·34
Pointer	0·048	2·226
Shaft	0·312	—
Total	2·36	10·416
Say	2·5 gm.	10·5 gm.-cm.²

Assuming a 90° scale, the torque per radian is $\frac{0{\cdot}25 \times 981 \times 2}{\pi} =$ 156 dyne-cm. per radian, and the undamped periodic time is

$$2\pi\sqrt{\frac{10{\cdot}5}{156}} = 1{\cdot}63 \text{ sec.}$$

For critical damping we have $D^2 = 4sk$, and therefore $N = 2\sqrt{10\cdot5 \times 156} = 81$. It is shown in Chapter 3, page 139, for a disc moving between the poles of a permanent magnet, that $D = \frac{\Phi^2 a^2 x}{1{,}000 A k \rho}$, and, if we make the radius of action of the magnet 80% of the radius of the sector, or 1·92 cm. from the axis of rotation, then $a^2 = 3\cdot72$; ρ for commercial aluminium may be taken as 3, k as 9, and x is 0·1.

Now in order to keep the field well inside the disc we must keep the pole area A small: let us provisionally make it circular, 0·25 sq. cm., or 0·565 cm. diameter. Then:

$$\Phi = \sqrt{\frac{81 \times 1{,}000 \times 0\cdot25 \times 9 \times 3}{3\cdot72 \times 0\cdot1}} = 1{,}210$$

This value is high, but is not higher than that sometimes reached in well-designed damping magnets. It is also evident that eddy-current damping is much better suited to this class of instrument than air damping, since it is obviously difficult to work with the small clearance demanded above, and even when this is done the air-damped instrument has only 40% of critical damping, while there is no great difficulty in getting critical damping in the last case.

Table XLI gives a summary of some tests on various electrostatic instruments.

This chapter cannot be completed without a brief mention of the ellipsoid voltmeter of Dr. Thornton. This was devised as an instrument for the absolute measurement of high voltages. The principle is briefly as follows: An ellipsoid is suspended between two plane parallel plates which are at different potentials. This ellipsoid is polarized by the field, and if it is placed at an angle to the field, a torque will be developed to bring it into line. The accuracy of this instrument is dependent on a knowledge of the physical dimensions.

The torque on the suspension of an insulated conducting ellipsoid, the major axis making an angle θ with the electrostatic field F, can be shown to be $B \sin\theta \,.\, \cos\theta$, where $B = F^2V(L - N)/LN$, where V is the volume of the ellipsoid and L and N are constants which depend only on its shape. For a prolate spheroid of eccentricity e the longitudinal coefficient

$$N = 4\pi\left(\frac{1}{e^2} - 1\right)\left(\frac{1}{2e}\log\frac{1+e}{1-e} - 1\right)$$

and the transverse coefficient

$$L = 2\pi \frac{1}{e^2} - \frac{1 - e^2}{2e^3} \log \frac{1 + e}{1 - e}$$

If the ellipsoid is deflected steadily an angle θ, by a twist φ of the suspension, the torque for small values of θ is

$$\begin{aligned} 2A\varphi &= B \sin 2\theta \\ &= F^2 V \frac{(L - N)}{LN} \sin 2\theta \end{aligned}$$

or

$$F^2 = 2A\varphi \frac{LN}{L - N} \frac{1}{V \sin 2\theta} \quad . \quad . \quad . \quad (1)$$

A is found by experiment, and the field is then determined by observing the angle of torsion φ required to hold the ellipsoid at an angle θ to the field.

If the ellipsoid makes small oscillations about the line of the field, the equation of motion is

$$K\ddot{\theta} + A\theta + B \sin\theta \, . \cos\theta = O \quad . \quad . \quad (2)$$

K being the moment of inertia about the suspension.

A can be determined from the free period of oscillation T_o with no field, i.e. $B = O$, and

$$T_o = 2\pi \sqrt{\frac{K}{A}} \text{ or } A = 4\pi^2 n_o^2 K \quad . \quad . \quad (3)$$

n_o being the number of swings per second $= 1/T_o$.

For most applications the amplitude of swing is limited to a few degrees, and the equation of motion to a close approximation becomes

$$K\ddot{\theta} + (A + B)\theta = O \quad . \quad . \quad . \quad (4)$$

The number of swings per second is

$$n = \frac{1}{2\pi} \sqrt{\frac{A + B}{K}}$$

or

$$\begin{aligned} (A + B) &= 4\pi^2 n^2 K \\ B &= 4\pi^2 (n^2 - n_o^2) K \end{aligned}$$

$$F^2 = \frac{B}{V} \frac{LN}{(L - N)} = 4\pi^2 \frac{K}{V} \frac{LN}{(L - N)} (n^2 - n_o^2)$$

or

$$F = k (n^2 - n_o^2)^{\frac{1}{2}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

where k (the constant of the instrument) is

$$2\pi \left[\frac{K}{V}\frac{LN}{L-N}\right]^{\frac{1}{2}}$$

It will be seen that k is dependent only on the physical constants of the ellipsoid, which can be known with great precision.

When the field strength is very high, the difference between n and n_0 may be so great that the latter may be neglected, and to an accuracy of 1% the working formula is $F = kn$.

The torque on the ellipsoid is proportional to the square of the field and the instrument thus gives root mean square values, independent of wave-form. It is independent of temperature within normal ranges.

A full account of this instrument will be found in the *Journal of The Institution of Electrical Engineers*, Vol. 71, 1932, pp. 1–23.

Later Developments.

As a conclusion to this chapter, the following is a brief note of some recent developments in electrostatic instruments. There has been a great demand for small instruments to measure voltages from about 100 volts to many kilovolts. These small instruments, of sizes ranging from 2½ in. diameter to 6 in. diameter, lend themselves readily to the production of instruments with ranges up to a few hundred volts, and the latest developments include the means of extending the range to high potentials.

The construction of a typical instrument by Ernest Turner Electrical Instruments is shown in Figs. 10.25 and 10.26. These illustrations show an instrument with dial plate and one half of damper-box cover plate removed, and it will be seen that the construction consists essentially of three fixed vanes, spaced apart and secured to one terminal. These vanes are shaped to provide the desired scale divisions. The moving element is a pivoted duralumin staff on which are mounted two flat moving vanes which swing midway between the fixed vanes, the hair spring and pointer and a light air paddle to provide damping. This swings in a closed compartment at the bottom of the case. The dial, top bridge, moving system and all metal work are connected to the earth terminal. An adjustable spark gap is provided to afford protection to the moving vanes in the event of an overload when the instrument is used on D.C. This spark gap can be seen at the top of the base.

TABLE XLI.—*Early Electrostatic Instruments*

Maker.	Type.	Maximum volts indicated.	Period of movement (secs.).	Time to come to rest (secs.).	Length of pointer (cm.).	System of damping.	System of control.
Robt. W. Paul	Ayrton-Mather	120	5	18	10·5	None	Spring
,, ,,	,, ,,	130	7	17	10	,,	,,
Kelvin & Jas. White	Multicellular	240	?	40	8·1	Disc in oil	Torsion of wire
Everett, Edgcumbe & Co.	Leakage indicator	2,200	2·3	165	9·5	None	Gravity
Nalder Bros. & Thompson	Ayrton-Mather	2,400	1·5	7	12·7	Magnetic	,,
Robt. W. Paul	,, ,,	2,000	2·2	28	13·7	,,	,,
Kelvin & Jas. White	Vertical	4,000	4·5	900	18·5	None	,,
,, ,,	,,	24,000	2·5	800	19·2	,,	,,

Maker.	Type.	Weight of movement (gm.).	Torque (gm.-cm.).	Torque / Weight.	Capacity ($\mu\mu$ farads). Min.	Capacity ($\mu\mu$ farads). Max.	Minimum clearance (cm.).	Friction error % at ½ scale.
Robt. W. Paul	Ayrton-Mather	2·883	0·012	0·00402	23·5	105·5	0·025	0·1
,, ,,	,, ,,	1·805	0·00625	0·00346	16	53	0·1	0·2
Kelvin & Jas. White	Multicellular	23·24	0·00235	0·000101	34·5	51·5	0·38	..
Everett, Edgcumbe & Co	Leakage indicator	5·16	0·0945	0·0132	15·35	20·95	0·51	1·4
Nalder Bros. & Thompson	Ayrton-Mather	5·98	0·3435	0·0575	5·5	15·9	0·375	0·267
Robt. W. Paul	,, ,,	3·17	0·1535	0·5485	12·55	18·6	0·57	1
Kelvin & Jas. White	Vertical	12·92	0·985	0·0762	17	21	1	0·1
,, ,,	,,	15·17	2·19	0·1442	16	19	1·5	0·15

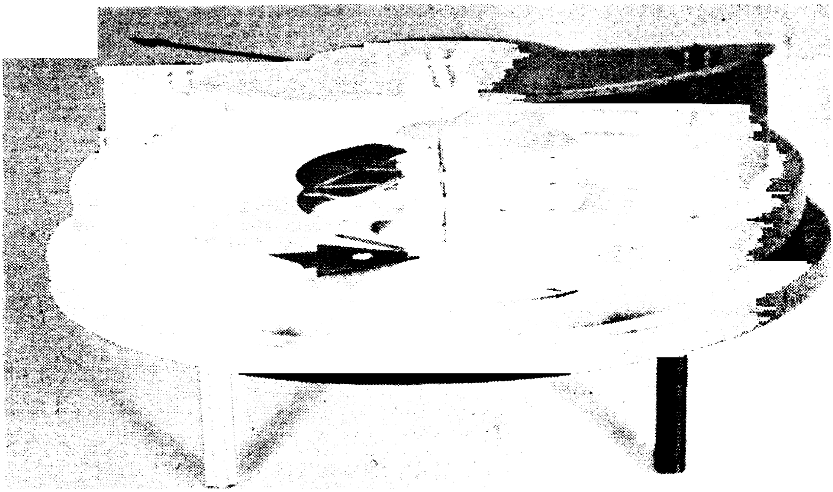


FIG. 10.25.—Modern electrostatic meter (Ernest Turner).

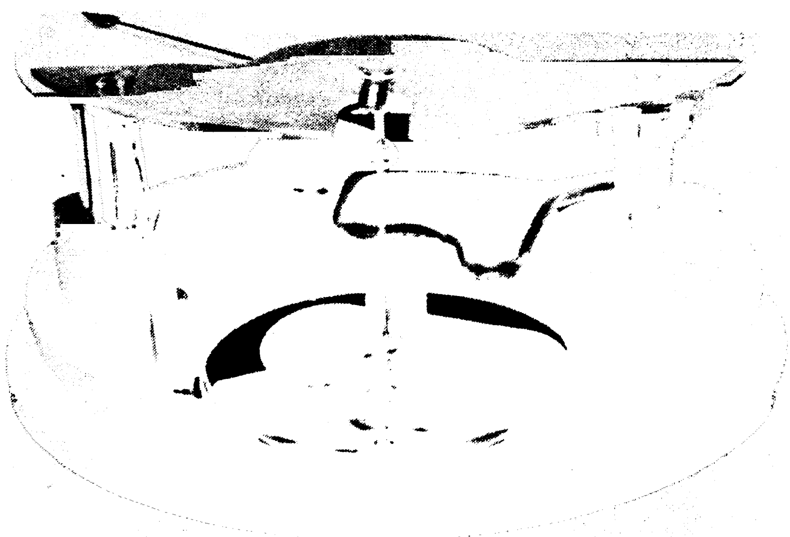


FIG. 10.26.—Meter for higher voltage (Ernest Turner).

The construction of an instrument for a higher range in the same size case is shown in Fig. 10.26. The single fixed vane has been increased in thickness, with a mirror-finish and radiused edges. The two halves of the cover of the damper box have been treated in the same way, the screws holding these being inserted from the back of the case to eliminate all sharp edges which might cause brush or

FIG. 10.27—High voltage electrostatic meter with improved vanes (Ernest Turner).

corona discharge. The moving vanes which replace the thin strips previously used are hollow copper shells.

The illustration in Fig. 10.27 shows this firm's latest assembly for high-voltage instruments. One terminal has been lengthened to prevent surface tracking, the fixed vane is still more generously radiused, and the moving system has a thicker staff with the moving vanes dished, made from aluminium.

APPENDIX

ABSOLUTE UNITS

In this book definitions are given of the International Units of current, potential difference, resistance etc. These are based on material standards. It was decided by the International Committee of Weights and Measures in October, 1946, that the system of units should be changed to the so-called absolute units derived from the centimetre, gramme and second. Since 1st January, 1948, these absolute units are employed at the National Physical Laboratory.

The differences between the units in the two systems are small, a maximum of about 5 parts in 10,000, and in everyday practical measurements can be neglected. With standard apparatus or instruments of high precision the differences must be taken into account. The relationship between the two systems of units is as follows :

1 international ohm	=	1·00049 absolute ohms.
1 international volt	=	1·00034 absolute volts.
1 international ampere	=	0·99985 absolute ampere.
1 international watt	=	1·00019 absolute watts.
1 international henry	=	1·00049 absolute henries.
1 international farad	=	0·99951 absolute farad.

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